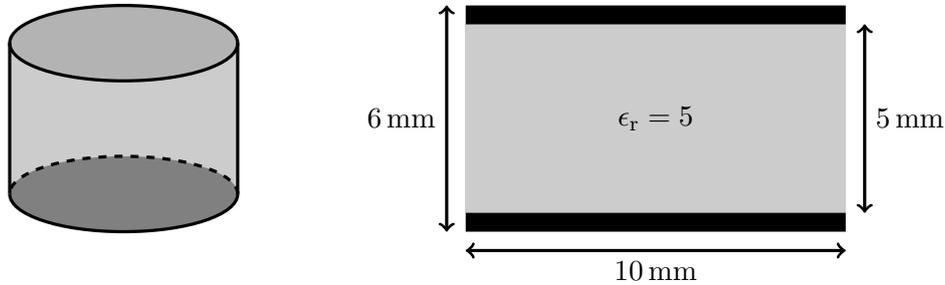


A. Computation of capacitance

Let us consider the capacitance of the cylindrical plate capacitor pictured below:

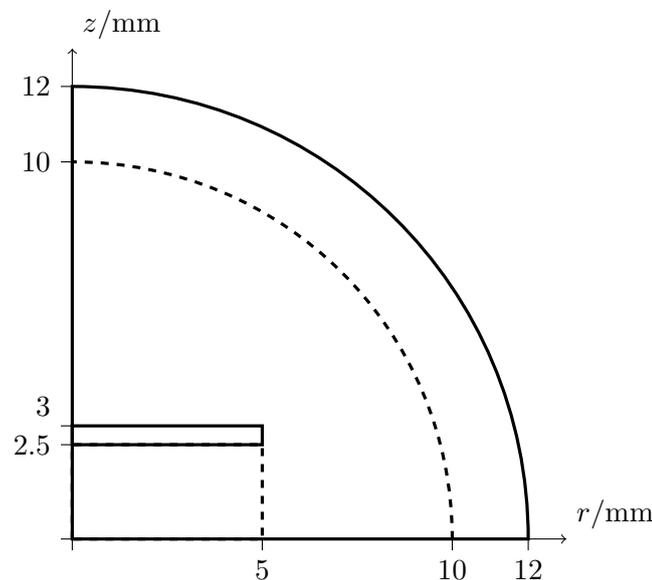


In the idealized case, the capacitance would be

$$C = \epsilon_r \epsilon_0 \frac{\pi r^2}{h} \approx 695.4 \text{ fF.}$$

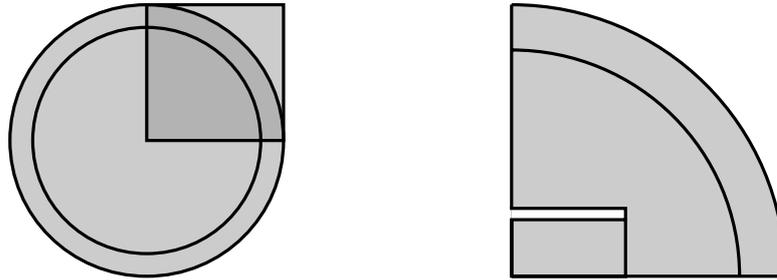
Using Comsol Multiphysics, the task is to compute the capacitance more accurately by also taking the stray fields into account. **Answer the given questions in a separate answer sheet which is returned to the course assistant in the end of the exercise session.**

1. Start COMSOL MULTIPHYSICS and choose the following:
Axial symmetry (2D) → AC/DC Module → Statics, Electric → Electrostatics
2. Make use of the symmetry and draw the following geometry:



First, draw two concentric circles at (0,0) with radii 10 mm and 12 mm. Choose them both by holding down Ctrl key and clicking with left mouse key and create the **union** of these two circles. Then, draw a 12 mm × 12 mm square with its corner in the origin. Choose both the square and the union of the circles and create the **intersection** of these objects. Draw a 5 mm × 2.5 mm rectangle with its corner at (0,0). Draw another rectangle with measures 5 mm × 0.5 mm right above the previous rectangle. This rectangle is then cut away

from the computational domain. Choose both the upper rectangle and the background domain and choose the **difference**.



3. Set the material parameters by choosing **Subdomain Settings** from the **Physics** menu. Set the relative permittivity $\epsilon_r = 5$ for the domain inside the capacitor. Leave all other domains with $\epsilon_r = 1$.
4. Set the following boundary conditions from the **Physics** menu:
 - **Axial symmetry** for the boundaries along the z -axis.
 - **Ground** for the boundaries along the $z = 0$ plane.
 - **Port** for the boundaries of the electrode. For the input property choose **Energy method**.
 - **Ground** for the circular outer boundary.
 - **Continuity** for the interior boundaries (Default setting, needs not to be changed.)
5. Create the mesh by **Initialize Mesh** and **Solve** the problem.
6. See how to visualize the solution in different ways by choosing **Plot Parameters** from the **Postprocessing** menu. (Try, for example, surface plot or contour plot for the potential and streamline plot for the electric field.)
7. Compute the capacitance by choosing **Data Display** \rightarrow **Global** from the **Postprocessing** menu (Capacitance matrix, element 11 (C11_emes)). Notice that we have made use of the vertical symmetry and computed only one half of the capacitor. Therefore, the actual capacitance in question is the series capacitance of the two halves. That is, to get the total capacitance, the obtained capacitance must be divided by 2. We can also ask Comsol to evaluate expression C11_emes/2.

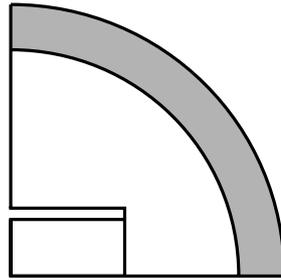
QUESTION 1: What is the obtained capacitance value?

8. The previous result was computed using the default mesh. Study different ways to refine the mesh to increase the accuracy of the computation. You can just click **Refine Mesh** (a couple of times) or see **Free Mesh Parameters**. One possibility is to use **Adaptive mesh refinement** from **Solver Parameters**. Create a denser mesh and solve the problem again. Compute the capacitance.

QUESTION 2: What is the obtained capacitance value?

9. The boundary condition (Ground) is probably not the most optimal choice on the outer boundary as it is relevantly close to the capacitor. Change the boundary condition of the outer circular boundary to **Zero charge/symmetry**, keep the previous dense mesh and solve the problem again. Visualize the solution to see the effect of changing the boundary condition. Compute the capacitance.

QUESTION 3: What is the obtained capacitance value?



10. Let us try to further diminish the effect of the outer boundary by introducing the infinite elements. That is a coordinate transformation which, in theory, moves the boundary infinitely far away. Change the boundary condition of the outer boundary back to **Ground**. Then, from **Subdomain Settings** set **Spherical Infinite Elements** for the domain between the two outer circular boundaries. (This explains why the 10 mm circle was needed.) Solve the problem one more time using the same mesh and compute the capacitance.

QUESTION 4: What is the obtained capacitance value?

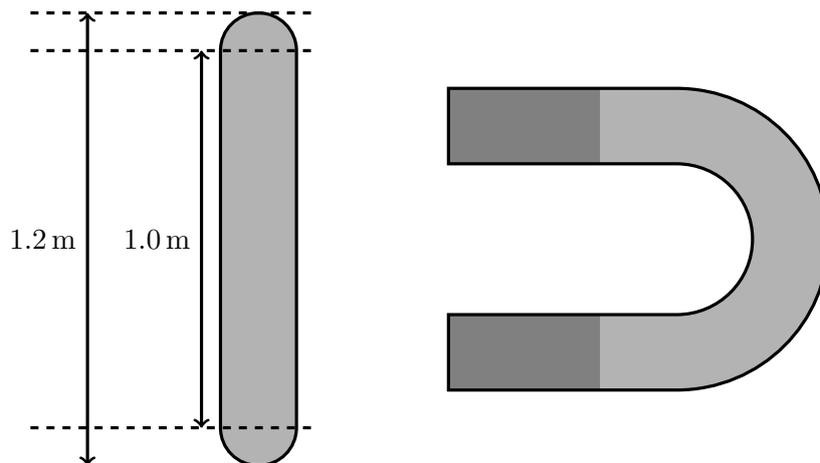
B. Computation of magnetostatic force

ACDC_Module/Tutorial_Models/rod_and_permanent_magnet

The task is to study a tutorial model from Comsol's own model library. In this problem, we compute the force which is exerted to an iron rod by a horseshoe shaped permanent magnet. The documentation of the model is given in the attached paper. These instructions tell how to model the geometry, set the correct boundary and subdomain settings, create the mesh, visualize the solution and compute the force. The obtained total force (x -component) is 592 N.

Originally, the rod is 1.0 m long. Now, we want increase its length up to 1.2 m. See the new measures of the iron rod in the picture below. **By following the attached instructions, model the geometry using these new measures and compute the magnetostatic force.**

QUESTION 5: What is the obtained force value?



C. Optional

If you finish early and you are still hungry for more you may familiarize yourself with other examples given in Comsol's **Model Library**. You may also try modeling your own examples (Different geometries, materials, boundary conditions, excitations, ...).