Decay identification
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Let us define some Constants the decay/rise *k*, the fundamental frequency *f*, the mean value *Fm*, Start and stop times *T0* and *Tmax*, and the time period DT for the plots

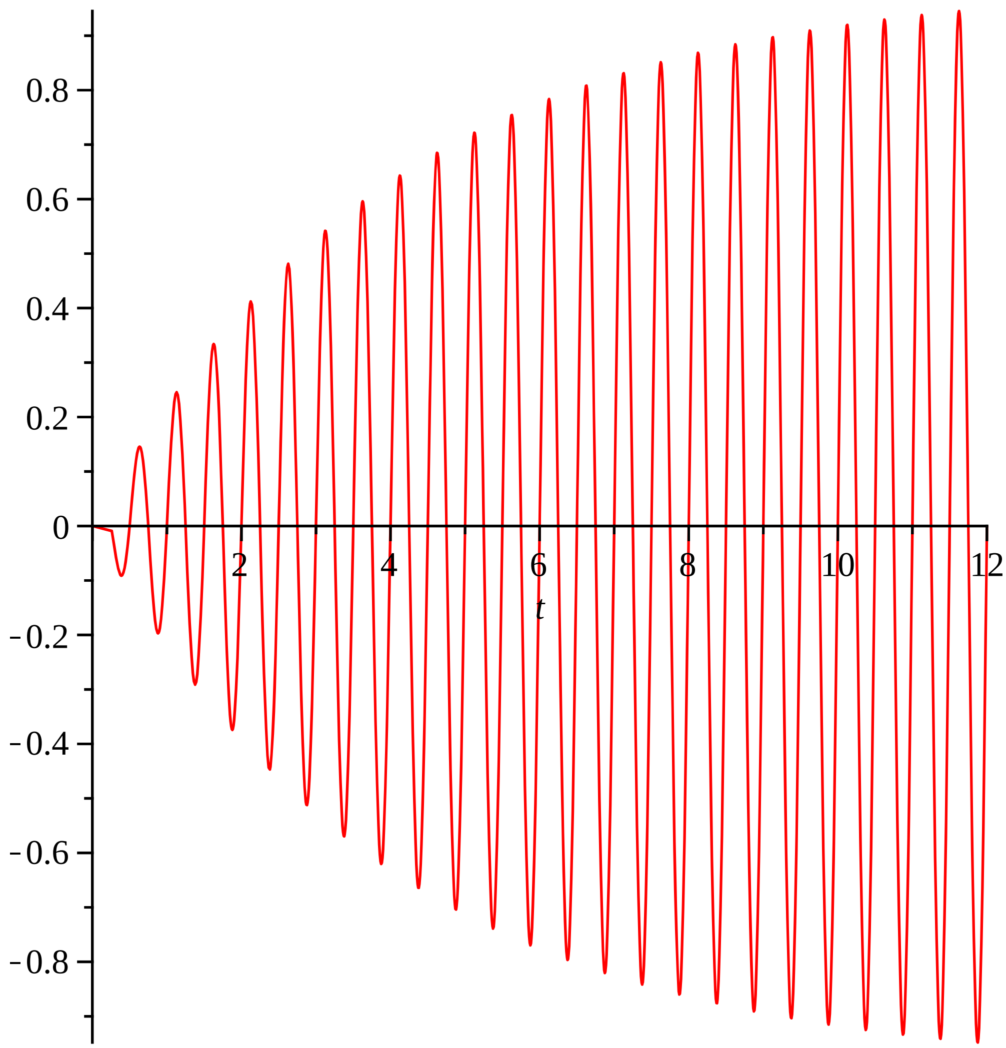
$k := 0.25 : f := 2 : Fm := 0.0 : T0 := 0.0 : Tmax := 12 :$
$\Delta T := \mathrm{T0}..\mathrm{Tmax} :$

Our nice oscillating function, with no noise and zero average/mean is

$F := t \rightarrow (1 - \exp(-k\,t))\cdot\sin(2\cdot\pi\cdot f\cdot t) + Fm$

$$t \rightarrow \left(1 - e^{-k t}\right) \sin\left(2\,\pi\,f\,t\right) + Fm \qquad \textbf{(1)}$$

$plot(F(t), t = \Delta T)$

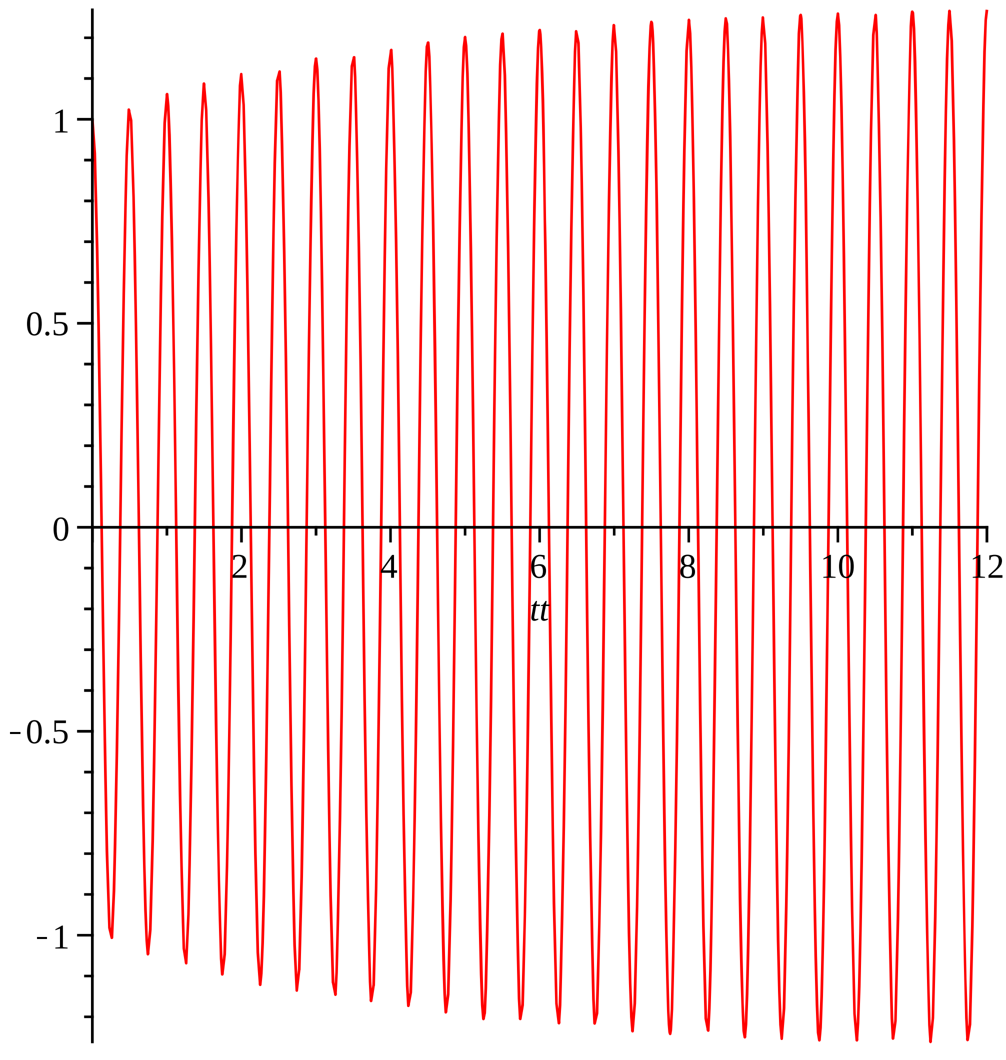


The autocorrelation is then

$$AC := tt \rightarrow \frac{\int_{T0}^{Tmax} F(\mathrm{t}+\mathrm{tt}) \cdot F(t) \, \mathrm{d}t}{\int_{T0}^{Tmax} F(t)^2 \, \mathrm{d}t}$$

$$tt \rightarrow \frac{\int_{T0}^{Tmax} F(t+tt) \, F(t) \, \mathrm{d}t}{\int_{T0}^{Tmax} F(t)^2 \, \mathrm{d}t} \tag{2}$$

$plot(\mathrm{AC}(\mathrm{tt}), \mathrm{tt} = \Delta T)$

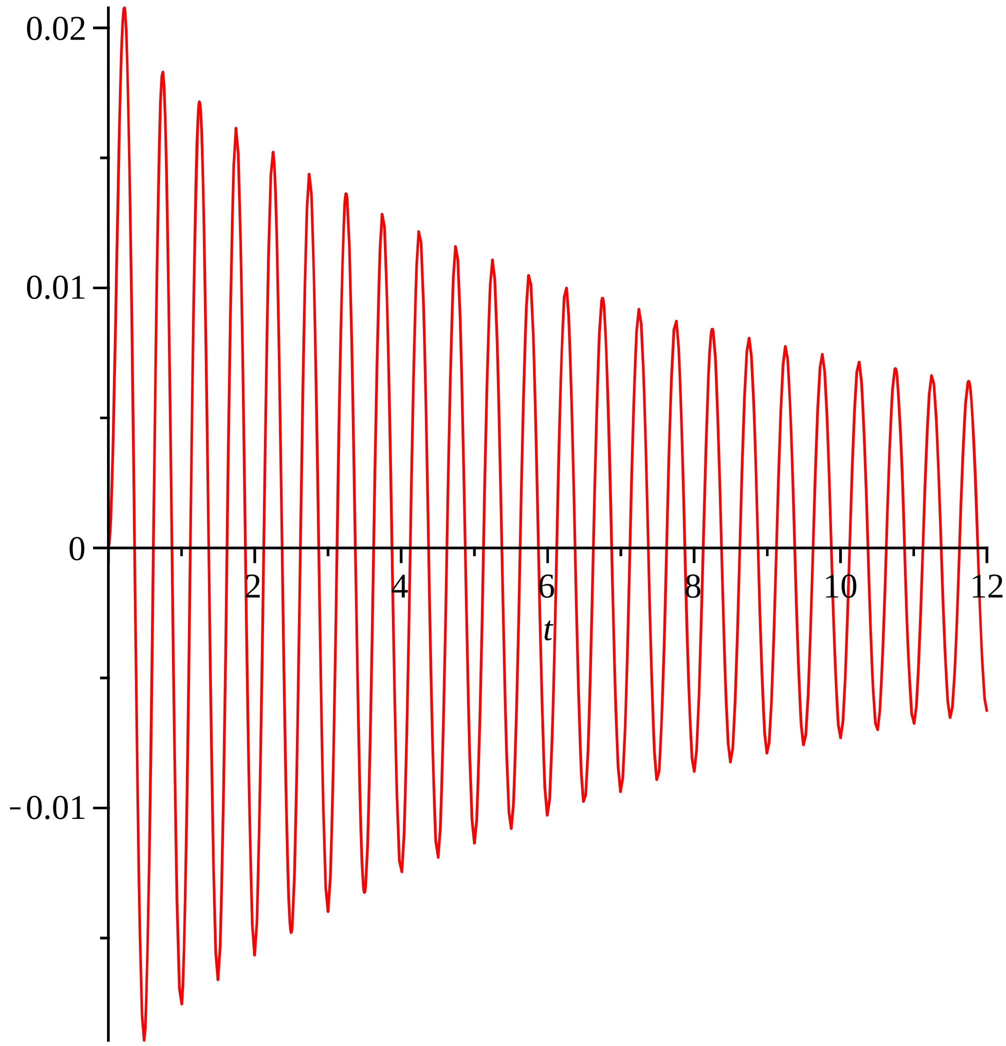


The mean as it would be caluated by COMSOL is then

$$Fmean := t \rightarrow \frac{1}{(t - T0)} \int_{T0}^{t} F(t) \mathrm{d}t$$

$$t \rightarrow \frac{\int_{T0}^{t} F(t)\, dt}{t - T0} \qquad \textbf{(3)}$$

$plot(\text{Fmean}(t), t = \Delta T)$

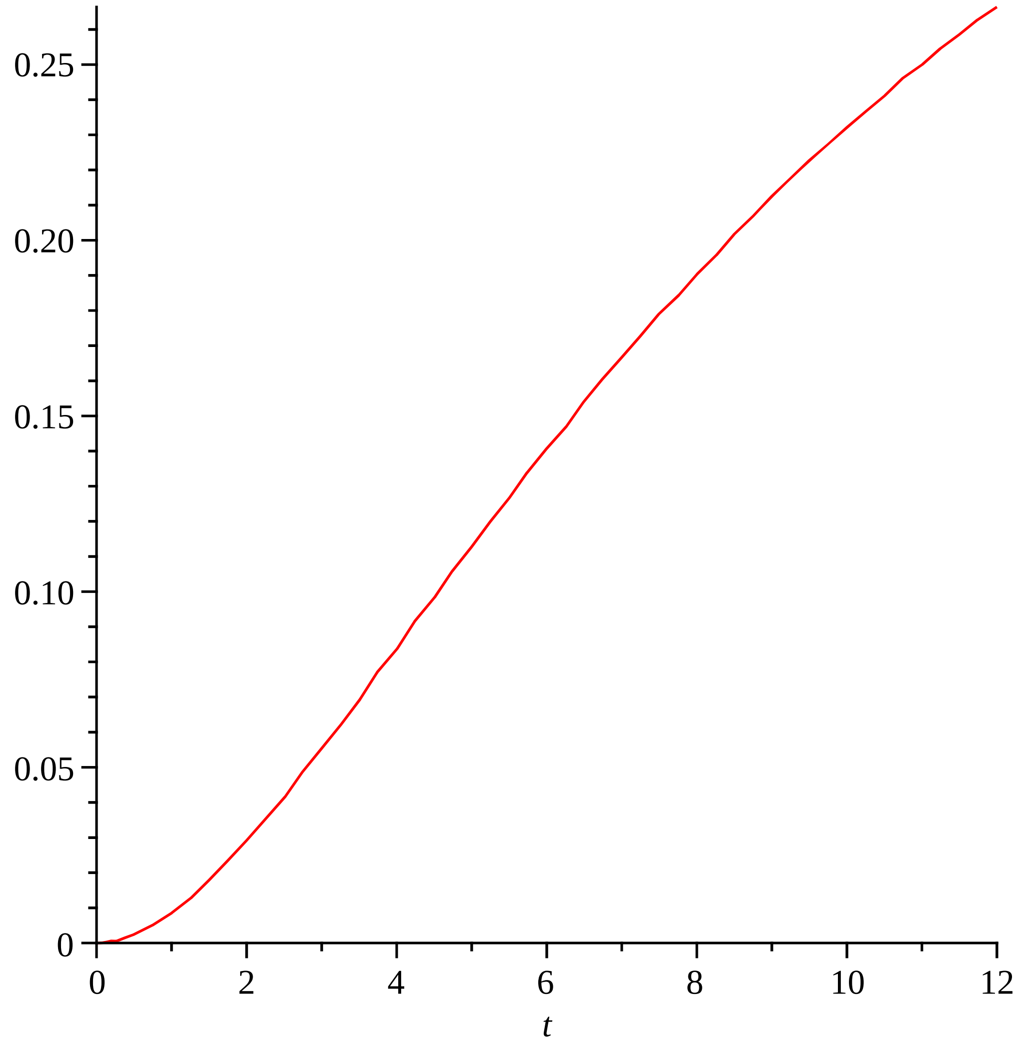


By integrating the square of the function normalised once, we get a nice sloope

$$F12 := t \rightarrow \left(\frac{1}{(t - T0)} \int_{T0}^{t} (F(t))^2\, dt \right)$$

$$t \rightarrow \frac{\int_{T0}^{t} F(t)^2\, dt}{t - T0} \qquad \textbf{(4)}$$

$plot(\text{F12}(t), t = \Delta T)$

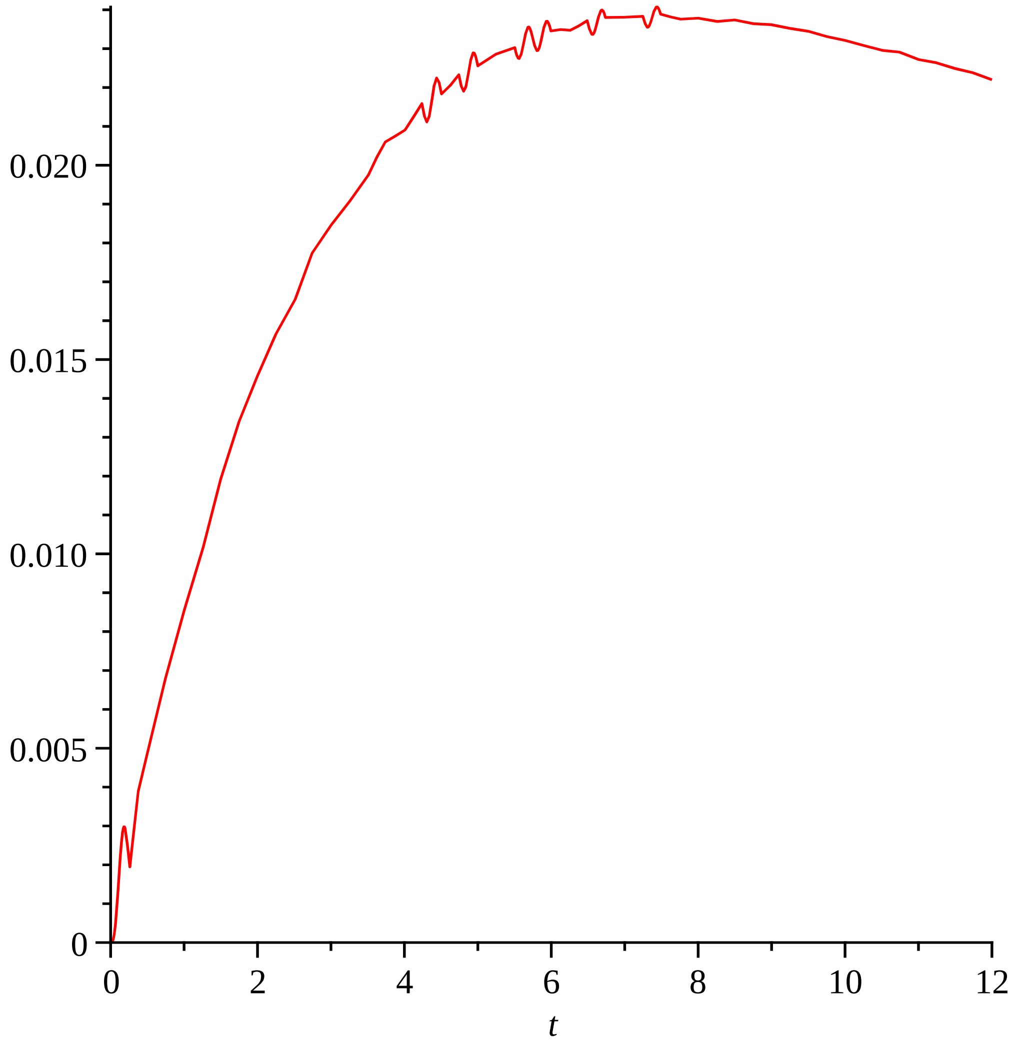


By integrating the square of the function normalised by the square, we can detect a maximum

$$F22 := t \rightarrow \left(\frac{1}{(t - T0)^2} \int_{T0}^{t} (F(t))^2 \, dt \right)$$

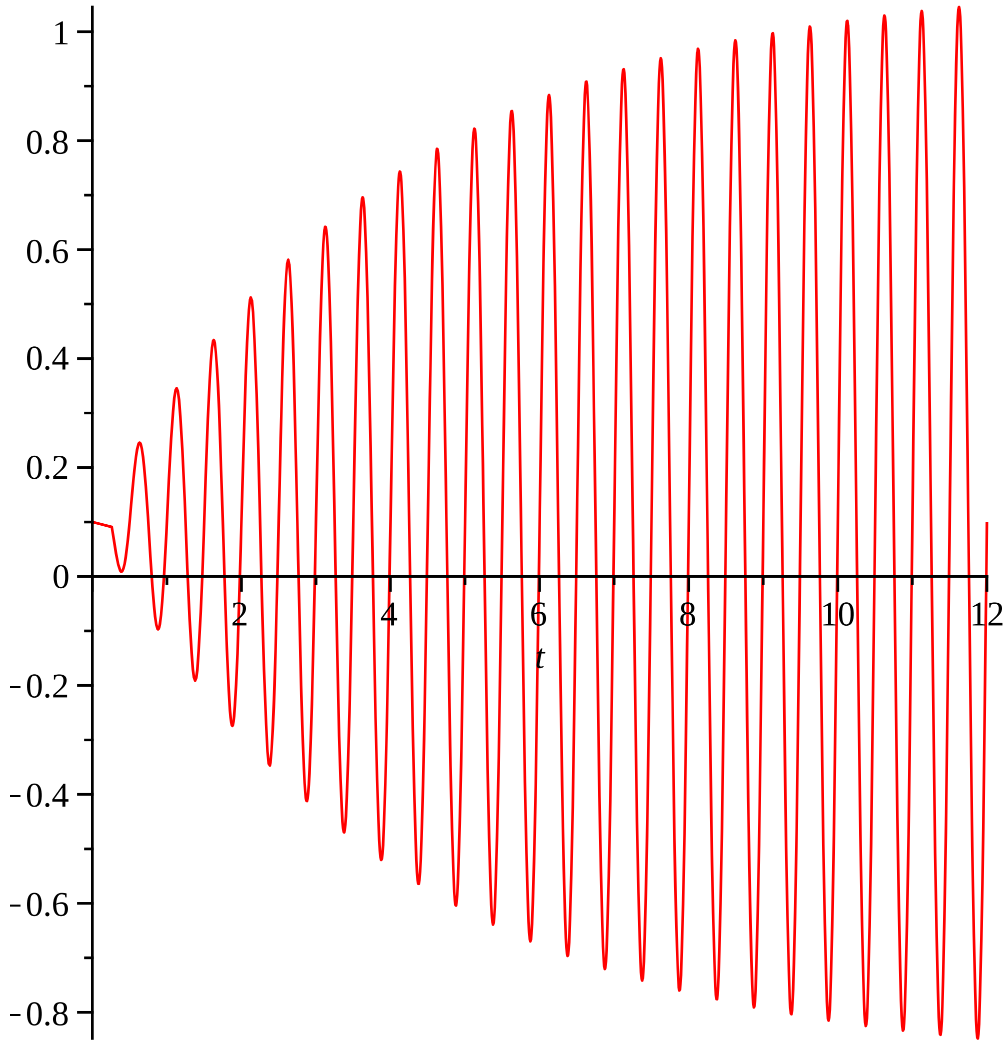
$$t \rightarrow \frac{\int_{T0}^{t} F(t)^2 \, dt}{(t - T0)^2} \qquad \textbf{(5)}$$

$plot(\mathrm{F22}(\mathrm{t}), \mathrm{t} = \Delta T)$

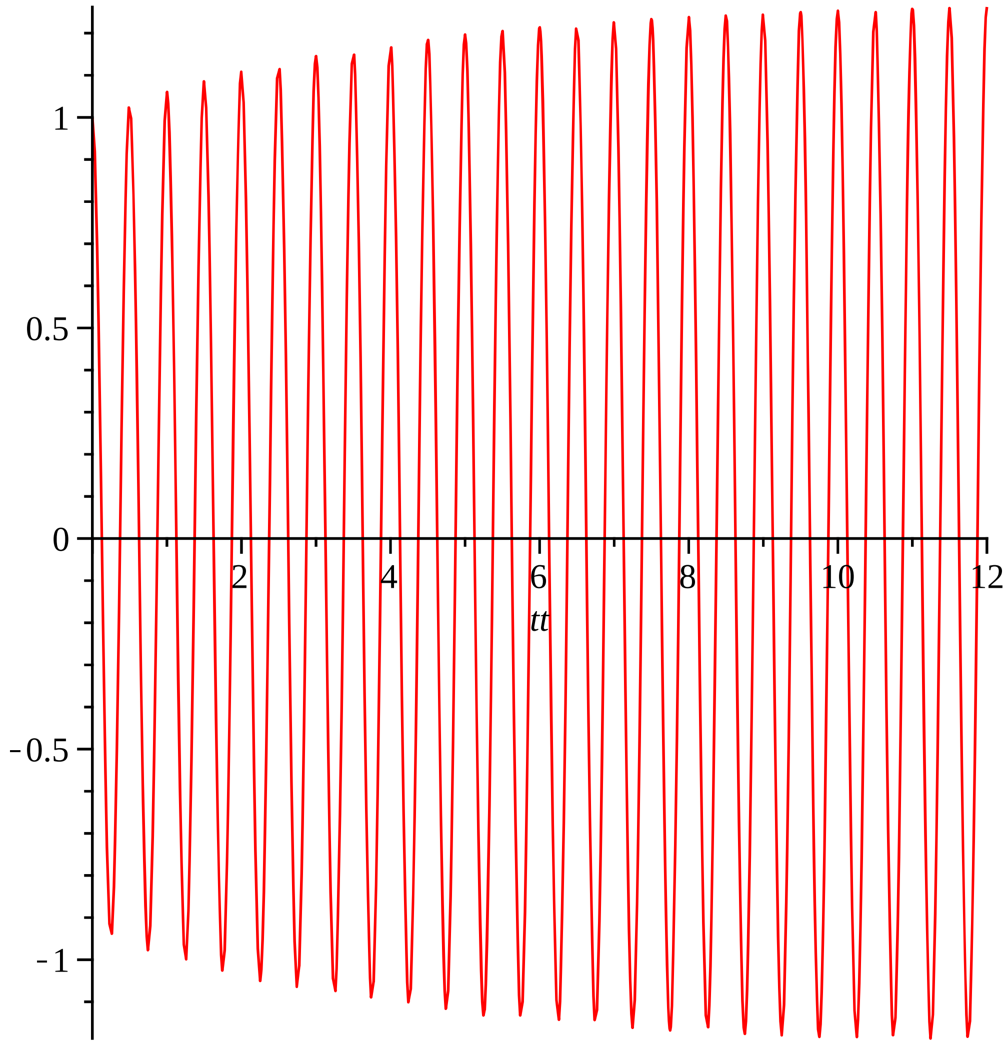


If we add an average value, we have some issues on the F22 with the vertical scale at t=0

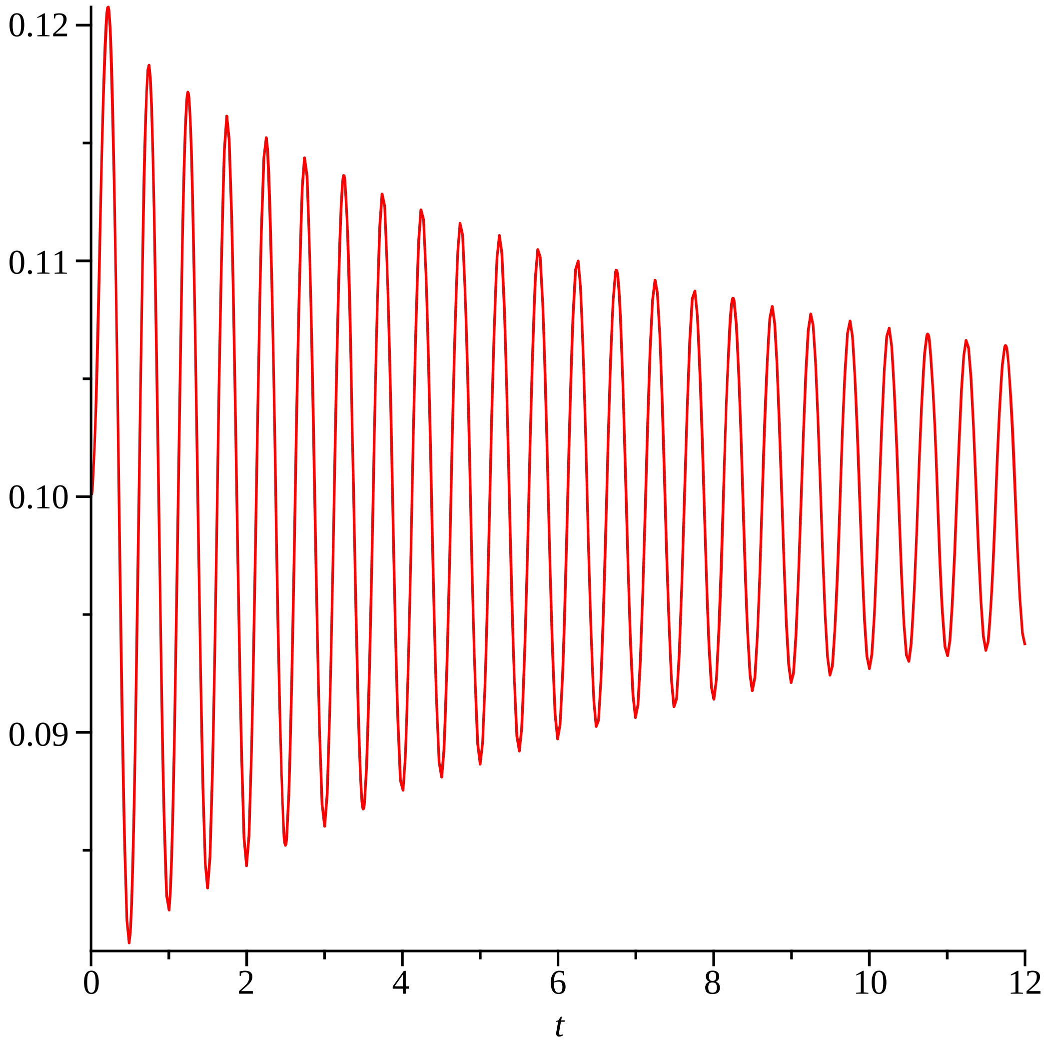
$k := 0.25 : f := 2 : Fm := 0.1 : T0 := 0.0 : Tmax := 12 :$
$\Delta T := \mathrm{T0}..\mathrm{Tmax} :$
$plot(\mathrm{F}(\mathrm{t}), \mathrm{t} = \Delta T)$



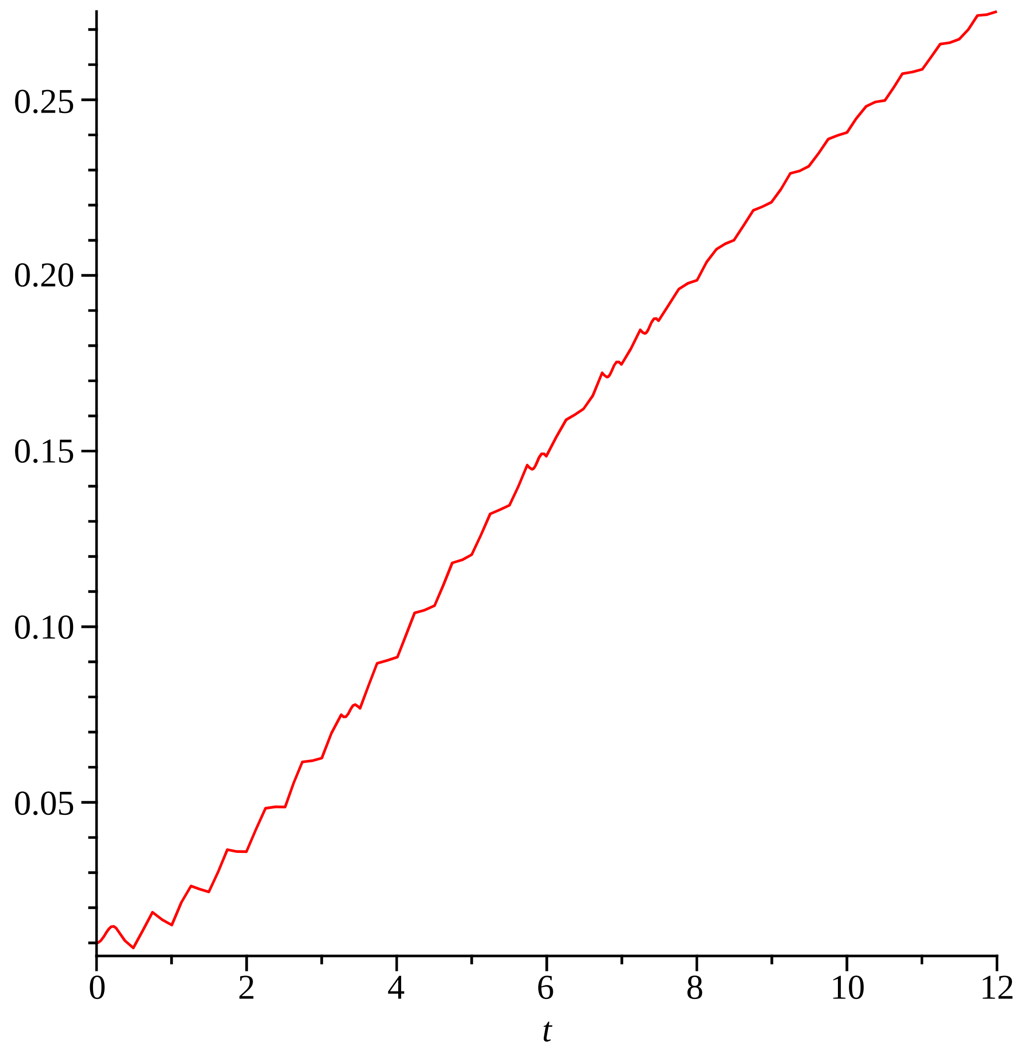
$plot\left(\mathrm{AC}(\mathrm{tt}), \mathrm{tt} = \Delta T\right)$



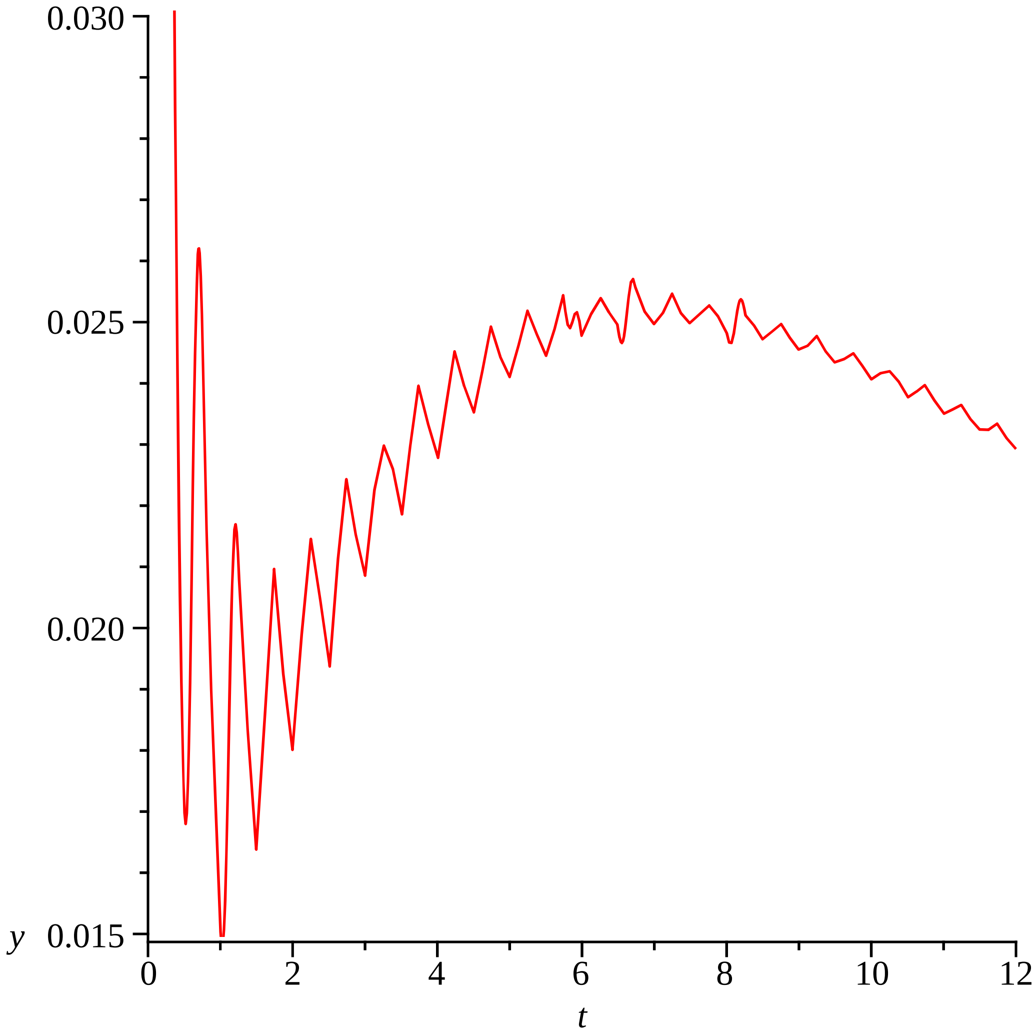
$plot(\mathrm{Fmean}(t), t = \Delta T)$



$plot(\mathrm{F12}(\mathrm{t}), \mathrm{t} = \Delta T)$



plot(F22(t), t = ΔT, y = 0.015 ..0.03)



If we add an average value and some higher frequency oscillations (should use random noise)
We need also to clip the starting values of F22

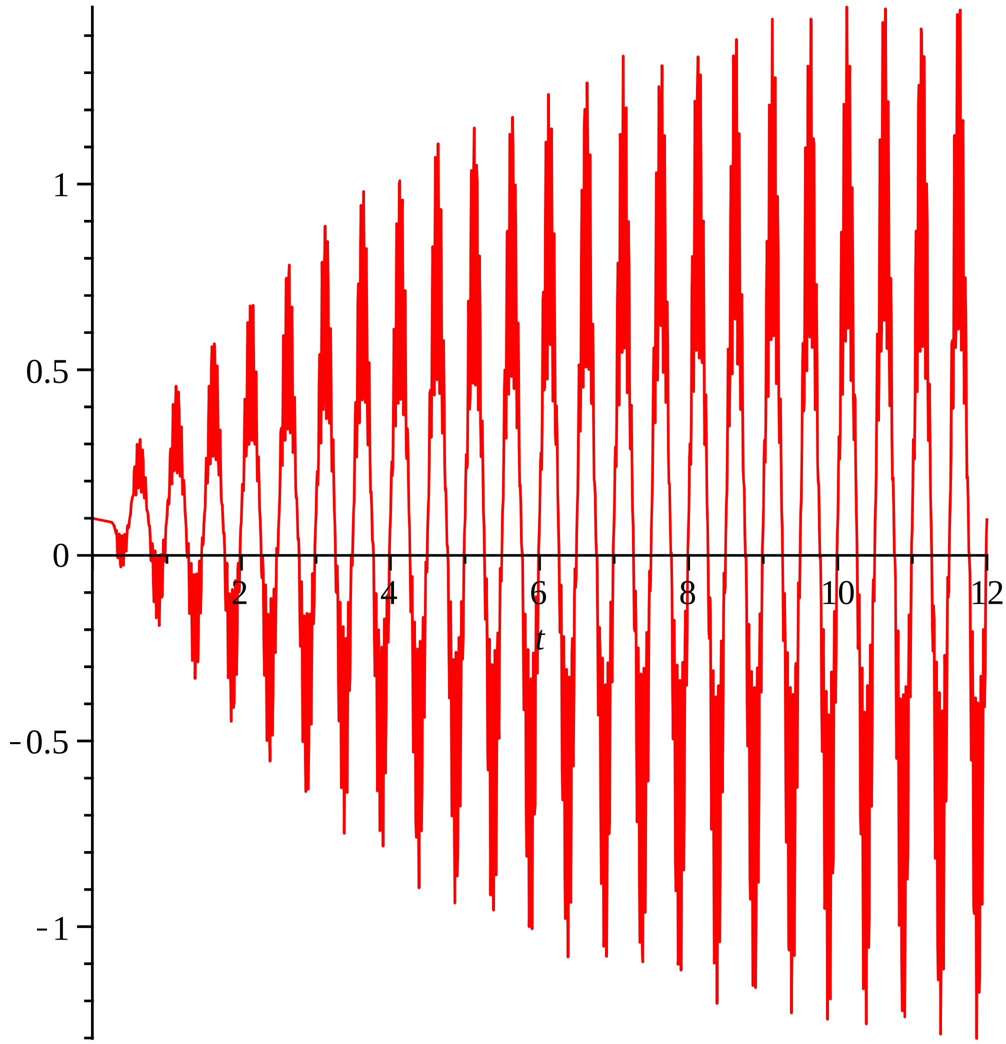
$k := 0.25 : f := 2 : Fm := 0.1 : T0 := 0.0 : Tmax := 12 :$

$\Delta T := \mathrm{T0}..\mathrm{Tmax} :$

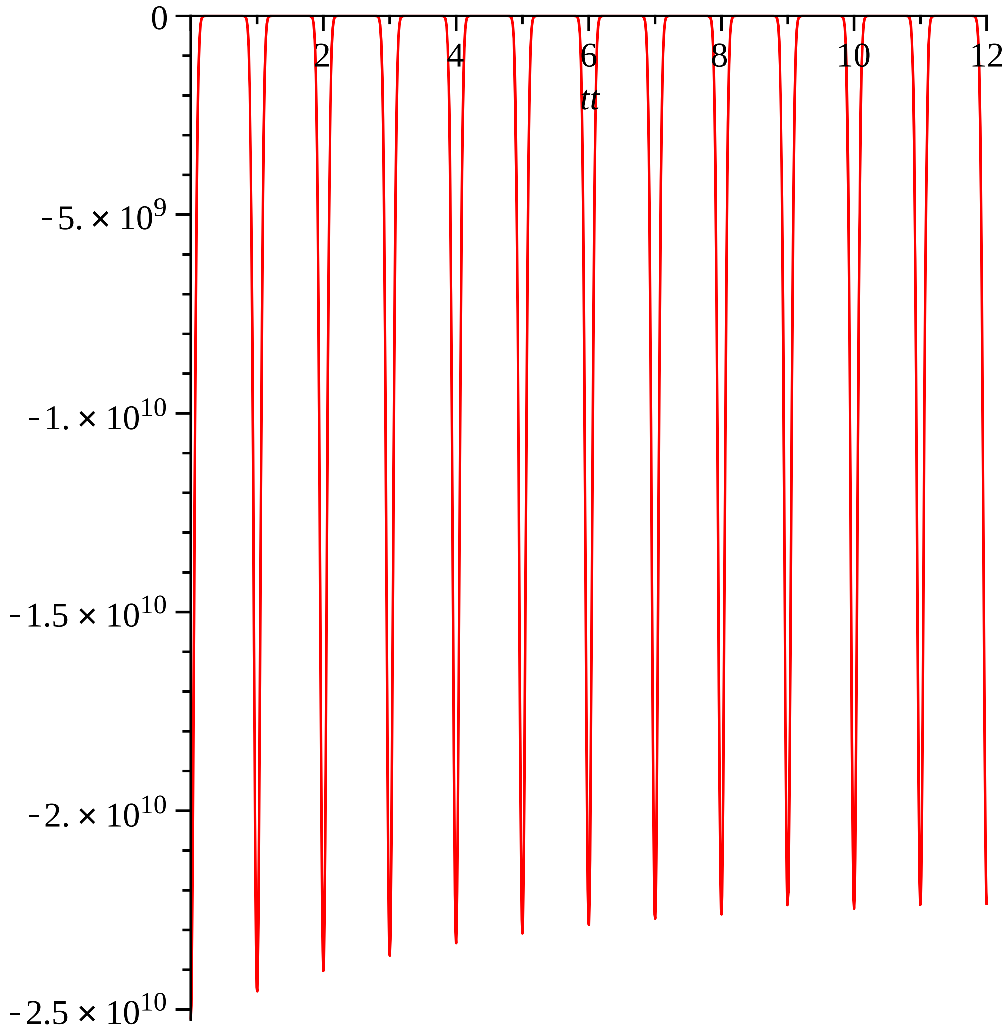
$F := t \rightarrow (1 - \exp(-\mathrm{k} \cdot t)) \cdot \sin(2 \cdot \pi \cdot f \cdot t) \cdot (1 + 0.5 \cdot \sin(2 \cdot \pi \cdot 27 \cdot t)) + \mathrm{Fm}$

$$t \rightarrow (1 - \mathrm{e}^{-k t}) \sin(2 \pi f t) (1 + 0.5 \sin(54 \pi t)) + Fm \tag{6}$$

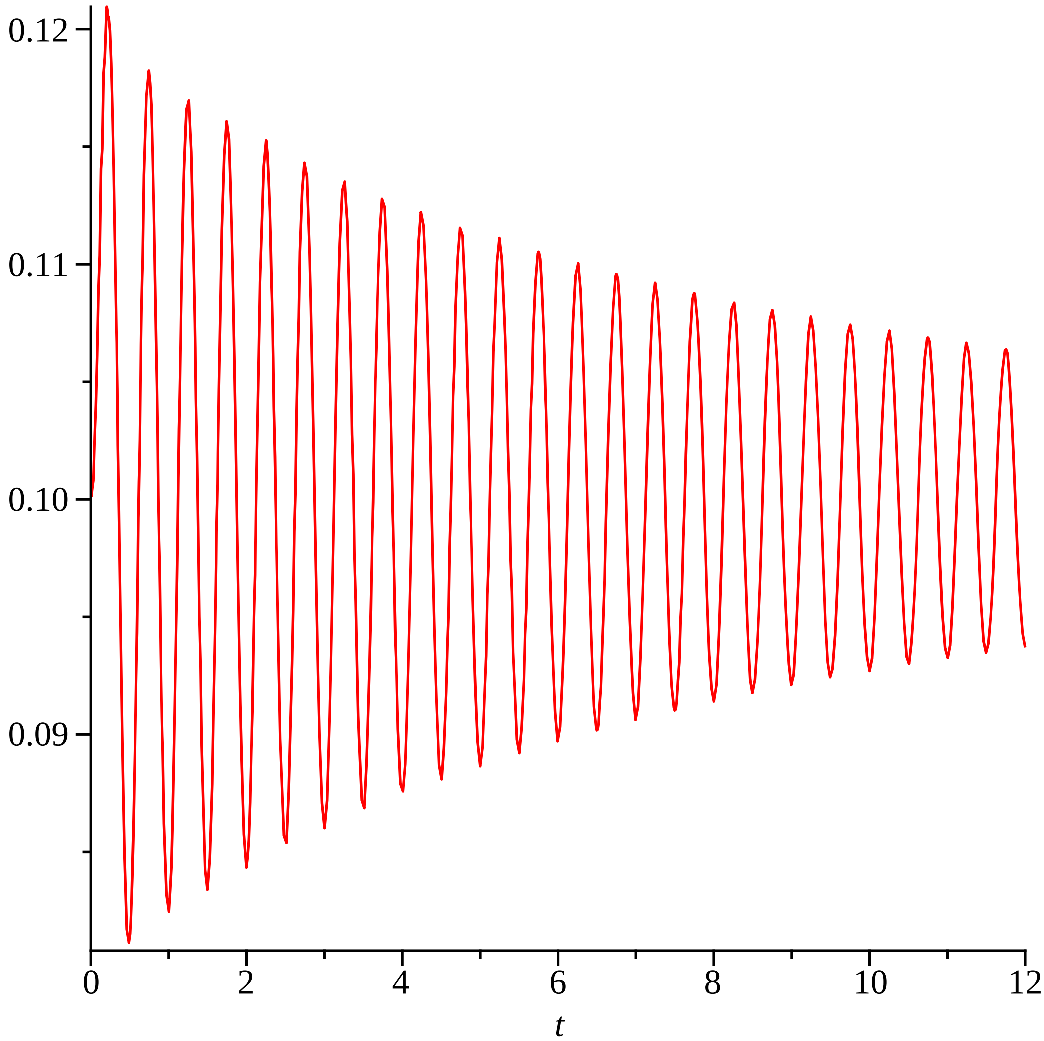
$plot(\mathrm{F}(\mathrm{t}), \mathrm{t} = \Delta T)$



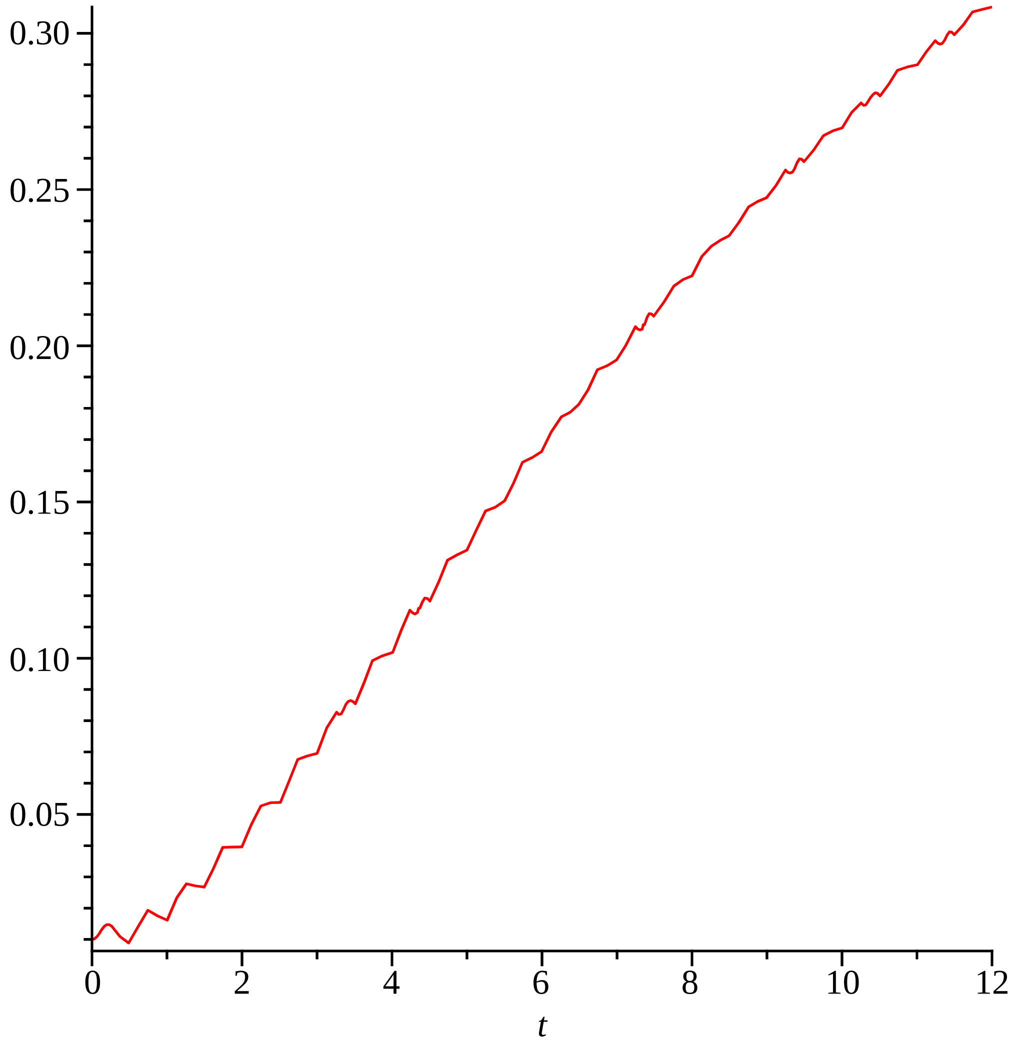
$plot(\mathrm{AC}(\mathrm{tt}), \mathrm{tt} = \Delta T)$



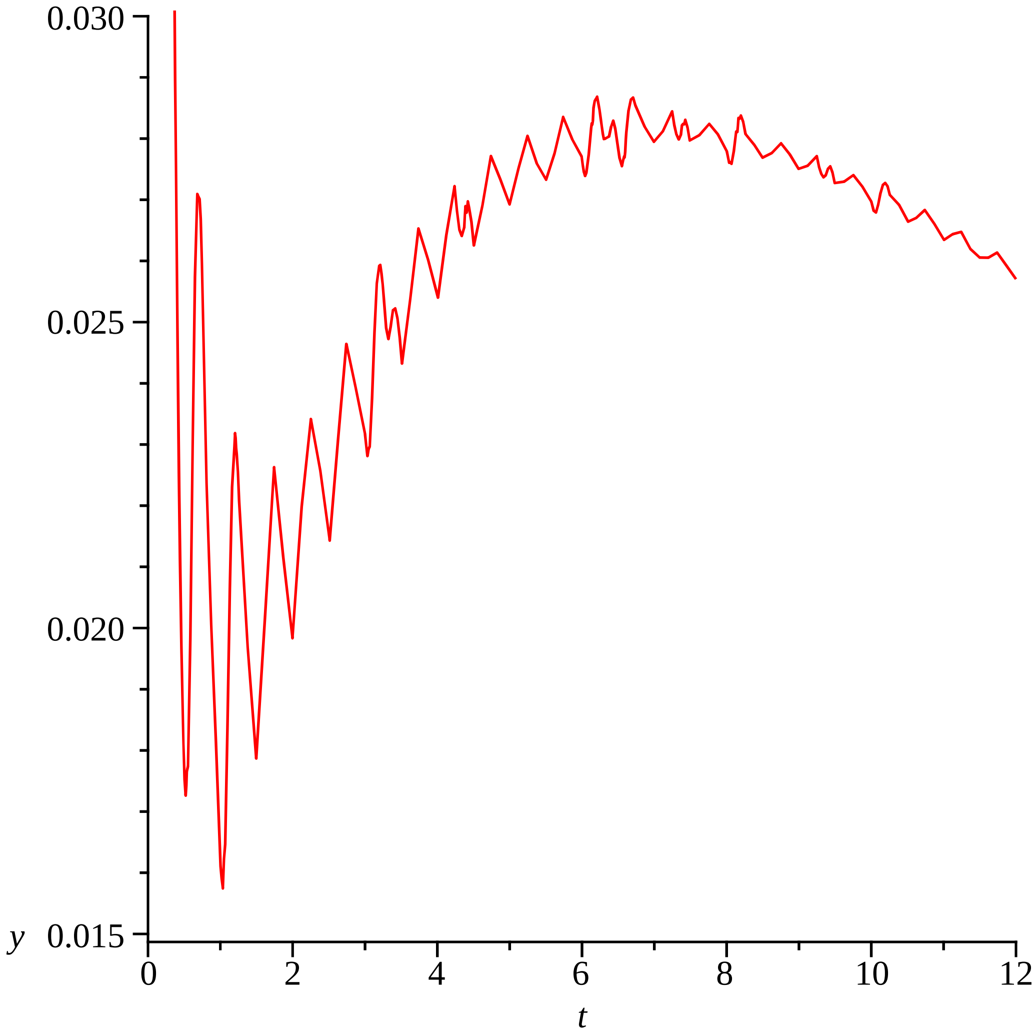
plot(Fmean(t), t = ΔT)



$plot(\mathrm{F12}(t), t = \Delta T)$



plot(F22(t), t = ΔT, y = 0.015 ..0.03)


0.030
0.025
0.020
y 0.015
0
2
4
6
8
10
12
t