

The non-linear PDE is :

$$\left(1 - \varphi + \frac{\varphi}{(1+u)^2}\right) \left(\frac{\partial u}{\partial \tau} + \frac{\varepsilon}{1-\tau} \frac{\partial u}{\partial \varepsilon}\right) - \left(\frac{1}{(1-\tau)^2} \frac{\partial}{\partial \varepsilon} \left((1-\varphi) \frac{\partial u}{\partial \varepsilon}\right)\right) + \frac{(\gamma - u)}{(1-\tau)} \delta(\varepsilon - \varepsilon p) = 0$$

Where,

$$\varphi = k_1 + k_2 * H(\varepsilon - \varepsilon p);$$

" $H(\varepsilon - \varepsilon p)$ " is the Heaviside step function.

$$\varepsilon p = \frac{(1 - k_3 * \tau)}{(1 - \tau)};$$

" ε " is dimensionless length: " τ " is dimensionless time;

$$\gamma = \frac{k_4 * u}{k_5 + u};$$

k_1, k_2, k_3, k_4, k_5 are constants.

$\delta(\varepsilon - \varepsilon p)$ is the Dirac delta function.

" u " is the dimensionless dependent variable.

Boundary Conditions:

$$\varepsilon = 0 \quad \frac{\partial u}{\partial \varepsilon} = 0$$

$$\varepsilon = 1 \quad (1 - \varphi) \frac{\partial u}{\partial \varepsilon} = ((1 - \varphi)u + \gamma \varphi)(1 - \tau)$$

$$\tau = 0 \quad u = 1$$