

# Corona Discharge

## *Introduction*

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This model simulates a corona discharge occurring between two co-axially fashioned conductors. The negative electric potential is applied to the inner conductor and the exterior conductor is grounded. The discharge gas simulated is argon at atmospheric pressure.

## *Model Definition*

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[Figure 1](#) shows a cross section of the model. By considering a long and uniform coaxial conductor configuration, the model can be viewed as axisymmetric and thus simplified to a 1D problem.

The model presented in the following section is used to simulate the ionization of the neutral gas (Ar) as well as the flux of charged particles (Ar<sup>+</sup> and electrons) when the negative electric potential is applied at the inner conductor (cathode). The high electric field generated by the combination of high potential and small conductor curvature radius (inner conductor,  $r_i$ ) causes electron drift and ionization of the neutral gas surrounding the cathode. The resulting ions generate more electrons through secondary emission at the cathode surface. These electrons are accelerated through a small region away from the cathode, where they can acquire significant energy. This can lead to ionization which creates new electron-ion pairs. The secondary ions will migrate towards the cathode where they eject more secondary electrons. This process is responsible for sustaining the discharge.

The model is based on the fluid equations for electrons and ions as well as on Poisson's equation. Secondary electrons generated by ion bombardment of the cathode surface are taken into account. The model uses a Scharfetter-Gummel upwind scheme to eliminate numerical instabilities in the number density of the charged particles associated with the finite element method. This is needed, in particular close to the cathode, where the ion flux is particularly high.

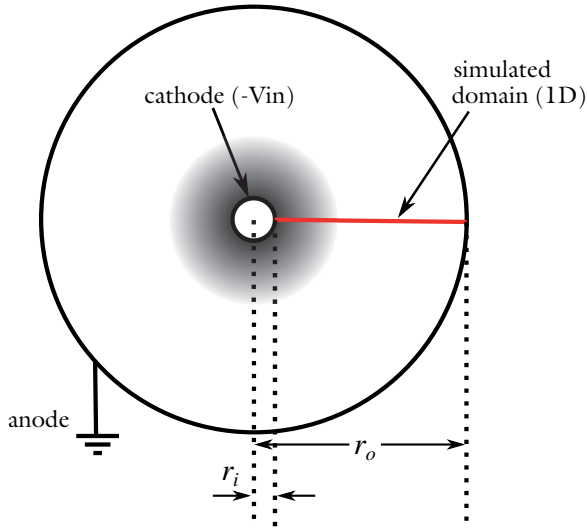


Figure 1: Not-to-scale cross section of the co-axial configuration. The negative potential ( $-V_{in}$ ) is applied at the inner conductor (cathode) and the outer electrode is grounded (anode). The shaded area represents the ionization region created by the positive space charge distribution generated in the vicinity of the cathode.

### DOMAIN EQUATIONS

The electron density and mean electron energy are computed by solving the drift-diffusion equations for the electron density and mean electron energy. Convection of electrons due to fluid motion is neglected. For more detailed information on electron transport see the section [Theory for the Drift Diffusion Interface](#) in the *Plasma Module User's Guide*.

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot [-n_e(\boldsymbol{\mu}_e \cdot \mathbf{E}) - \mathbf{D}_e \cdot \nabla n_e] = R_e$$

$$\frac{\partial}{\partial t}(n_\epsilon) + \nabla \cdot [-n_\epsilon(\boldsymbol{\mu}_\epsilon \cdot \mathbf{E}) - \mathbf{D}_\epsilon \cdot \nabla n_\epsilon] + \mathbf{E} \cdot \boldsymbol{\Gamma}_e = R_\epsilon$$

The electron source  $R_e$  and the energy loss due to inelastic collisions  $R_\epsilon$  are defined later. The electron diffusivity, energy mobility, and energy diffusivity are computed from the electron mobility using the relations

$$\mathbf{D}_e = \mu_e T_e, \mu_\epsilon = \frac{5}{3} \mu_e, \mathbf{D}_\epsilon = \mu_\epsilon T_e$$

The source coefficients in the above equations are determined by the plasma chemistry using rate coefficients. Suppose that there are  $M$  reactions that contribute to the growth or decay of electron density and  $P$  inelastic electron-neutral collisions. In general  $P \gg M$ . In the case of rate coefficients, the electron source term is given by

$$R_e = \sum_{j=1}^M x_j k_j N_n n_e$$

where  $x_j$  is the mole fraction of the target species for reaction  $j$ ,  $k_j$  is the rate coefficient for reaction  $j$  (SI unit:  $\text{m}^3/\text{s}$ ), and  $N_n$  is the total neutral number density (SI unit:  $1/\text{m}^3$ ). The electron energy loss is obtained by summing the collisional energy loss over all reactions:

$$R_\varepsilon = \sum_{j=1}^P x_j k_j N_n n_e \Delta\varepsilon_j$$

Here  $\Delta\varepsilon_j$  is the energy loss from reaction  $j$  (SI unit: V). The rate coefficients can be computed from cross-section data as the integrals

$$k_k = \gamma \int_0^\infty \varepsilon \sigma_k(\varepsilon) f(\varepsilon) d\varepsilon$$

where  $\gamma = (2q/m_e)^{1/2}$  (SI unit:  $\text{C}^{1/2}/\text{kg}^{1/2}$ ),  $m_e$  is the electron mass (SI unit: kg),  $\varepsilon$  is the energy (SI unit: V),  $\sigma_k$  is the collision cross section (SI unit:  $\text{m}^2$ ), and  $f$  is the electron energy distribution function. In this case, a Maxwellian EEDF is assumed.

For non-electron species, the following equation is solved for the mass fraction of each species:

$$\rho \frac{\partial}{\partial t} (w_k) + \rho (\mathbf{u} \cdot \nabla) w_k = \nabla \cdot \mathbf{j}_k + R_k$$

For detailed information on the transport of the non-electron species see the section [Theory for the Heavy Species Transport Interface](#) in the *Plasma Module User's Guide*.

The electrostatic field is computed using the equation

$$-\nabla \cdot \varepsilon_0 \varepsilon_r \nabla V = \rho$$

The space charge density  $\rho$  is automatically computed based on the plasma chemistry specified in the model using the formula

$$\rho = q \left( \sum_{k=1}^N Z_k n_k - n_e \right)$$

For detailed information about electrostatics see [Theory for the Electrostatics Interface](#) in the *Plasma Module User's Guide*.

### *Boundary Conditions*

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Electrons are lost to the wall due to random motion within a few mean free paths of the wall and gained due to secondary emission effects, resulting in the boundary condition

$$-\mathbf{n} \cdot \Gamma_e = \left( \frac{1}{2} v_{e, \text{th}} n_e \right) - \sum_p \gamma_p (\Gamma_p \cdot \mathbf{n}) \quad (1)$$

for the electron flux and

$$-\mathbf{n} \cdot \Gamma_\varepsilon = \left( \frac{5}{6} v_{e, \text{th}} n_\varepsilon \right) - \sum_p \varepsilon_p \gamma_p (\Gamma_p \cdot \mathbf{n}) \quad (2)$$

for the electron energy flux. The second term on the right-hand side of [Equation 1](#) is the gain of electrons due to secondary emission effects,  $\gamma_p$  being the secondary emission coefficient. The second term in [Equation 2](#) is the secondary emission energy flux,  $\varepsilon_p$  being the mean energy of the secondary electrons. For the heavy species, ions are lost to the wall due to surface reactions and the fact that the electric field is directed towards the wall:

$$-\mathbf{n} \cdot \mathbf{j}_k = M_w R_k + M_w c_k Z_k \mu_k (\mathbf{E} \cdot \mathbf{n}) [Z_k \mu_k (\mathbf{E} \cdot \mathbf{n}) > 0] \quad (3)$$

The discharge is driven by the electric potential applied to the inner conductor of the coaxial geometry (at coordinate  $r = r_i$ ).

$$V = V_0 \tanh\left(\frac{t}{\tau}\right) \quad (4)$$

where the function  $\tanh(t/\tau)$  is used to generate the voltage step function ( $-1000$  V). The other boundary (at coordinate  $r = r_o$ ) is grounded.

Note that, during the simulation, the cathode is submitted to an intense flux of ions that generate an important amount of secondary electrons which, in turn, increase the cathode ion bombardment and so on. In order to prevent this avalanche from increasing indefinitely, a RC circuit has been added in series with the system. In order to model the circuit addition, the voltage at the inner conductor is modified using the differential equation

$$V = V_0 - \left( I_p R_b + R_b C_b \frac{\partial V}{\partial t} \right) \quad (5)$$

where  $I_p$  is defined as

$$I_p = - \int \left( \mathbf{n} \cdot \mathbf{J}_i + \mathbf{n} \cdot \mathbf{J}_e + \frac{\partial}{\partial t} (\mathbf{n} \cdot \mathbf{D}) \right) dS \quad (6)$$

and where  $\mathbf{n} \cdot \mathbf{J}_i$  is the normal component of the total ion current density at the wall,  $\mathbf{n} \cdot \mathbf{J}_e$  is the normal component of the total electron current density at the wall, and  $\mathbf{n} \cdot \mathbf{D}$  the normal electrical displacement at the surface.

### PLASMA CHEMISTRY

Argon is an attractive gas to use in a benchmark problem because only a handful of reactions and a few species need to be considered. [Table 1](#) lists the chemical reactions considered.

TABLE 1: TABLE OF COLLISIONS AND REACTIONS MODELED.

Reaction	Formula	Type	$\Delta E$ (eV)	$k_f$ ( $m^3/(s \cdot mol)$ )
1	$e + Ar \Rightarrow e + Ar$	Elastic	0	-
2	$e + Ar \Rightarrow e + Ar^*$	Excitation	11.5	-
3	$e + Ar^* \Rightarrow e + Ar$	Superelastic	-11.5	-
4	$e + Ar \Rightarrow 2e + Ar^+$	Ionization	15.8	-
5	$e + Ar^* \Rightarrow 2e + Ar^+$	Ionization	4.24	-
6	$Ar + Ar^* \Rightarrow Ar + Ar$	Reaction	-	1807
7	$Ar^* + Ar^* \Rightarrow Ar^* + Ar$	Reaction	-	$2.3 \cdot 10^7$

Initially a small number of seed electrons are present. These are necessary in order to initiate the discharge. In addition to the volumetric reactions, the following surface reactions are implemented:

TABLE 2: TABLE OF SURFACE REACTIONS.

Reaction	Formula	Sticking coefficient
1	$\text{Ar}_s \Rightarrow \text{Ar}$	1
2	$\text{Ar}^+ \Rightarrow \text{Ar}$	1

When the ion atoms reach the wall, they are assumed to change back to neutral argon atoms and donate their charge to the wall. Note that the secondary emission coefficient is set to 0.2 on the cathode boundary (at coordinate  $r = r_i$ ) and to 0 at the outer electrode (at coordinate  $r = r_o$ ). The mean electron energy of the secondary electron is set to 4 eV.

### *Results and Discussion*

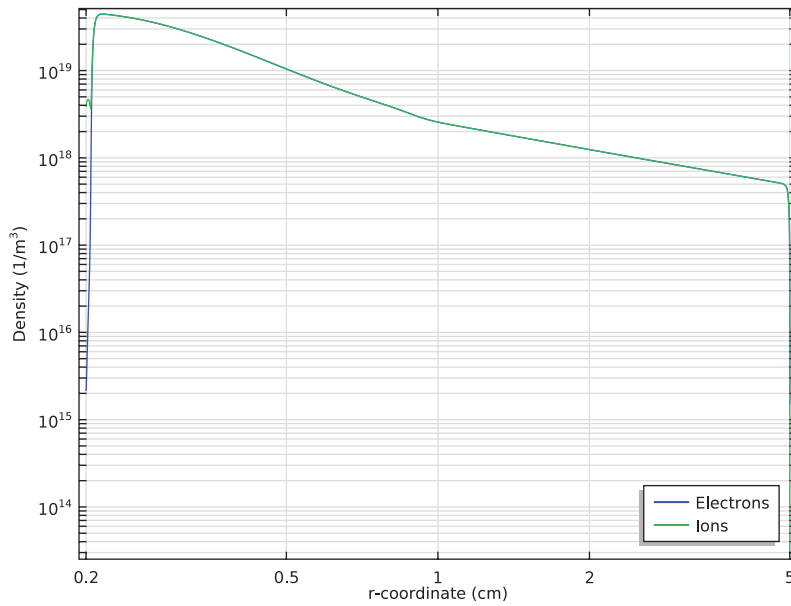
When solving the problem, you can choose to plot the electron and ion density as the solver runs. Doing so allows the behavior of the charged species to be observed as the model is solving.

Looking at the evolution of the densities during the solving process shows that the gas is initially weakly ionized (electrons and ions having relatively low densities compared to neutral atoms). As the negative voltage is applied to the cathode, the highly mobile electrons are accelerated towards the anode leaving behind a positively charged gas in the cathode vicinity. With increasing negative potential, ion bombardment becomes more persistent on the cathode surface, which generates more secondary electrons, ionizing more neutral atoms, and engendering greater ion current. As the negative potential rises, the population of charged particles becomes larger as a consequence of this avalanche effect.

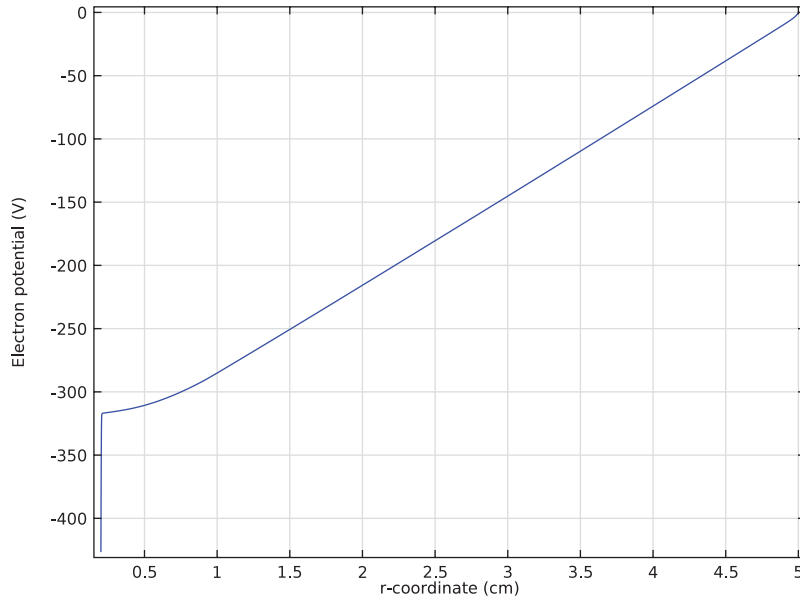
As the ion current becomes more significant, the RC circuit reduces the cathode negative potential such that an equilibrium is reached between the generation of the charged particles, preventing the transition of the plasma into an arcing regime.

Figure 2 shows the electron and ion densities at the end of the simulation ( $t = 0.1$  s). Note that the figure is plotted on a log-log scale. In the figure, one can see that the ion density is approximately three orders of magnitude higher than the electron density in the vicinity of the cathode. The tiny positive space charge distribution generated by the ion density defines an ionization region that screens the cathode potential from the anode. This can be observed by displaying the electric potential along the radius of the

coaxial assembly; see [Figure 3](#).



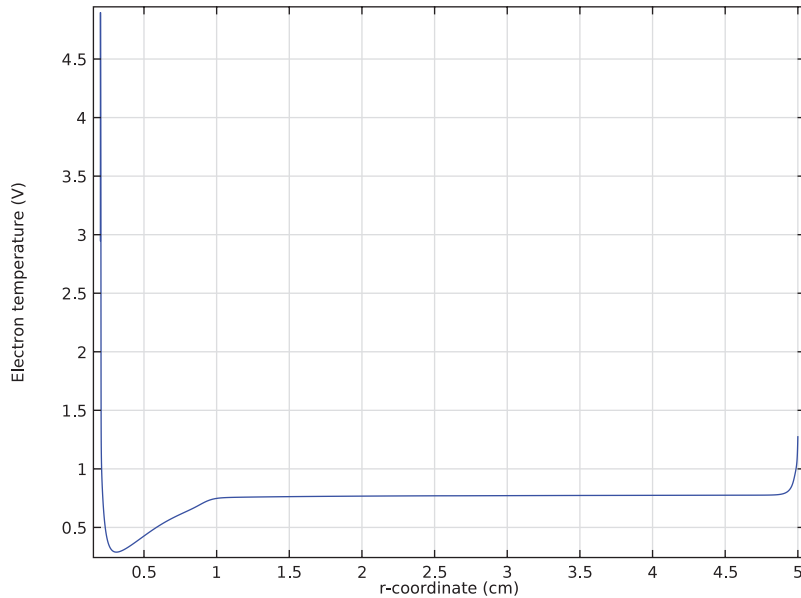
*Figure 2: Plot of electron (blue) and ion (green) number density at the end of the simulation ( $t = 0.1$  s).*



*Figure 3: Electric potential along the radius of the coaxial assembly at the end of the simulation ( $t = 0.1$  s).*

The high voltage and important current density at the cathode enhance the electron temperature near the electrode, thus boosting ionization of the neutral atoms in the ionization zone. This can be seen in [Figure 4](#), which displays the electron temperature at the end of the simulation.





*Figure 4: Electron temperature along the radius of the assembly at the end of the simulation ( $t = 0.1$  s).*

To see the effect of the RC circuit on the system, plot the secondary electron flux as a function of time at the cathode surface; see [Figure 5](#). A close look at the figure shows a rapid rise of the flux that tends to stabilize as the circuit sees higher currents. [Figure 6](#) also shows the potential at both electrodes as a function of time. Comparing figure [Figure 5](#) with [Figure 6](#) reveals the effect of the circuit on both potential and secondary emission at the cathode surface. One can also observe a similar effect by plotting the electron current density at the electrodes, [Figure 6](#). Doing so shows a direct relation between the cathode secondary emission (positive current flow), the anode recombination (negative flow), and the RC circuit. [Figure 7](#) displays a surface plot of the logarithm of the electron density at the last time step.

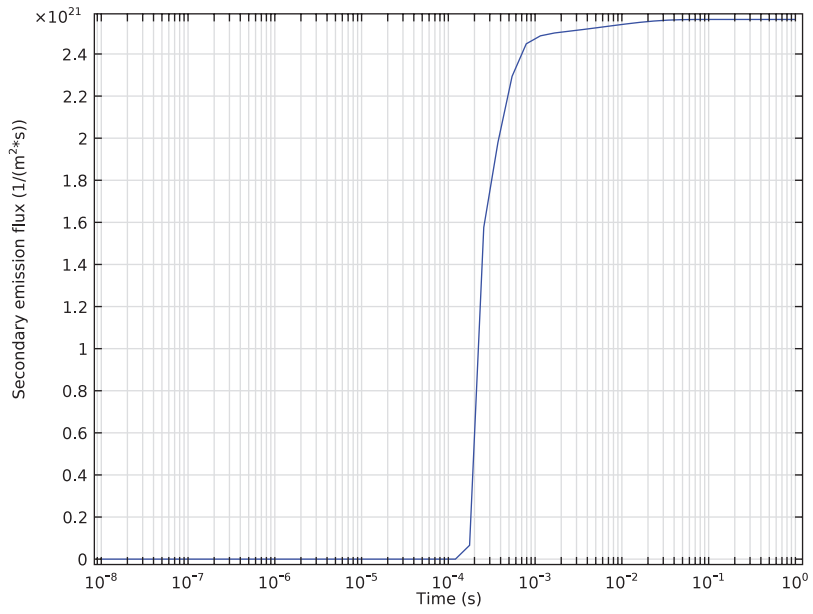


Figure 5: Secondary emission flux along time at the cathode.

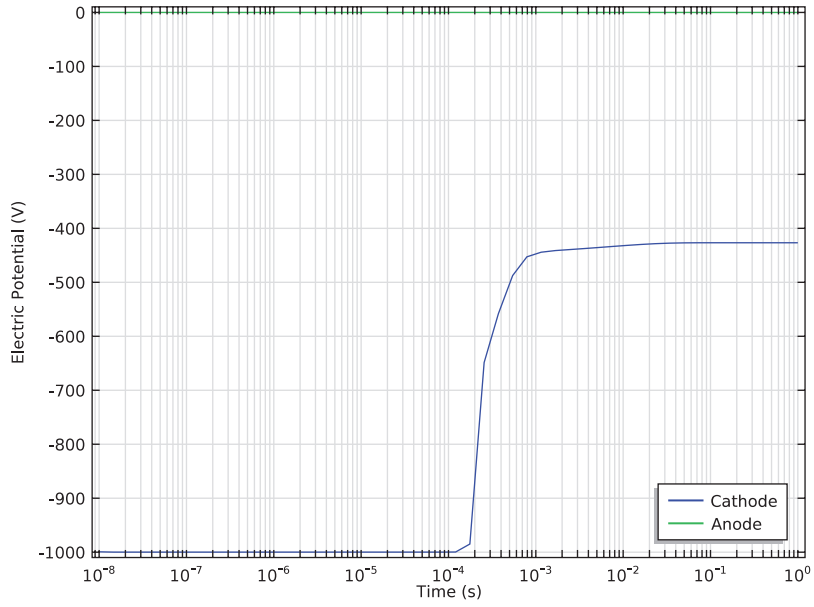


Figure 6: Potential at the electrodes as a function of time.

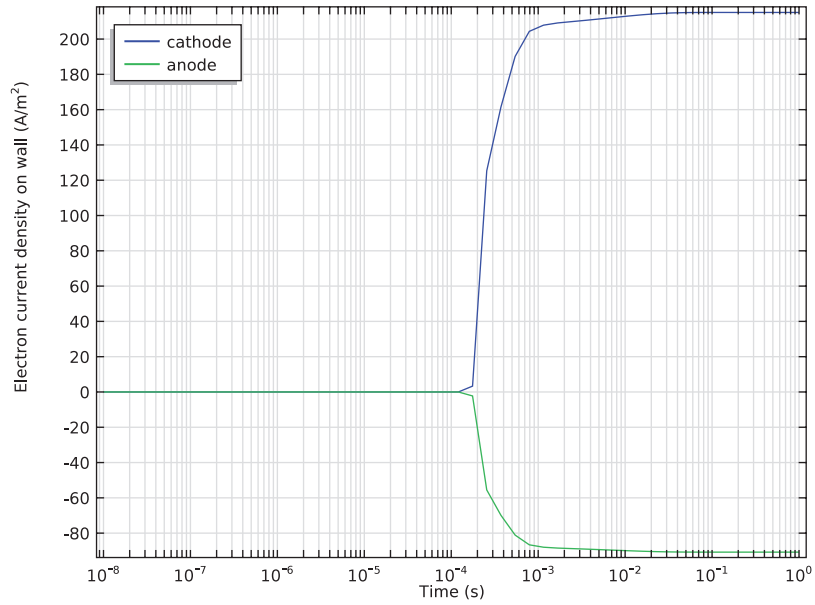


Figure 7: Electron current density at the electrodes as a function of time.

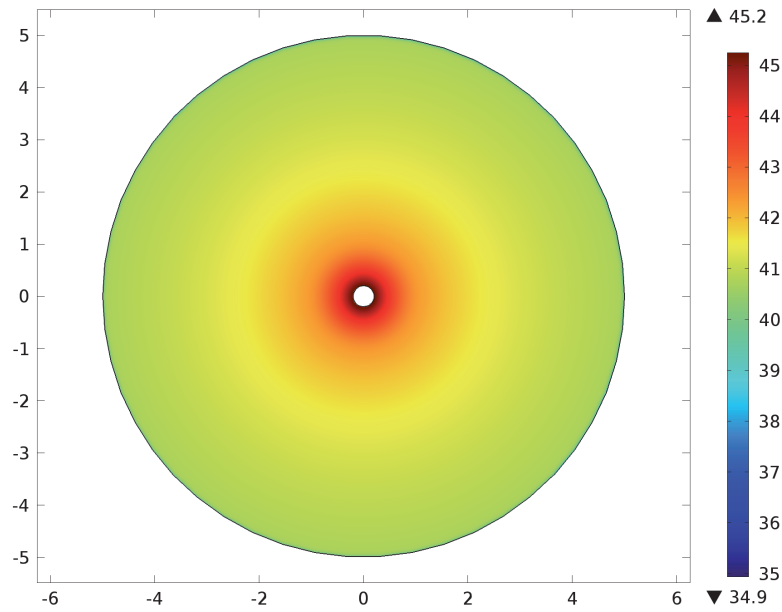


Figure 8: 2D electron density (log of  $n_e$ ) at the last time step.

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**Model Library path:** Plasma\_Module/Direct\_Current\_Discharges/  
corona\_discharge\_1d

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### *Modeling Instructions*

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From the **File** menu, choose **New**.

#### **NEW**

1 In the **New** window, click **Model Wizard**.

#### **MODEL WIZARD**

- 1 In the **Model Wizard** window, click **1D Axisymmetric**.
- 2 In the **Select physics** tree, select **Plasma>DC Discharge (dc)**.
- 3 Click **Add**.

- 4 Click **Study**.
- 5 In the **Select study** tree, select **Preset Studies>Time Dependent**.
- 6 Click **Done**.

To set up this model, you need access to some advanced settings.

#### DC DISCHARGE (DC)

Solve this model using the finite volume method and a Scharfetter-Gummel scheme. You can switch from the finite element method to the finite volume method by suitable choices in the **Advanced Settings** section.

- 1 In the **Model Builder** window's toolbar, click the **Show** button and select **Discretization** in the menu.
- 2 In the **Model Builder** window, under **Component 1 (comp1)** click **DC Discharge (dc)**.
- 3 In the **Settings** window for DC Discharge, click to expand the **Discretization** section.
- 4 From the **Formulation** list, choose **Finite volume (constant shape function)**.

#### DEFINITIONS

##### *Parameters*

- 1 On the **Model** toolbar, click **Parameters**.
- 2 In the **Settings** window for Parameters, locate the **Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
V0	1000[V]	1000.0 V	Voltage

#### GEOMETRY I

Follow the steps below to create the model geometry: a simple 1D geometry consisting of a single domain bounded by the cathode (left, inner conductor) and the anode (right, outer conductor).

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for Geometry, locate the **Units** section.
- 3 From the **Length unit** list, choose **cm**.

##### *Interval 1 (i1)*

- 1 On the **Geometry** toolbar, click **Interval**.
- 2 In the **Settings** window for Interval, locate the **Interval** section.

- 3 In the **Left endpoint** text field, type 0.2.
- 4 In the **Right endpoint** text field, type 5.
- 5 Click the **Build All Objects** button.

### DC DISCHARGE (DC)

Load the argon cross sections from file. They form the basis of the plasma chemistry under investigation. To fix the species data, load the argon preset for each species.

#### *Cross Section Import I*

- 1 On the **Physics** toolbar, click **Global** and choose **Cross Section Import**.
- 2 In the **Settings** window for Cross Section Import, locate the **Cross Section Import** section.
- 3 Click **Browse**.
- 4 Browse to the model's Model Library folder and double-click the file `Ar_xsecs.txt`.

When solving any type of reacting flow problem one species must always be chosen to fulfill the mass constraint. This should be taken as the species with the largest mass fraction.

#### *Species: Ar*

- 1 In the **Model Builder** window, under **Component 1 (comp1)>DC Discharge (dc)** click **Species: Ar**.
- 2 In the **Settings** window for Species, locate the **Species Formula** section.
- 3 Select the **From mass constraint** check box.

When solving a plasma problem the plasma must initially be charge neutral. COMSOL automatically computes the initial concentration of a selected ionic species such that the initial electroneutrality constraint is satisfied. Now let the initial number density of argon ions be the same as the initial number of electrons. This forces the plasma to be initially charge neutral.

#### *Species: Ar+*

- 1 In the **Model Builder** window, under **Component 1 (comp1)>DC Discharge (dc)** click **Species: Ar+**.
- 2 In the **Settings** window for Species, locate the **Species Formula** section.
- 3 Select the **Initial value from electroneutrality constraint** check box.
- 4 Locate the **General Parameters** section. From the **Preset species data** list, choose **Ar**.

*Species: Ar*

- 1 In the **Model Builder** window, under **Component 1 (comp1)>DC Discharge (dc)** click **Species: Ar**.
- 2 In the **Settings** window for Species, locate the **General Parameters** section.
- 3 From the **Preset species data** list, choose **Ar**.

*Species: Ars*

- 1 In the **Model Builder** window, under **Component 1 (comp1)>DC Discharge (dc)** click **Species: Ars**.
- 2 In the **Settings** window for Species, locate the **General Parameters** section.
- 3 From the **Preset species data** list, choose **Ar**.

Now add two more regular reactions that describe how electronically excited argon atoms are consumed on the volumetric level. The rate coefficients for these reactions are taken from the literature.

*Reaction 1*

- 1 On the **Physics** toolbar, click **Domains** and choose **Reaction**.
- 2 In the **Settings** window for Reaction, locate the **Reaction Formula** section.
- 3 In the **Formula** text field, type  $\text{Ar} + \text{Ars} \Rightarrow \text{Ar} + \text{Ar}$ .
- 4 Locate the **Kinetics Expressions** section. In the  $k^f$  text field, type 1807.

*Reaction 2*

- 1 On the **Physics** toolbar, click **Domains** and choose **Reaction**.
- 2 In the **Settings** window for Reaction, locate the **Reaction Formula** section.
- 3 In the **Formula** text field, type  $\text{Ars} + \text{Ars} \Rightarrow \text{Ars} + \text{Ar}$ .
- 4 Locate the **Kinetics Expressions** section. In the  $k^f$  text field, type  $2.3 \times 10^7$ .

Surface reactions must always be included in a plasma model because they describe how ionic, excited, and radical species interact with the wall.

*Surface Reaction 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Surface Reaction**.
- 2 In the **Settings** window for Surface Reaction, locate the **Reaction Formula** section.
- 3 In the **Formula** text field, type  $\text{Ars} \Rightarrow \text{Ar}$ .
- 4 Locate the **Boundary Selection** section. From the **Selection** list, choose **All boundaries**.

Secondary emission of electrons is important to sustain a DC discharge. In this example, add a secondary emission coefficient on the left wall (cathode).



*Surface Reaction 2*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Surface Reaction**.
- 2 In the **Settings** window for Surface Reaction, locate the **Reaction Formula** section.
- 3 In the **Formula** text field, type  $\text{Ar}^+ \Rightarrow \text{Ar}$ .
- 4 Select Boundary 1 only.
- 5 Locate the **Secondary Emission Parameters** section. In the  $\gamma_i$  text field, type 0.2.
- 6 In the  $\varepsilon_i$  text field, type 4.

*Surface Reaction 3*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Surface Reaction**.
- 2 In the **Settings** window for Surface Reaction, locate the **Reaction Formula** section.
- 3 In the **Formula** text field, type  $\text{Ar}^+ \Rightarrow \text{Ar}$ .
- 4 Select Boundary 2 only.
- 5 In the **Model Builder** window, click **DC Discharge (dc)**.
- 6 In the **Settings** window for DC Discharge, locate the **Plasma Properties** section.
- 7 Select the **Use reduced electron transport properties** check box.
- 8 Locate the **Out-of-Plane Thickness** section. In the  $d_z$  text field, type 0.02 [m].

*Plasma Model 1*

- 1 In the **Model Builder** window, under **Component 1 (comp1) > DC Discharge (dc)** click **Plasma Model 1**.
- 2 In the **Settings** window for Plasma Model, locate the **Electron Density and Energy** section.
- 3 In the  $\mu_e N_n$  text field, type 4e24.

*Ground 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Ground**.
- 2 Select Boundary 2 only.

Now, add an RC circuit in series with the system. The role of the circuit is to limit the current at the cathode and avoid the arcing regime.

*Metal Contact 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Metal Contact**.
- 2 Select Boundary 1 only.
- 3 In the **Settings** window for Metal Contact, locate the **Terminal** section.
- 4 In the  $V_0$  text field, type -V0.

- 5 Locate the **Quick Circuit Settings** section. From the **Quick circuit type** list, choose **Series RC circuit**.

#### *Wall 1*

- 1 On the **Physics** toolbar, click **Boundaries** and choose **Wall**.
- 2 In the **Settings** window for Wall, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **All boundaries**.

#### **MESH 1**

Meshing is a critical step in any plasma model. A fine mesh is needed close to the electrodes to capture the separation of space charge between the electrons and ions close to the wall.

#### *Edge 1*

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Edge**.

#### *Distribution 1*

- 1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** right-click **Edge 1** and choose **Distribution**.
- 2 In the **Settings** window for Distribution, locate the **Distribution** section.
- 3 From the **Distribution properties** list, choose **Predefined distribution type**.
- 4 In the **Number of elements** text field, type 500.
- 5 In the **Element ratio** text field, type 400.
- 6 From the **Distribution method** list, choose **Geometric sequence**.
- 7 Select the **Symmetric distribution** check box.
- 8 Click the **Build All** button.

#### **STUDY 1**

Compute an initial solution to generate the default plots and then set them up to show the electron and ion density while the solver runs.

- 1 In the **Model Builder** window, right-click **Study 1** and choose **Get Initial Value for Step**.

#### **RESULTS**

#### *Electron Density (dc)*

Set up an electron and ion density plot.

- 1 In the **Settings** window for 1D Plot Group, locate the **Plot Settings** section.
- 2 Select the **x-axis label** check box.
- 3 Select the **y-axis label** check box.
- 4 In the associated text field, type  $\text{Density} \left( \frac{1}{\text{m}^3} \right)$ .
- 5 Click to expand the **Legend** section. From the **Position** list, choose **Lower right**.
- 6 In the **Model Builder** window, expand the **Electron Density (dc)** node, then click **Line Graph 1**.
- 7 In the **Settings** window for Line Graph, click to expand the **Legends** section.
- 8 Select the **Show legends** check box.
- 9 From the **Legends** list, choose **Manual**.
- 10 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 11 Locate the **Legends** section. In the table, enter the following settings:

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**Legends**


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Electrons

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- 12 Right-click **Results>Electron Density (dc)>Line Graph 1** and choose **Rename**.
- 13 In the **Rename Line Graph** dialog box, type **Electrons** in the **New label** text field.
- 14 Click **OK**.
- 15 Right-click **Results>Electron Density (dc)>Electrons** and choose **Duplicate**.
- 16 Right-click **Results>Electron Density (dc)>Electrons 1** and choose **Rename**.
- 17 In the **Rename Line Graph** dialog box, type **Ions** in the **New label** text field.
- 18 Click **OK**.
- 19 In the **Settings** window for Line Graph, locate the **y-Axis Data** section.
- 20 In the **Expression** text field, type  $\text{dc.n\_wAr\_1p}$ .
- 21 Locate the **Legends** section. In the table, enter the following settings:

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**Legends**


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Ions

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- 22 In the **Settings** window for 1D Plot Group, locate the **Data** section.
- 23 From the **Time selection** list, choose **Last**.
- 24 Right-click **Electron Density (dc)** and choose **Rename**.

**25** In the **Rename 1D Plot Group** dialog box, type **Densities (dc)** in the **New label** text field.

**26** Click **OK**.

**27** Click the **y-Axis Log Scale** button on the **Graphics** toolbar.

**28** Click the **x-Axis Log Scale** button on the **Graphics** toolbar.

Now set up the last two default plots.

#### *Electron Temperature (dc)*

**1** In the **Settings** window for 1D Plot Group, locate the **Data** section.

**2** From the **Time selection** list, choose **Last**.

**3** In the **Model Builder** window, expand the **Electron Temperature (dc)** node, then click **Line Graph 1**.

**4** In the **Settings** window for Line Graph, click to expand the **Title** section.

**5** From the **Title type** list, choose **None**.

#### *Electric Potential (dc)*

**1** In the **Model Builder** window, under **Results** click **Electric Potential (dc)**.

**2** In the **Settings** window for 1D Plot Group, locate the **Data** section.

**3** From the **Time selection** list, choose **Last**.

**4** In the **Model Builder** window, expand the **Electric Potential (dc)** node, then click **Line Graph 1**.

**5** In the **Settings** window for Line Graph, locate the **Title** section.

**6** From the **Title type** list, choose **None**.

**7** Locate the **y-Axis Data** section. Select the **Description** check box.

**8** In the associated text field, type **Electron potential**.

### **STUDY 1**

#### *Step 1: Time Dependent*

**1** In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.

**2** In the **Settings** window for Time Dependent, locate the **Study Settings** section.

**3** In the **Times** text field, type **0** .

**4** Click **Range**.

**5** In the **Range** dialog box, choose **Number of values** from the Entry method list.

**6** In the **Start** text field, type **-8**.

- 7 In the **Stop** text field, type 0.
- 8 In the **Number of values** text field, type 50.
- 9 From the **Function to apply to all values** list, choose **exp10**.
- 10 Click **Add**.
- 11 In the **Settings** window for Time Dependent, click to expand the **Results while solving** section.
- 12 Locate the **Results While Solving** section. Select the **Plot** check box.
- 13 On the **Model** toolbar, click **Compute**.

## RESULTS

### *Electric Potential (dc)*

Now plot the electric potential at the boundaries.

#### *1D Plot Group 4*

- 1 On the **Model** toolbar, click **Add Plot Group** and choose **ID Plot Group**.
- 2 On the **ID plot group** toolbar, click **Point Graph**.
- 3 In the **Settings** window for Point Graph, locate the **Selection** section.
- 4 From the **Selection** list, choose **All boundaries**.
- 5 Locate the **y-Axis Data** section. In the **Expression** text field, type  $V$ .
- 6 Click to expand the **Legends** section. Select the **Show legends** check box.
- 7 From the **Legends** list, choose **Manual**.
- 8 In the table, enter the following settings:

Legends
Cathode
Anode

- 9 Locate the **y-Axis Data** section. Select the **Description** check box.
- 10 In the associated text field, type **Electric Potential**.
- 11 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 12 Click the **x-Axis Log Scale** button on the **Graphics** toolbar.
- 13 In the **Model Builder** window, click **ID Plot Group 4**.
- 14 In the **Settings** window for ID Plot Group, locate the **Legend** section.
- 15 From the **Position** list, choose **Lower right**.

**16** Right-click **ID Plot Group 4** and choose **Rename**.

**17** In the **Rename ID Plot Group** dialog box, type **Electric potential at boundaries** in the **New label** text field.

**18** Click **OK**.

#### *Electric potential at boundaries*

**1** In the **Model Builder** window, collapse the **Results>Electric potential at boundaries** node.

Plot the electron current density at the boundaries as follows:

#### *ID Plot Group 5*

**1** On the **Model** toolbar, click **Add Plot Group** and choose **ID Plot Group**.

**2** On the **ID plot group** toolbar, click **Point Graph**.

**3** In the **Settings** window for Point Graph, locate the **Selection** section.

**4** From the **Selection** list, choose **All boundaries**.

**5** Locate the **y-Axis Data** section. In the **Expression** text field, type  $dc.nJe$ .

**6** Locate the **Legends** section. Select the **Show legends** check box.

**7** From the **Legends** list, choose **Manual**.

**8** In the table, enter the following settings:

<b>Legends</b>
cathode
anode

**9** Locate the **Title** section. From the **Title type** list, choose **None**.

**10** Click the **x-Axis Log Scale** button on the **Graphics** toolbar.

**11** In the **Model Builder** window, click **ID Plot Group 5**.

**12** In the **Settings** window for ID Plot Group, locate the **Legend** section.

**13** From the **Position** list, choose **Upper left**.

**14** Right-click **ID Plot Group 5** and choose **Rename**.

**15** In the **Rename ID Plot Group** dialog box, type **Electron current density at boundaries** in the **New label** text field.

**16** Click **OK**.

### *Electron current density at boundaries*

- 1 In the **Model Builder** window, collapse the **Results>Electron current density at boundaries** node.

Next, plot the secondary electron emission at the cathode.

### *1D Plot Group 6*

- 1 On the **Model** toolbar, click **Add Plot Group** and choose **ID Plot Group**.
- 2 On the **ID plot group** toolbar, click **Point Graph**.
- 3 Select Boundary 1 only.
- 4 In the **Settings** window for Point Graph, locate the **y-Axis Data** section.
- 5 In the **Expression** text field, type `dc.sf1ux`.
- 6 Locate the **Title** section. From the **Title type** list, choose **None**.
- 7 In the **Model Builder** window, right-click **ID Plot Group 6** and choose **Rename**.
- 8 In the **Rename ID Plot Group** dialog box, type Secondary emission electron flux in the **New label** text field.
- 9 Click **OK**.

### *Secondary emission electron flux*

- 1 In the **Model Builder** window, collapse the **Results>Secondary emission electron flux** node.
- 2 Click the **x-Axis Log Scale** button on the **Graphics** toolbar.  
Finally, create a nice plot for the model thumbnail.

### *Data Sets*

- 1 On the **Results** toolbar, click **More Data Sets** and choose **Revolution 1D**.
- 2 In the **Model Builder** window, collapse the **Data Sets** node.

## **RESULTS**

### *2D Plot Group 7*

- 1 On the **Results** toolbar, click **2D Plot Group**.
- 2 In the **Model Builder** window, under **Results** right-click **2D Plot Group 7** and choose **Surface**.
- 3 In the **Settings** window for Surface, locate the **Expression** section.
- 4 In the **Expression** text field, type `log(dc.ne)`.
- 5 In the **Model Builder** window, right-click **2D Plot Group 7** and choose **Rename**.

- 6** In the **Rename 2D Plot Group** dialog box, type  $\log(N_e)$  in the **New label** text field.
- 7** Click **OK**.

*log(N<sub>e</sub>)*

- 1** In the **Model Builder** window, collapse the **Results>log(N<sub>e</sub>)** node.
- 2** On the **2D plot group** toolbar, click **Plot**.