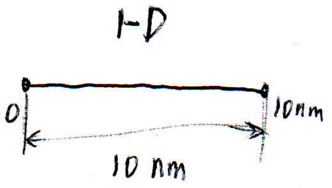
PNP Equation — 1D

$$
\begin{cases}
\dfrac{\partial C_+}{\partial t} - \dfrac{\partial}{\partial x}\left(D_+ \dfrac{\partial C_+}{\partial x} + \dfrac{1}{k_B \cdot T}\cdot(+1)\cdot C_+ \cdot \dfrac{\partial \phi}{\partial x}\right) = 0 & ① \\[2ex]
\dfrac{\partial C_-}{\partial t} - \dfrac{\partial}{\partial x}\left(D_- \dfrac{\partial C_-}{\partial x} + \dfrac{1}{k_B \cdot T}\cdot(-1)\cdot C_- \cdot \dfrac{\partial \phi}{\partial x}\right) = 0 & ② \\[2ex]
\varepsilon \dfrac{\partial^2 \phi}{\partial x^2} + (C_+ - C_-)\cdot \underset{\substack{\downarrow \\ \text{elementary charge}}}{e} = 0. & ③
\end{cases}
$$



BCs: 1. No ionic fluxes at both boundaries

2. $\phi|_{x=0} = 0$,

$\phi|_{x=10nm} = 2V$.

①: $\dfrac{\partial C_+}{\partial t} - \dfrac{\partial}{\partial x}\left(D_+ \dfrac{\partial C_+}{\partial x} + \dfrac{D_+ \cdot C_+}{k_B \cdot T} \dfrac{\partial \phi}{\partial x}\right) = 0$

$v \to$ test function.

$$\int \frac{\partial C_+}{\partial t}\cdot v \cdot dx - \int v \cdot d\left(D_+ \frac{\partial C_+}{\partial x} + \frac{D_+ \cdot C_+}{k_B \cdot T}\frac{\partial \phi}{\partial x}\right) = 0.$$

$$\int \frac{\partial C_+}{\partial t}\cdot v \cdot dx - \left[\cancel{v\cdot\left(D_+ \frac{\partial C_+}{\partial x} + \frac{D_+ C_+}{k_B \cdot T}\frac{\partial \phi}{\partial x}\right)\Bigg|_{B_0}^{B_1}} - \int\left(D_+ \frac{\partial C_+}{\partial x} + \frac{D_+ C_+}{k_B \cdot T}\frac{\partial \phi}{\partial x}\right)\cdot v' \cdot dx\right] = 0.$$

no flux at boundaries.

Weak form:
$$\int \frac{\partial C_+}{\partial t}\cdot \underset{\substack{\downarrow \\ \text{test}(C_+)}}{v} \cdot dx + \int\left(D_+ \frac{\partial C_+}{\partial x} + \frac{D_+ \cdot C_+}{k_B \cdot T}\frac{\partial \phi}{\partial x}\right)\cdot v' \cdot dx = 0.$$

$\Downarrow$

$$0 = \int\left[-\frac{\partial C_+}{\partial t}\cdot v - \left(D_+ \frac{\partial C_+}{\partial x} + \frac{D_+ \cdot C_+}{k_B \cdot T}\cdot\frac{\partial \phi}{\partial x}\right)\cdot v'\right]\cdot dx \quad ☆$$

②:
Weak form:
$$\int \frac{\partial C_-}{\partial t}\cdot v \cdot dx + \int\left(D_- \frac{\partial C_-}{\partial x} - \frac{D_- \cdot C_-}{k_B \cdot T}\frac{\partial \phi}{\partial x}\right)\cdot v' \cdot dx = 0$$

$\Downarrow$

$$0 = \int\left[-\frac{\partial C_-}{\partial t}\cdot v - \left(D_- \frac{\partial C_-}{\partial x} - \frac{D_- \cdot C_-}{k_B \cdot T}\cdot\frac{\partial \phi}{\partial x}\right)\cdot v'\right]\cdot dx \quad ☆$$