Transient heat conduction in solids irradiated by a moving heat source

N.Bianco[1], O.Manca[2], S. Nardini[2] and S. Tamburrino[2]
[1]Dipartimento di Energetica, Termofluidodinamica applicata e Condizionamenti ambientali, Università degli Studi di Napoli Federico II, Piazzale Tecchio 80, 80125 Napoli, Italia
[2]Dipartimento di Ingegneria Aerospaziale e Meccanica, Seconda Università degli Studi di Napoli, via Roma 29, 81031 Aversa (CE), Italia

1. Abstract

Transient three-dimensional temperature distribution in a solid irradiated by a moving Gaussian laser beam was investigated numerically by means of COMSOL Multiphysics 3.2. Convection and radiaton from the work-piece surfaces as well as variable thermophysical properties are accounted for. The work-piece is considered infinite or semi-infinite along the motion direction.

2. Introduction

Moving and stationary heat sources are frequently employed in many manufacturing processes and contact surfaces. In recent years applications of localized heat sources have been related to the development of laser and electron beams in material processing, such as welding, cutting, heat treatment of metals and manufacturing of electronic components [1-2]. Analytical and numerical models for the prediction of the thermal fields induced by the stationary or moving heat sources are useful tools for studying the afore mentioned problems [2]. In some laser beam applications, such as surface heat treatment, the contribution of convective heat transfer must also be taken into account [3].

Quasi-steady state thermal fields induced by moving localized heart sources have been widely investigated [3,4], whereas further attention seems to be devoted to the analysis of temperature distribution in transient heat conduction.

The one-dimensional unsteady state temperature distribution in a moving semi-infinite solid subject to a pulsed Gaussian laser irradiation was investigated analytically by Modest and Abakians [5]. Shankar and Gnamamuthu [6] obtained a finite difference numerical solution to the three-dimensional transient heat conduction for a moving elliptical Gaussian heat source on a finite dimension solid. Rozzi et al. [7] carried out the experimental validation for a transient three-dimensional numerical model of the process by which a rotating silicon nitride work-piece is heated with a translating CO_2 laser beam, without material removal. In a companion paper Rozzi et al [8] used the afore mentioned transient three-dimensional numerical model to elucidate the effect of operating parameters on thermal conditions within the work-piece. Rozzi et al. [9, 10] extended the above referred numerical and experimental investigation to the transient three-dimensional heat transfer in a laser assisted machining of a rotating silicon nitride work-piece heated by a translating CO_2 laser and material removing by a cutting tool. Transient and steady state analytical solutions in a solid due to both stationary and moving plane heat sources of different shapes and heat intensity distributions were derived in [11], by using the Jaeger's heat source method. Yilbas et al. [12] presented a numerical study for the transient heating of a titanium work-piece irradiated by a pulsed laser beam, with an impinging turbulent nitrogen jet. Gutierrez and Araya [13] carried out the numerical simulation of the temperature distribution generated by a moving laser heat source, by the control volume approach. Radiation and convection effects were accounted for. Bianco et al. proposed two numerical models for two and three dimensional models in [14, 15] to evaluate transient conductive fields due to moving laser sources.

In this paper a three dimensional transient conductive fields are solved by COMSOL Multiphysics code. The investigated work-pieces are simple brick-type solids. A laser source with Gaussian distribution is considered moving with constant velocity along motion direction. The solid dimension along the motion direction is assumed to be infinite or semi-infinite and finite width and thickness are considered. Thermal properties are temperature dependent. Surface heat losses toward the ambient are taken into account. This paper extends the investigations given in [14, 15] to semi-infinite solids.

Keywords: Transient Heat Conduction, Laser Source, Manufacturing, Moving Sources

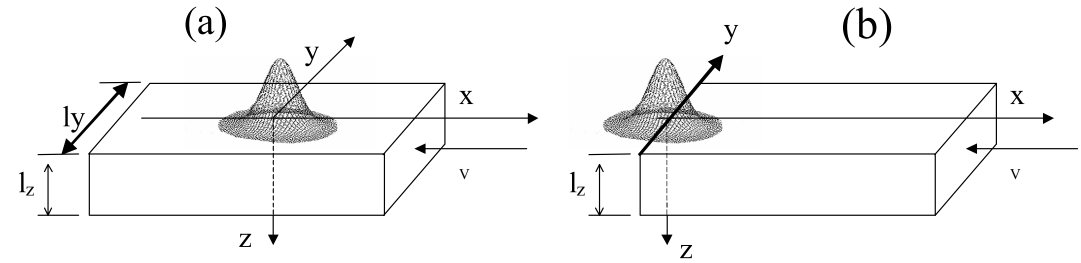


Figure 1: Sketch of the workpiece: (a) infinite; (b) semi-infinite

3. Mathematical Description

The mathematical formulation for the proposed model is reported in the following. A brick-type solid irradiated by a moving heat source is considered. The solid dimension along the motion direction is assumed to be infinite or semi-infinite, while finite thickness is assumed. A 2-D and a 3-D model are presented and the solid width is assumed infinite in the first case and finite in the second one. Radiative and convective heat losses are taken into account. The thermophysical properties of the material are assumed to be temperature dependent, except the density. The conductive model is assumed to be transient.

A sketch of the investigated configuration is reported in Fig. 1. If a coordinate system fixed to the heat source is chosen, according to the moving heat source theory [16], a mathematical statement of the three dimensional thermal conductive problem is:

$$\frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right)=\rho c\left(\frac{\partial T}{\partial \theta}-v\frac{\partial T}{\partial x}\right) \quad (1)$$

for $0 \le |x| < +\infty$ or $0 \le x$, $0 \le |y| \le l_y/2$, $0 \le z \le l_z$, $\theta > 0$.

The boundary and initial conditions are reported in the following:

$$T(x \to \pm\infty, y, z, \theta) = T_{in} \text{ or } T(x \to +\infty, y, z, \theta) = T_{in} \quad (1a)$$

for $0 \le |y| \le l_y/2$; $0 \le z \le l_z$; $\theta > 0$

$$\frac{\partial T(x, 0, z, \theta)}{\partial y} = 0 \quad (1b)$$

for $0 \le |x| < +\infty$ or $0 \le x$; $0 \le z \le l_z$; $\theta > 0$

$$-k\frac{\partial T(x, \pm l_y/2, z, \theta)}{\partial y} = h_l(x,y,\theta) \times\left[T(x, \pm l_y/2, z, \theta) - T_f\right] + \varepsilon_l\sigma\left[T^4(x, \pm l_y/2, z, \theta) - T_a^4\right] \quad (1c)$$

$0 \le |x| < +\infty$ or $0 \le x$; $0 \le z \le l_z$; $\theta > 0$

$$-k\frac{\partial T(x,y,0,\theta)}{\partial z} = q(x,y) - h_u(x,y,\theta)\left[T(x,y,0,\theta) - T_f\right] - \varepsilon_u\sigma\left[T^4(x,y,0,\theta) - T_a^4\right] \quad (1d)$$

for $0 \le |x| < +\infty$ or $0 \le x$; $0 \le |y| \le l_y/2$; $\theta > 0$

$$-k\frac{\partial T(x,y,l_z,\theta)}{\partial z} = h_b(x,y) \times\left[T(x,y,l_z,\theta) - T_f\right] + \varepsilon_b\sigma\left[T^4(x,y,l_z,\theta) - T_a^4\right] \quad (1e)$$

for $0 \le |x| < +\infty$ or $0 \le x$; $0 \le |y| \le l_y/2$; $\theta > 0$

$$T(x,y,z,0) = T_{in} \quad (1f)$$

for $0 \le |x| < +\infty$ or $0 \le x$; $0 \le |y| \le l_y/2$; $0 \le z \le l_z$

where the absorbed heat flux q(x,y) is:

$$q(x,y) = q_0 \exp\left[-\left(\frac{x^2+y^2}{r_G^2}\right)\right] \quad (2)$$

The solid is assumed to be infinite or semi-infinite along the motion direction (Eq. (1a)) and the problem is considered geometrically and thermally symmetric along the *y* direction (Eq. (1b)). Moreover convective and radiative heat losses on the lateral, upper and bottom surfaces are considered (Eqs. (1c-1e)).

The mathematical statement of the two dimensional problem can be obtained from Eqs. (1) by neglecting the temperature variations along the y direction.

The 2-D and 3-D conductive models are solved by means of the COMSOL Multiphysics 3.2 code.

For the thermal model *"Heat Transfer Module"* and "*Transient analysis"* in *"General Heat Transfer"* window have been chosen in order to solve the heat conduction equation.

Several different grid distributions have been tested to ensure that the calculated results are grid independent. Maximum temperature differences of the fields is less than 0.1 precent by doubling the mesh nodes. The grid mesh is unstructured.

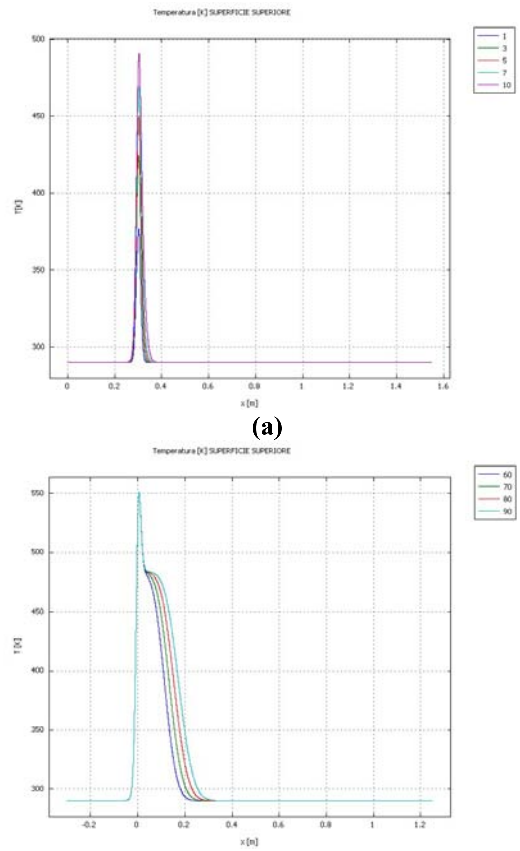
4. Results and Discussion

Results are presented for two cases: a) an infinite workpiece along the motion direction with constant heat transfer coefficients on the upper (h_u) and bottom (h_b) surfaces, b) a semi-infinite workpiece along the motion direction with negligible convective and radiative heat losses.

The spot radius r_G, the width and the height of the workpiece are equal to 0.0125 m. Temperature dependent thermophysical properties are taken from the Metals Handbook [17] for a 10-18 steel material and they are reported in Table 1. The absorbed laser heat flux is equal to 120 W/cm^2. For the case with constant heat transfer coefficients, these latter are assumed to be equal to 30 and 10 W/m^{2}K, for the upper and bottom surfaces, respectively. In this case also radiation heat losses are considered and the upper and lower surfaces are considered grey with a 0.8 wall emissivity. The workpiece velocity is equal to 2.0 10^{-3} m/s. The ambient temperature is assumed equal to 290 K.

Temperature profiles in the solid as a function of the coordinate along the motion direction, in the centerline (y=0) of the work-piece upper surface (z=0), are reported in Fig. 2. It shows that at the beginning of the heating process temperature profiles are nearly symmetrical with reference to x=0, Fig. 2a, whereas the longer the time the larger the shift of the x coordinate where the maximum surface temperature is reached. The figure points out no further increase in maximum temperature after about 60 s (Fig. 2b), thus denoting the attainment of a quasi-steady state condition, with constant surface temperatures in the proximity of the maximum value and increasing temperatures along the motion direction (Fig. 2b).

In fact, for higher time values temperature profiles along the x direction show, in Fig. 3a,



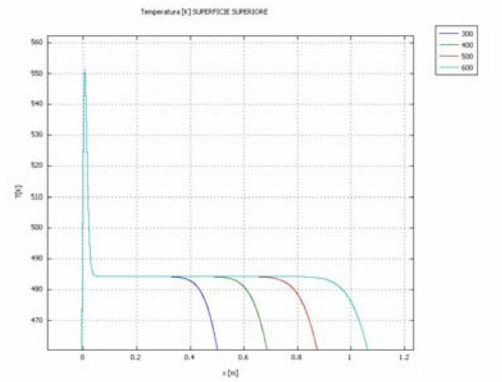
(a)

(b)

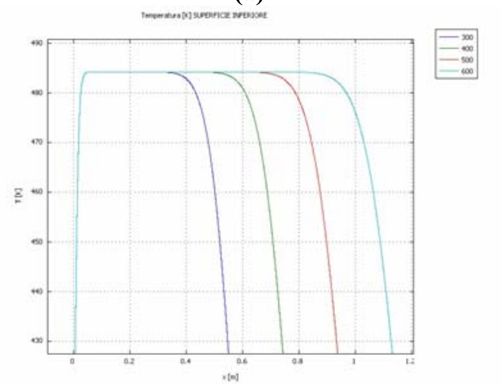
Figure 2: 3-D temperature profiles vs x, for y=0, z=0, at different time values: a) t=1-10 s, b) t=60-90 s.

the same dependence with the same maximum values. The decreasing part of the profiles are similar. Also along the lower surface, in Fig. 3b, for these time values temperature profiles show a similar function. It is interesting to observe that along this profile temperature values do not reach a maximum.

In Fig. 4 temperature fields are reported for 1 s, 50 s and 90 s. In the first thermal field it is observed that the diffusion does not penetrate completely inside the work-piece. In fact, the thermal disturbance, as shown in Fig. 4a, does not reach the lower adiabatic surface and temperature of this surface is still the initial one. Upward and downward the spot the solid is not thermally affected. After 50 s, Fig. 4b, the downward zone is thermally affected and it presents temperature values significantly higher than the initial one. At this time the lower surface temperatures are increased. The highest temperature differences are observed close to the spot zone whereas downstream temperature decreases along the motion direction but is almost uniform in the y-z planes. Similar trends are observed in Fig. 4c, for t=90s, with a greater extension of the thermal disturb downward the hot spot.



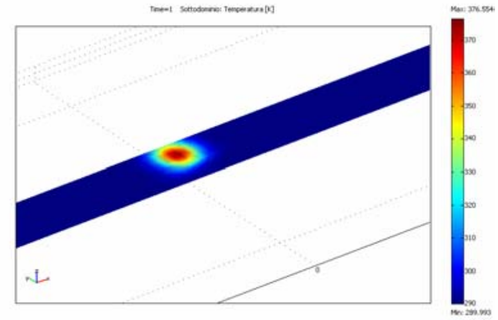
(a)



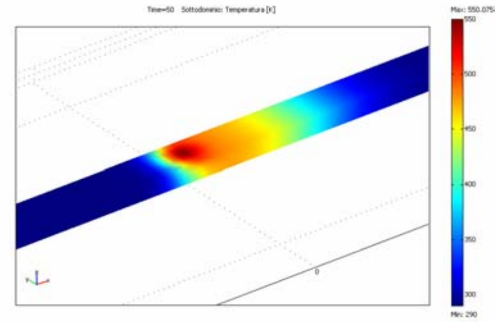
(b)

Figure 3: 3-D temperature profiles vs x coordinate, at different time values, for y=0 and a) z=0, b) z= l_z.

Temperature profiles in the solid for the 2-D infinite solid case as a function of the coordinate along the motion direction on the upper and lower surfaces, for several times, are reported in Figs. 5 and 6. It can be observed that the temperature profiles are nearly symmetrical with reference to x=0 at the beginning of the heating. As the time increases, the x coordinate at which the maximum temperature is reached increases and the temperature profiles are not symmetrical, but larger temperature values are obtained for x greater than zero. The maximum temperature value increases until a time equal to about 3.5 s. After this time no further increase can be observed and a local quasi-steady state condition is reached. The maximum temperature values at the quasi-steady state conditions is equal to 368 K. Figure 6a, related to the upper surface, points out that a decreasing temperature profile along the motion direction is obtained. This is due to the heat transfer coefficient imposed on the upper surface. It is worth observing that the slope of this curve is constant. Figure 6b, related to the lower surface, points out that the maximum temperature on the lower surface is lower than that reached on the upper surface. By comparing figures 6a and 6b one can notice that temperature values on the upper and lower



(a)



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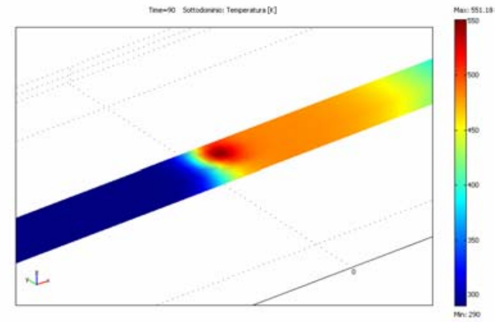


(c)

Figure 4: 3-D temperature fields at different time values: a) t=1.0 s, b) 50 s and c) 90 s for *y*=0, *z*=0.

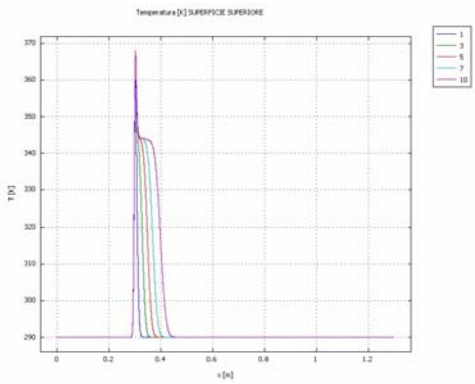
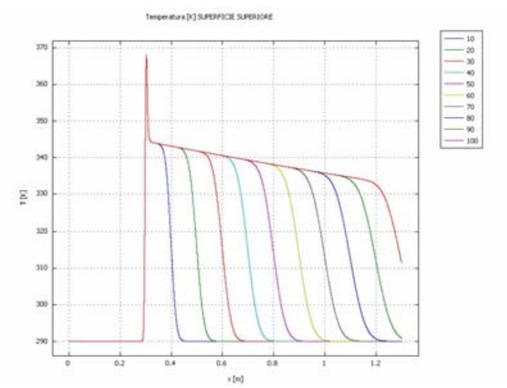
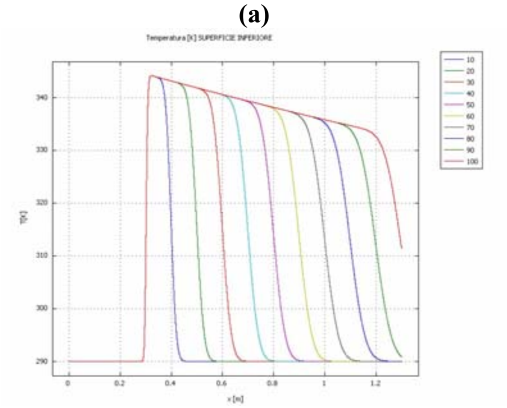


Figure 5: 2-D temperature profiles along x, at different time values.



(a)



(b)

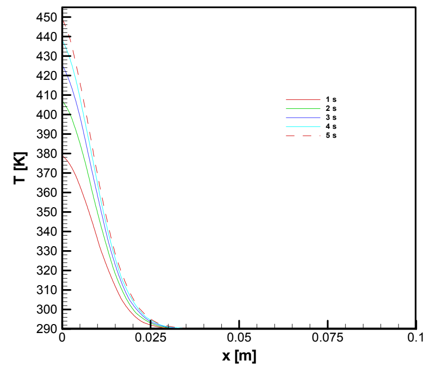
Figure 6: 2-D temperature profiles along x, at different time values: a) z=0, b) z=l_z.

surfaces are practically coincident for x greater than r_G. It can be observed that the temperature profiles along the motion direction on the lower surface does not present a local maximum value.

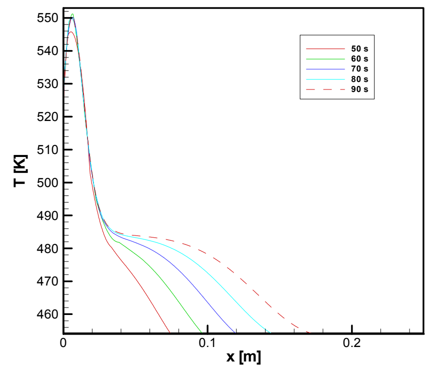
In Fig. 7 temperature profiles along x for 3-D semi-infinite model for different time values are reported. For lower time values, in Fig. 7a, the maximum temperature is attained at x=0 and increasing the x value temperature decreases significantly reaching its initial value at about x=0.03 m. For larger time values, in Fig. 7b, the maximum temperature is attained at x>0. Some differences are observed between the temperature profiles at different times, temperature profile is developing respect to the time. In Fig. 7c a quasi steady state is reached and the temperature profile in the maximum value zone is constant whereas it changes for x>0.075 m increasing its values.

Temperature profiles in the solid for the 2-D semi-infinite solid case as a function of the coordinate along the motion direction on the upper surface, for several times, are reported in Fig. 8. Fig. 8a shows the temperature rise at the beginning of the transient regime (θ<5 s). It can be observed that until this time the maximum temperature is attained at x=0 and its value is equal at about 440 K for θ=5 s. In this case the quasi-steady state conditions are reached for θ=90s, as it is shown in Fig. 8b. In this case the maximum temperature value is equal to 556 K and it is attained at x=0.017 m. It is interesting to observe that in this case the quasi-steady state conditions are reached much later than the 2-D case with infinite workpiece.

The thermal fields within the solid are reported in Figs 9a and 9b for θ=10 and 90 s, respectively. These figures clearly show how the thermal field develop within the structure, and, particularly, Fig. 9b points out that at 90s, when the steady state condition is reached, a uniform asymptotic temperature value is attained in the zone where the heat source is already passed. This value is equal at about 500 K.



(a)



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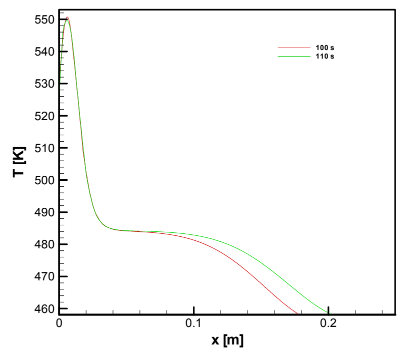
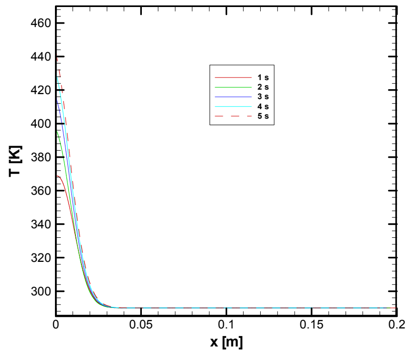
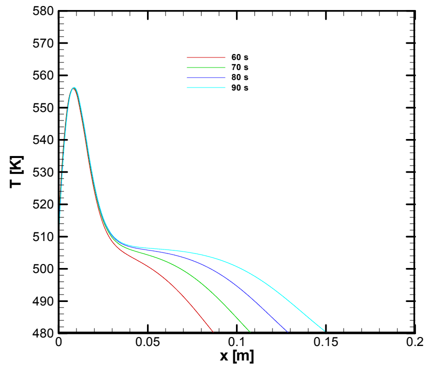


(c)

Figure 7: 3-D temperature profiles vs x, for *y*=0, *z*=0, at different time values: a) t=1-6 s, b) t=60-90 s, c) t=100-110 s.

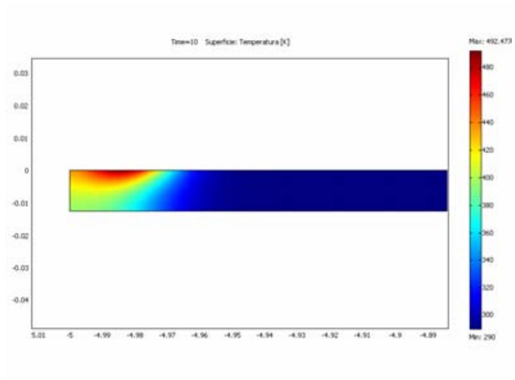


(a)

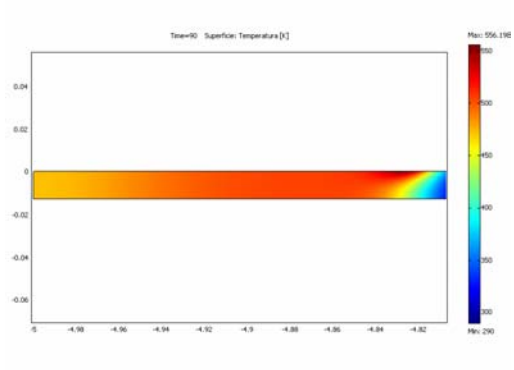


(b)

Figure 8: 2-D temperature profiles along x, at different time values: a) t=1-5 s, b) t=60-90 s.



(a)



(b)

Figure 9: 2-D temperature fields at different time values: a) t=10 s, b) 50 s and c) 90 s.

5. Conclusions

The present numerical investigation allowed to estimate two and three dimensional transient heat conductive fields in infinite and semi-infinite metallic solids due to a moving laser source. Temperature profiles and fields showed that a quasi steady state is reached. Maximum temperature value are attained at a time smaller than the quasi steady state one. Surface heat transfer strongly affected the temperature distributions in the considered solids.

6. Nomenclature

c specific heat (J kg^{-1} K^{-1})
h convective heat transfer coefficient (W m^{-2} K^{-1})
k thermal conductivity (W m^{-1} K^{-1})
l length (m)
q absorbed heat flux (W m^{-2})
r radius (m)
T temperature (K)
v velocity of the work-piece (m s^{-1})
x,y,z Cartesian coordinates (m)

6.1 Greek Letters

α thermal diffusivity (m^2 s^{-1})
ε emissivity;
θ time (s)
ρ density (kg m^{-3})

6.2 Subscripts

a ambient
b bottom surface
G Gaussian beam
in initial for $x \to +\infty$
u upper surface
x,y,z along axes.

Table 1: Thermophycal properties of the employed material

	k [W/mK]	ρ [kg m^{-3}]	c_p [J/kg K]
10-18 Steel	53.7-0.03714 (T-273.15)	7806	500.0 + 0.40 (T-273.15)

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