A Numerical Comparison of Dielectric based Measurement of Atmospheric Ice Using Comsol

U. N. Mughal and M. S. Virk

Narvik University College,

Narvik, Norway

Email : unm@hin.no

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Outline

- Atmospheric Icing
- Ice Detection Techniques
- Capacitive or dielectric sensing technique
- Debye Relations and some modifications
- Experimental results of dielectric constant for ice & snow
- Comsol Model
- Comsol Results
- Room for further research
- Conclusion.

Atmospheric Ice

- It occurs when water droplets freeze on object they contact.
- Liquid below 0^0C is called SUPERCOOLED which causes icing problem.
- Below -20^0C icing is rare
- Below -42^0C icing is impossible

Types of Atmospheric Icing

- **In-Cloud Icing**
  - **Rime**
    - **Hard Rime**
    - **Soft Rime**
  - **Glaze**
- **Precipitation Icing**
  - **Wet Snow**
  - **Freezing Rain**
- **Frost**

Common methods of ice detection and measurement

- ***Indirect methods of ice detection***
  - ✓ Involve measuring weather conditions, or measuring the variables that cause icing or variables that correlate with the occurrence of icing, such as cloud height and visibility
  - ✓ Empirical or deterministic models are then used to determine when icing is occurring.
- ***Direct methods of ice detection***

Based on the principle of detecting property changes caused by ice accretion. Examples of such properties include :

- ✓ Changes of a vibrating frequency
- ✓ Changes in electrical properties
- ✓ The load of ice (ISO 12494) .
- ✓ The growth rate of ice
- ✓ Optically

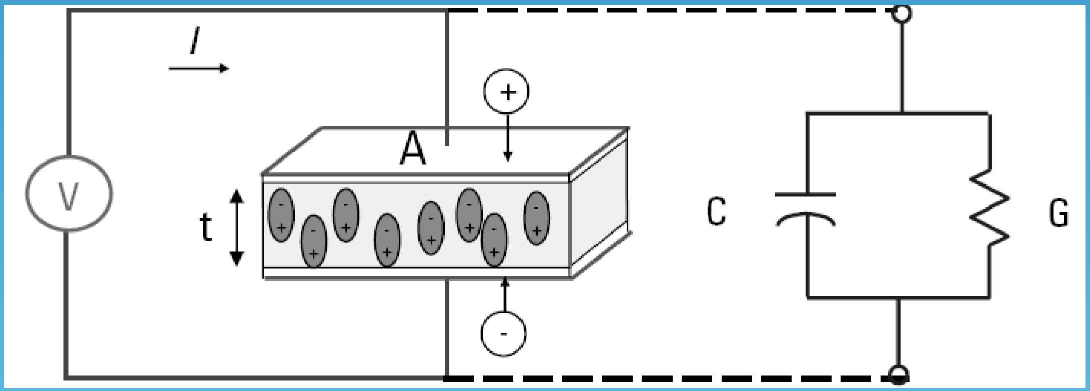
Capacitive or Dielectric Sensing

If I is the total current across the material, I_c is the charging current and I_l be the loss current then we can write,

$$I = I_c + I_l = V\left(j\omega C_0 \varepsilon_0 \varepsilon_r^{'} + G\right)$$

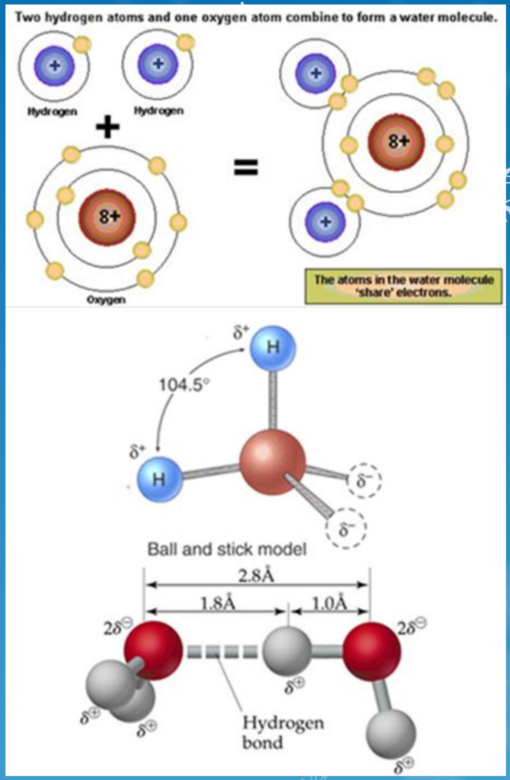
- Where ω is the supply frequency, C_0=A/t is the capacitance, $\varepsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$ is the permittivity of free space, and G is the conductance. Hence if we assume $G = \omega C_0 \varepsilon_0 \varepsilon_r^{''}$ then,

$$I = V(j\omega C_0 \varepsilon_0)\left(\varepsilon_r^{'} - j\varepsilon_r^{''}\right) = V(j\omega C_0 \varepsilon_0)(\varepsilon_r)$$



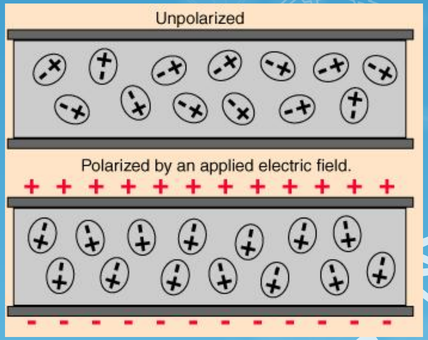
Relaxation time constant

- In the physical sciences, **relaxation** usually means the return of a perturbed system into equilibrium. Each relaxation process can be characterized by a **relaxation time** τ. The simplest theoretical description of relaxation as function of time *t* is an exponential law exp(-*t*/τ).



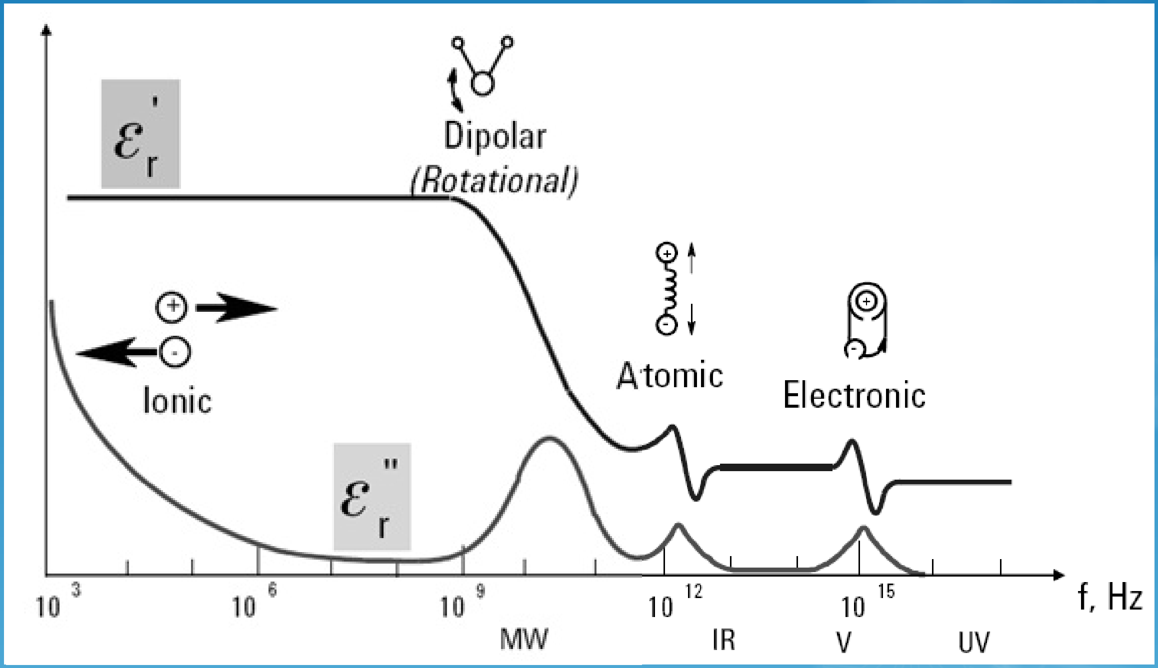
Dielectric relaxation time

- In DIELECTRIC materials, the dielectric polarization *P* depends on the electric field *E*. If *E* changes, *P(t)* reacts: the polarization *relaxes* towards a new equilibrium.



Dielectric Constant

- Dielectric mechanisms
  - Dipolar polarization
  - Atomic polarization
  - Electronic polarization
- Dipolar Polarization
  - Unequal electron sharing
  - Friction accompanying orientation
- Resonance Effect
- Relaxation Effect



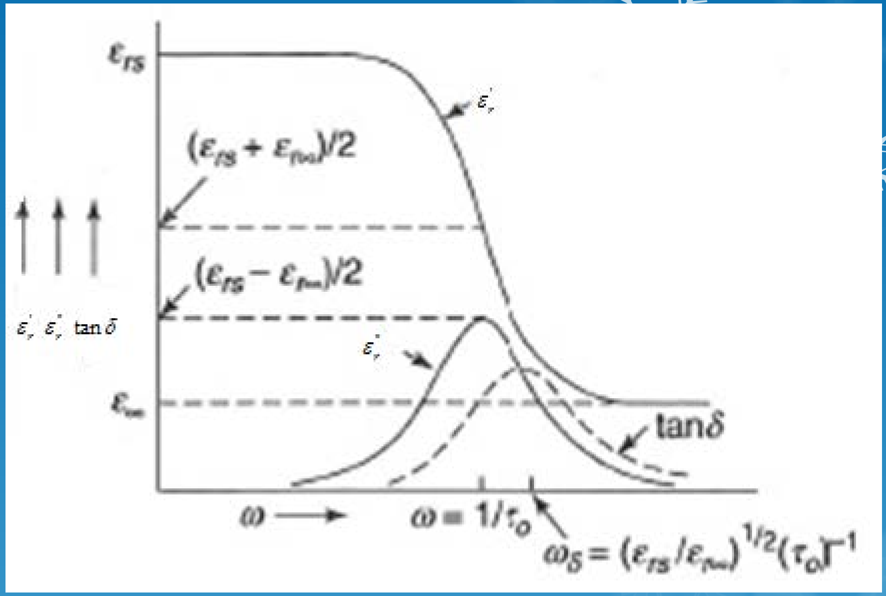
Debye Relations for the Parameters of Interest

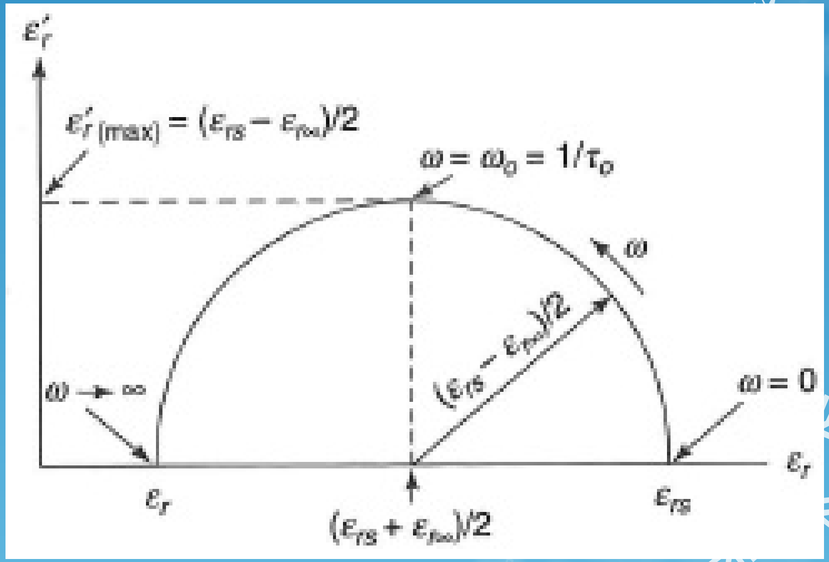
- Debye relations provides a mathematical insight to find ε_r' , ε_r'' and $\tan\delta\ (\varepsilon_r''/\varepsilon_r')$ as a function of excitation frequency and relaxation time.
- For single relaxation time without dc conductivity we have,

$$\varepsilon_r' = \varepsilon_{r\infty} + \frac{\varepsilon_{rs} + \varepsilon_{r\infty}}{1 + \omega^2\tau_0^2}$$

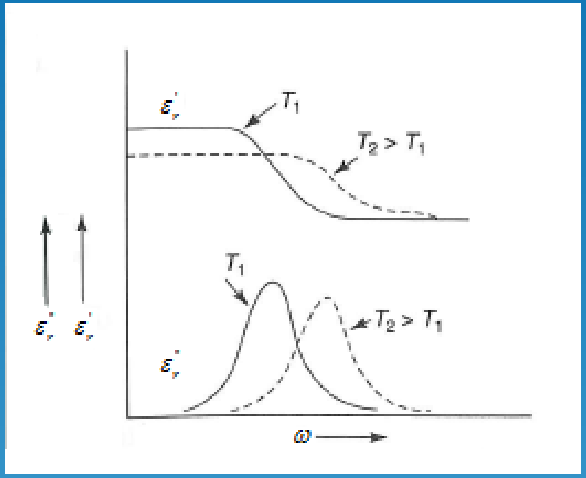
$$\varepsilon_r'' = \frac{(\varepsilon_{rs} - \varepsilon_{r\infty})\omega\tau_0}{1 + \omega^2\tau_0^2}$$

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'} = \frac{(\varepsilon_{rs} - \varepsilon_{r\infty})\omega\tau_0}{\varepsilon_{rs} + \varepsilon_{r\infty}\omega^2\tau_0^2}$$





Dielectric Variation with temperature



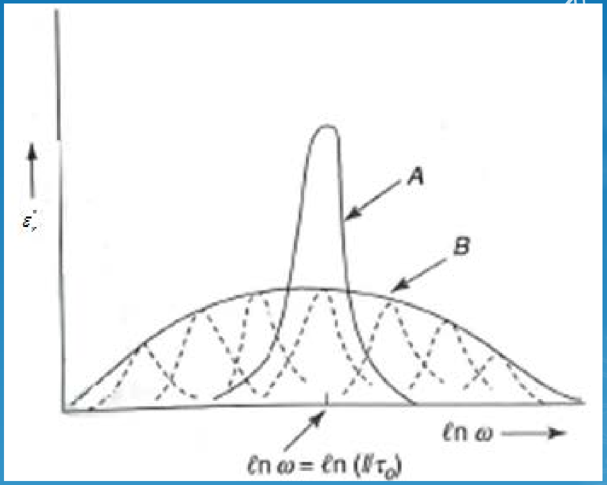
The ε_r' and ε_r'' functions of ω for fixed temperatures T_1 and T_2 with $T_2 > T_1$ REF 1

$$\tau_0 = \tau_h e^{\frac{H}{kT}}$$

$$\varepsilon_r' = \varepsilon_{r\infty}^T + \frac{\varepsilon_{rs}^T + \varepsilon_{r\infty}^T}{1 + \omega^2 \tau_h^2 e^{\frac{2H}{kT}}}$$

$$\varepsilon_r'' = \frac{(\varepsilon_{rs}^T - \varepsilon_{r\infty}^T)\omega\tau_h e^{\frac{H}{kT}}}{1 + \omega^2 \tau_h^2 e^{\frac{2H}{kT}}}$$

Dielectric Variation with relaxation times



The ε_r''-$\ln\omega$ curves (A) involving one relaxation time and (B) involving a set of relaxation times, each having Debye type loss peak. REF 2

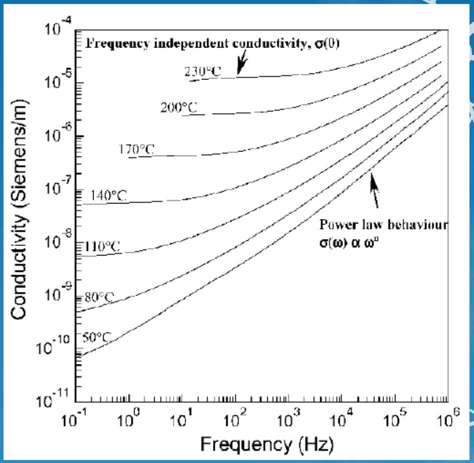
$$\varepsilon''_{r\max} = \lambda\left(\varepsilon_{rs} - \varepsilon_{r\infty}\right)$$

$$\varepsilon_r'' = \frac{\lambda\left(\varepsilon_{rs} - \varepsilon_{r\infty}\right)}{\left(\omega\tau\right)^{\lambda} + \left(\omega\tau\right)^{-\lambda}}$$

Dielectric Variation with conductivity

In the recent literature REF 9, the conductivity equation is given as,

$$\sigma(\omega) = \sigma(0) + A\omega^n$$

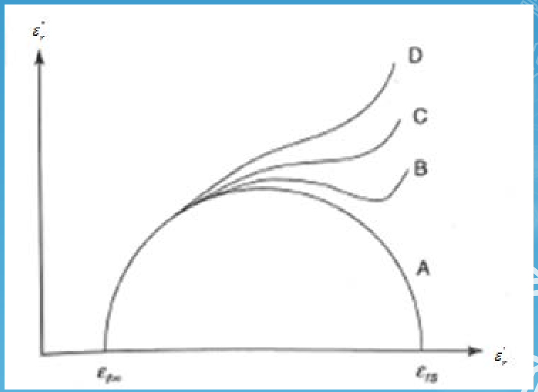


AC Conductivity of doped zirconia at diff. Temp.

In this $\sigma(0)$ and A are assumed to be empirical constants however the below given curves clearly reflect that they both need to be a function of temperature. I am presently working to find this relation which I will incorporate in the below mentioned equations.

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' - j\frac{\sigma}{\omega}$$

$$\varepsilon_r'' = \frac{(\varepsilon_{rs} - \varepsilon_{r\infty})\omega\tau_0}{1 + \omega^2\tau_0^2} + \frac{\sigma}{\omega\varepsilon_0}$$



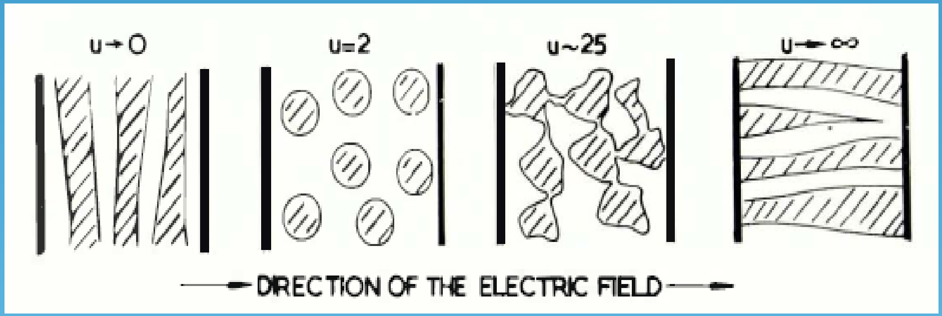
The effect of dc conductivity on $\varepsilon_r' - \varepsilon_r''$ arc. (A) $\sigma = 0$ (B) $\sigma = \sigma_1 > 0$ (C) $\sigma_2 > \sigma_1$ (D) $\sigma_3 > \sigma_2$

Dielectric mixture formulae

For a mixture of dielectric material we can use the Wiener relation which is given as,

$$\frac{\varepsilon_m - 1}{\varepsilon_m + u} = (\rho)\frac{\varepsilon_1 - 1}{\varepsilon_1 + u} + (1-\rho)\frac{\varepsilon_2 - 1}{\varepsilon_2 + u}$$

'ε_m' is the dielectric constant of the mixture,
'ε_1' and 'ε_2' are the dielectric constants of two materials,
'ρ' is the proportion of the total volume occupied by medium '1',
u is the Formzahl number



$$\varepsilon_d^{'} = \frac{\left(\rho u - u - \rho u \varepsilon_{ice}^{'} - \varepsilon_{ice}^{'}\right)\left(\rho \varepsilon_{ice}^{'} - \rho - u - \varepsilon_{ice}^{'}\right) - \left(\rho \varepsilon_{ice}^{''} - \varepsilon_{ice}^{''}\right)\left(\rho u \varepsilon_{ice}^{''} + \varepsilon_{ice}^{''}\right)}{\left(\rho \varepsilon_{ice}^{'} - \rho - u - \varepsilon_{ice}^{'}\right)^2 + \left(\rho \varepsilon_{ice}^{''} + \varepsilon_{ice}^{''}\right)^2}$$

$$\varepsilon_d^{''} = \frac{-\left(\rho u - u - \rho u \varepsilon_{ice}^{'} - \varepsilon_{ice}^{'}\right)\left(\rho \varepsilon_{ice}^{''} - \varepsilon_{ice}^{''}\right) - \left(\rho \varepsilon_{ice}^{'} - \rho - u - \varepsilon_{ice}^{'}\right)\left(\rho u \varepsilon_{ice}^{''} + \varepsilon_{ice}^{''}\right)}{\left(\rho \varepsilon_{ice}^{'} - \rho - u - \varepsilon_{ice}^{'}\right)^2 + \left(\rho \varepsilon_{ice}^{''} - \varepsilon_{ice}^{''}\right)^2}$$

Electrostatics (Used Phyics)

Why Electrostatics measurements can be done on ice?

- Electrostatics include high-voltage apparatus, electronic devices, and capacitors.
- The term "statics" is not to be interpreted literally but rather that the observation time or time scale at which the applied excitation changes is short compared to the charge relaxation time and that the electromagnetic wavelength and skin depth are very large compared to the size of the domain of interest.

Constitutive Relations

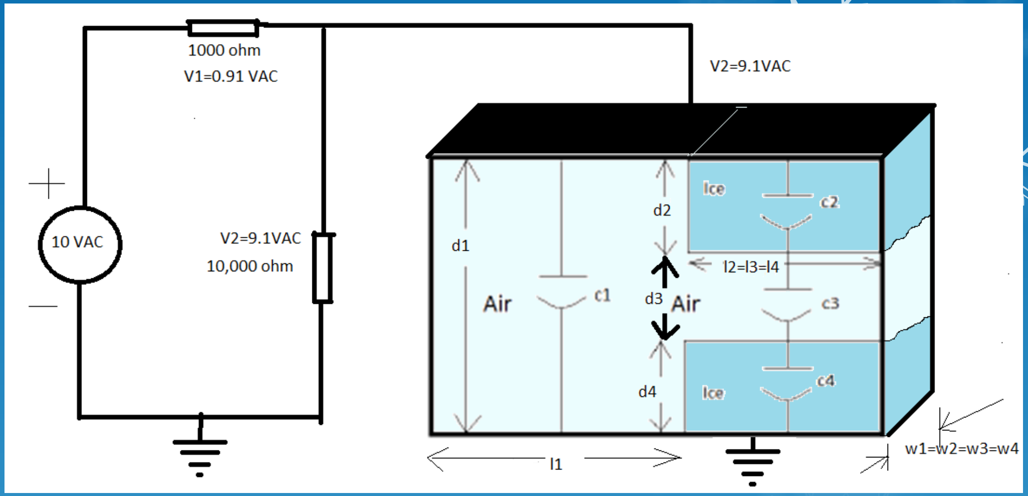
$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \quad \Rightarrow \quad \mathbf{D} = \varepsilon_0 \left(1 + \lambda_e\right) \mathbf{E} \quad \Rightarrow \quad \mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E}$$

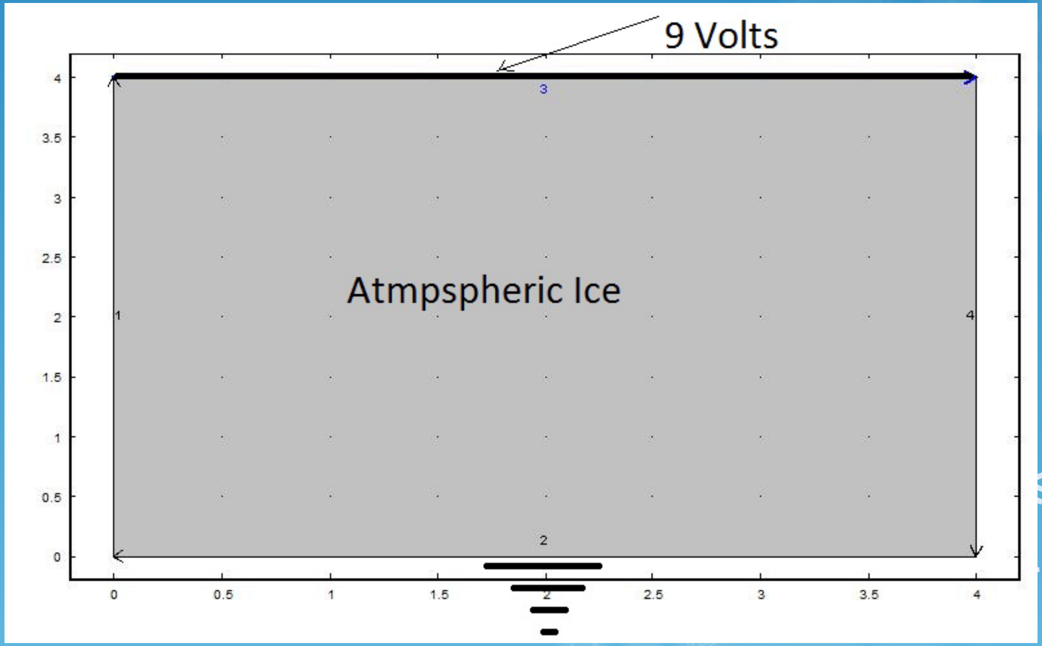
$$\&$$

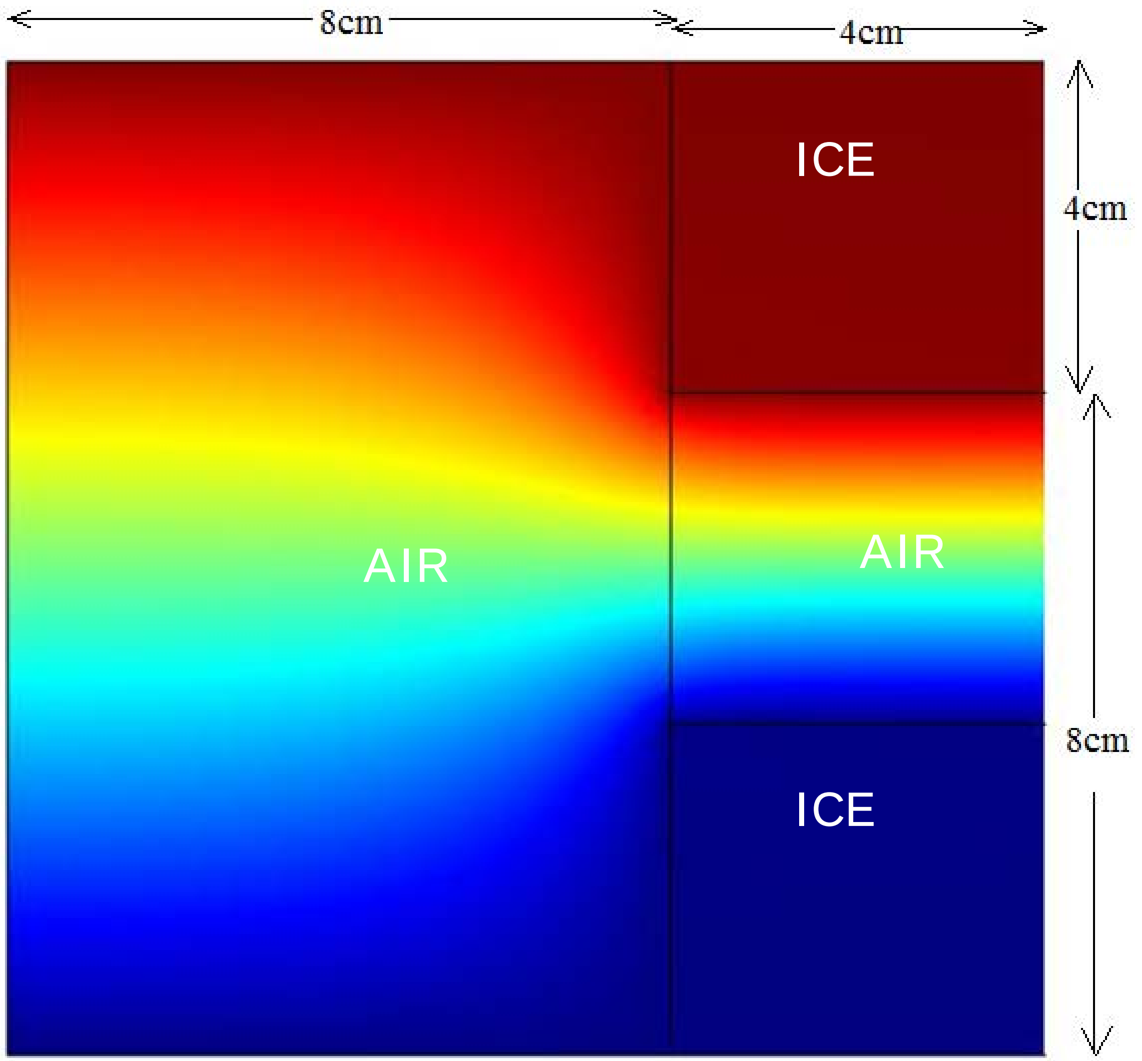
$$\mathbf{J} = \sigma \mathbf{E}$$

Comsol Model

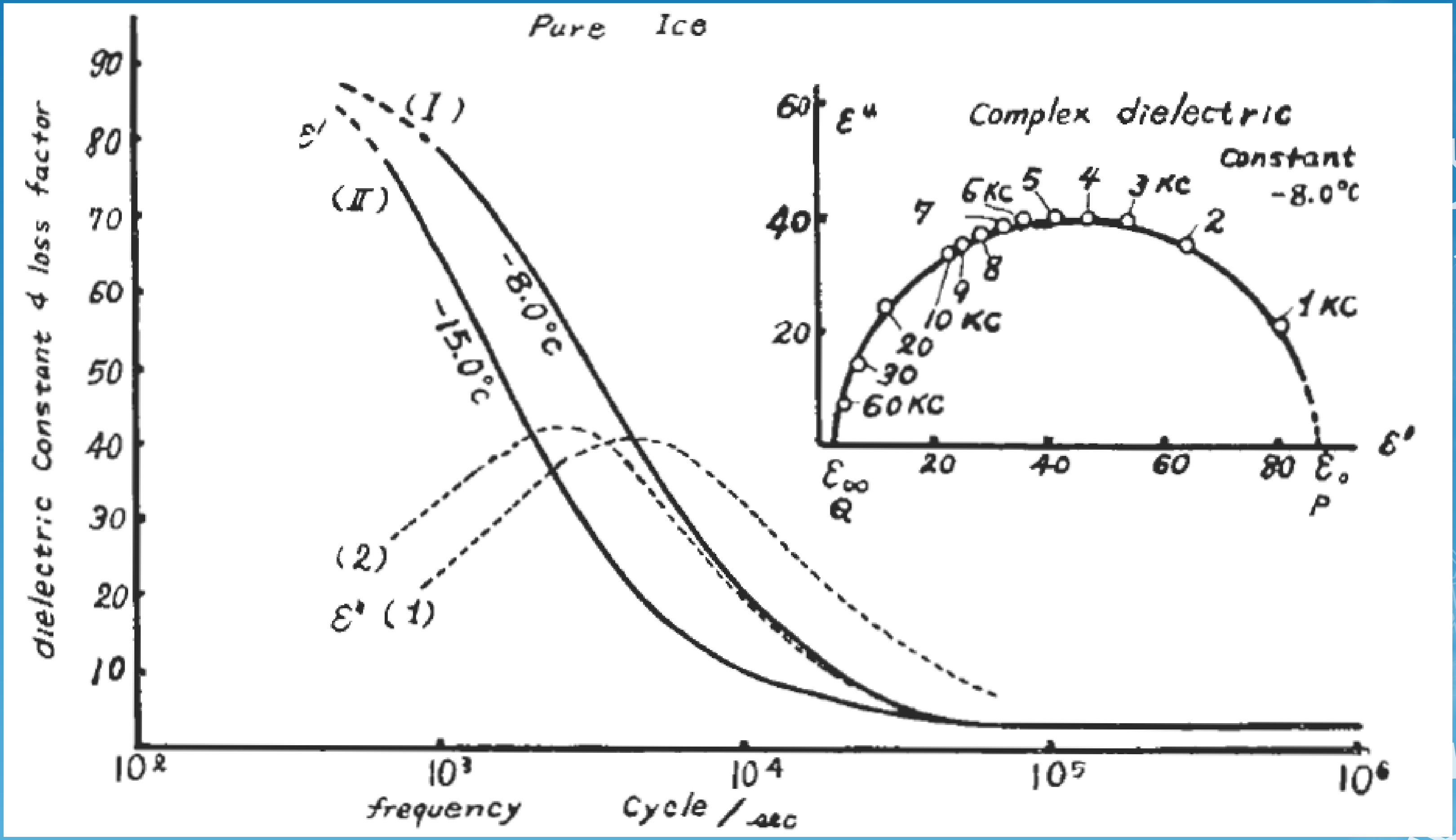
$L_1 = 4[cm]$
$W_1 = 4[cm]$
$\varepsilon_0 = 8.85e^{-12}$ [F/m]
$\varepsilon_{0\_ice\_freq} = 97.5$
$\varepsilon_{\infty\_ice\_freq} = 3.5$
$\tau_{0\_ice} = 5.3e^{-16}$[s]
ω = 0:0.1:100000 [Hz]
$\lambda = 1$
Activation Energy, H = 0.57[eV]
$k = 1.38e^{-23}$ [J/K]
ρ_d = 0.095, 0.132, 0.254
u = 40
temp = 272, 268, 258 [K]






8cm
4cm
ICE
4cm
AIR
AIR
8cm
ICE

Dielectric constant of pure ice (Experimental results)

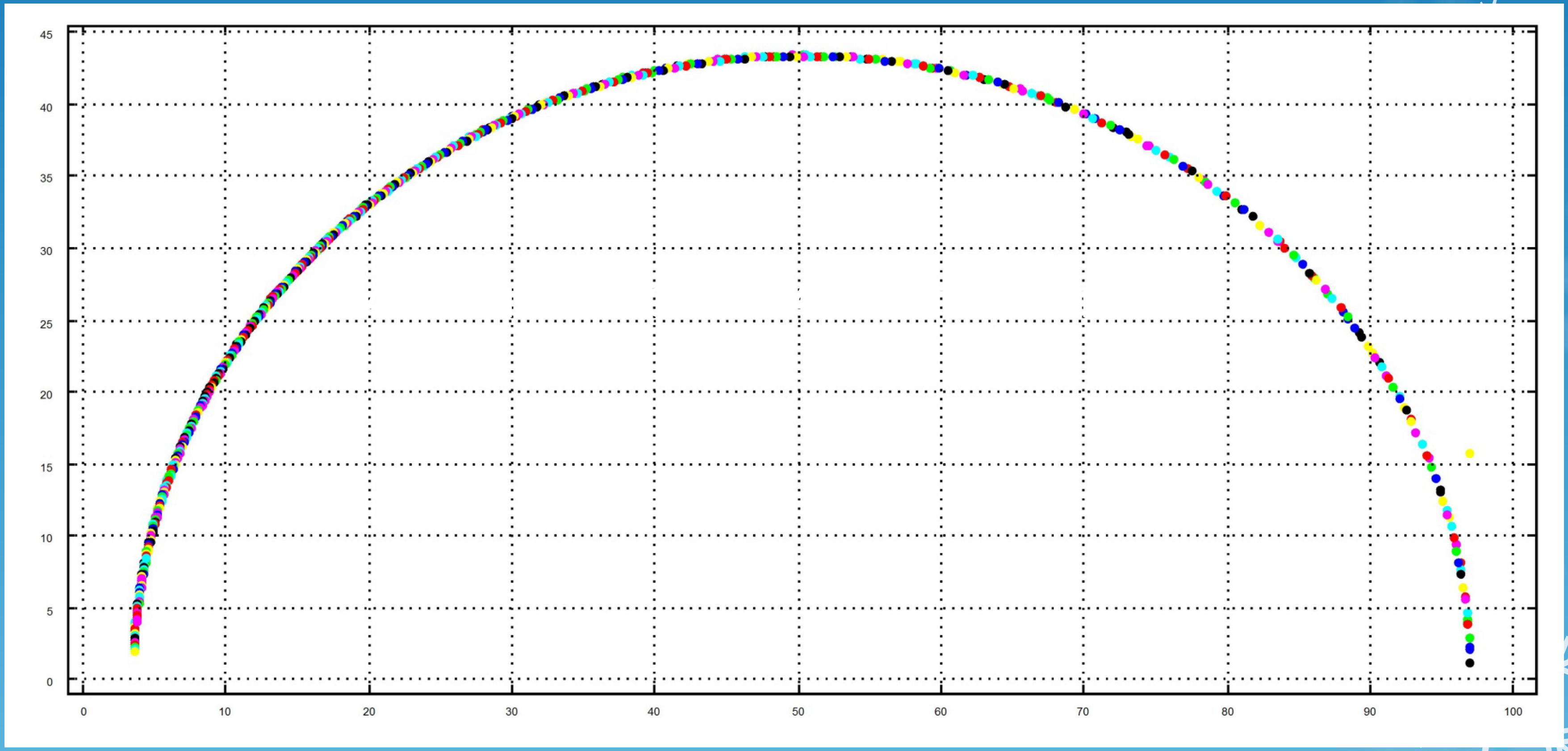


Dispersion of ε_r' and ε_r'' of pure ice REF 3

Comsol Results For Pure Ice

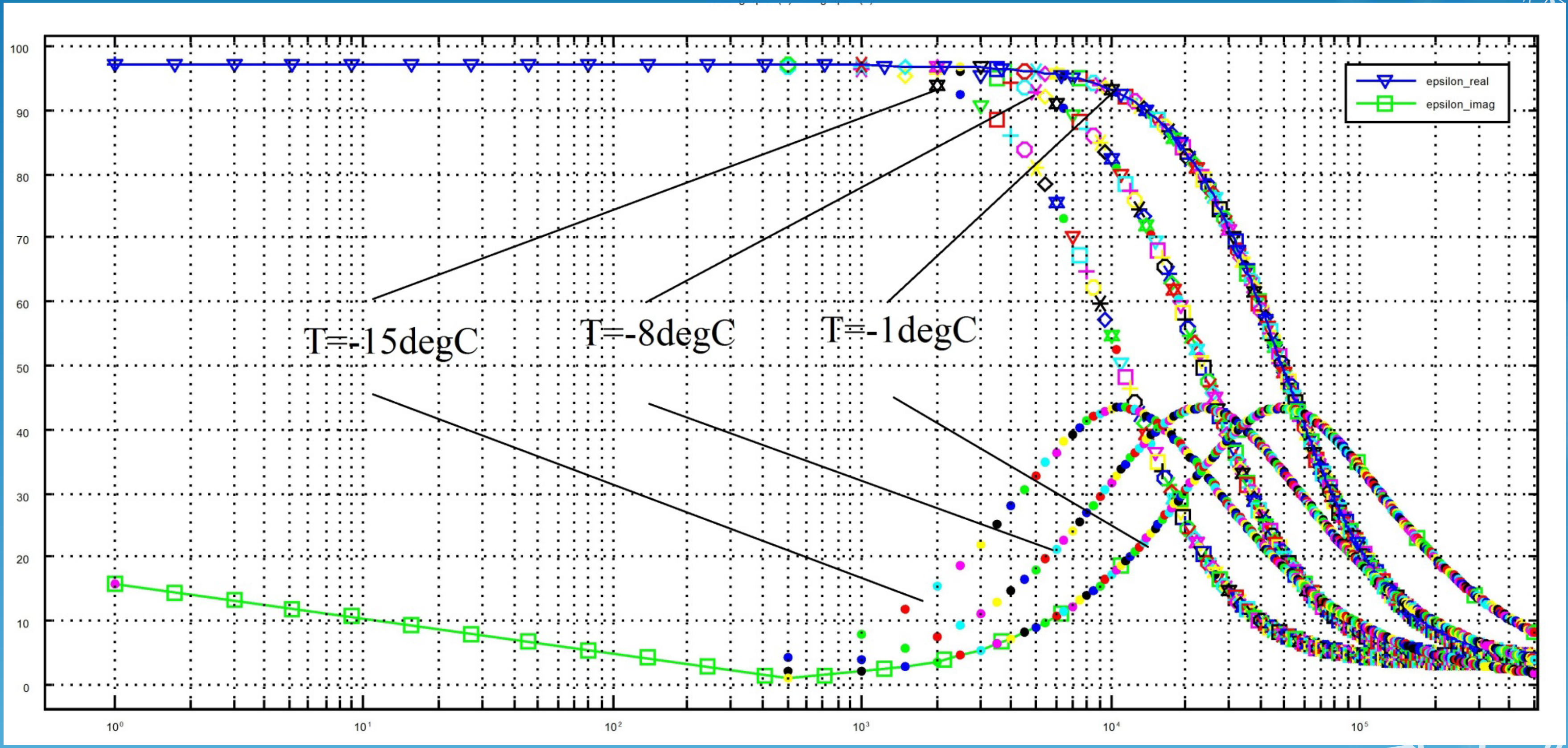
ARGAND DIAGRAM

ε_r''



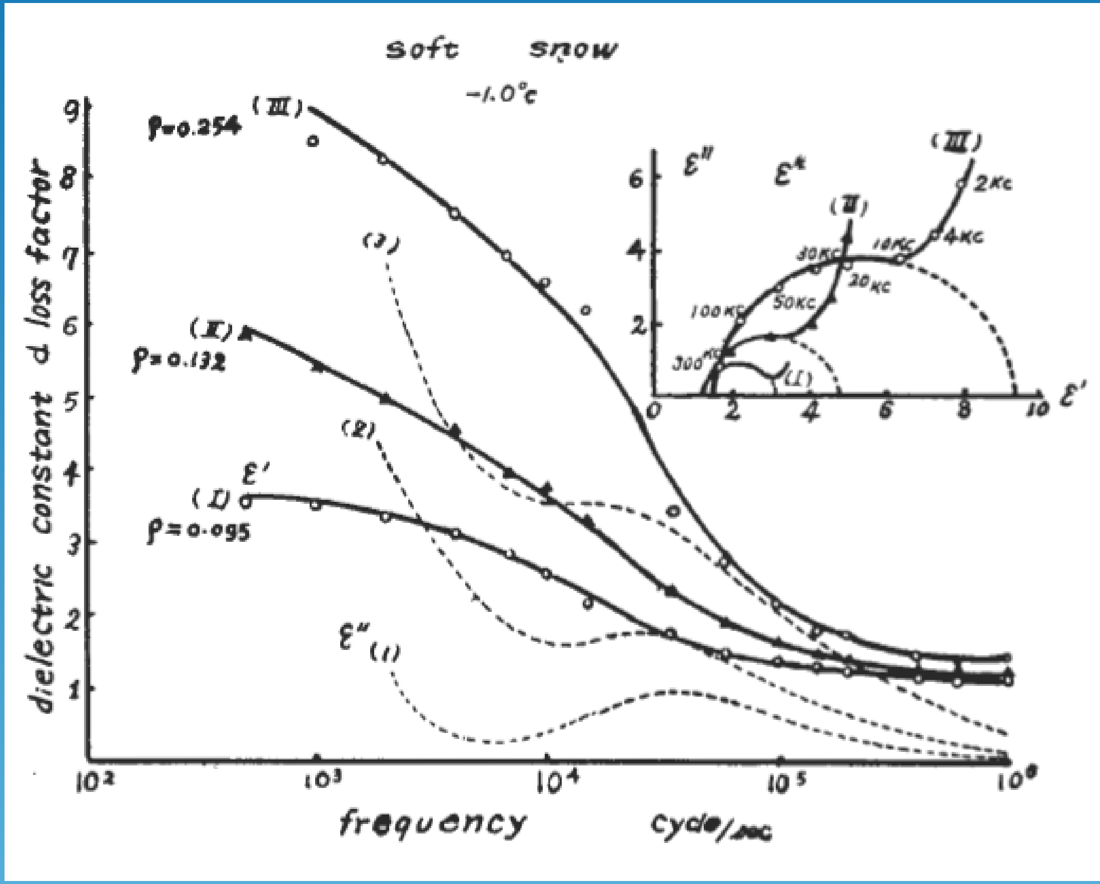
ε_r'

ε_r'
&
ε_r''



Log(ω)

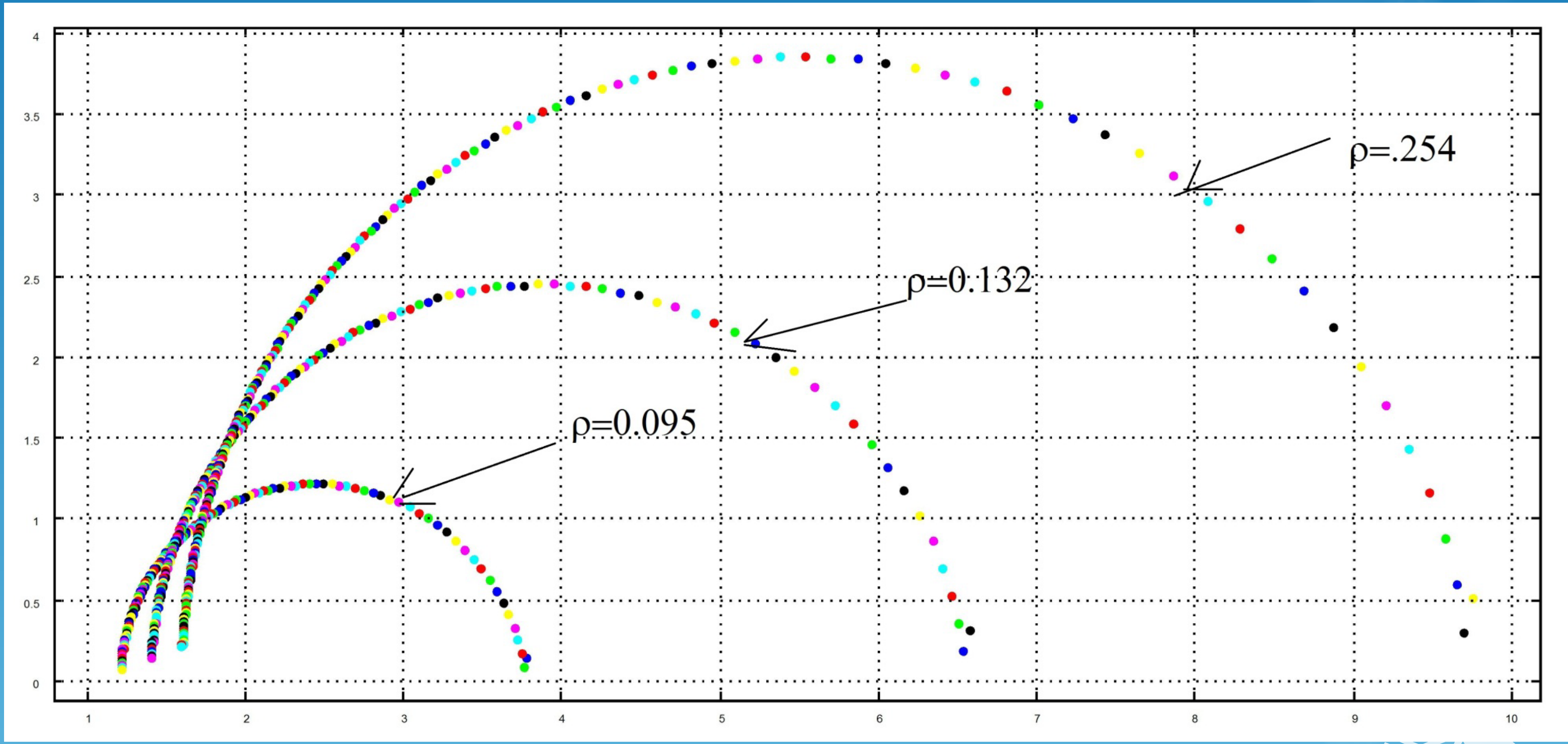
Dielectric constant of soft snow (Experimental Results)



Dispersion of ε_r' and ε_r'' of snow at different volumetric ratios

Comsol Results For Snow

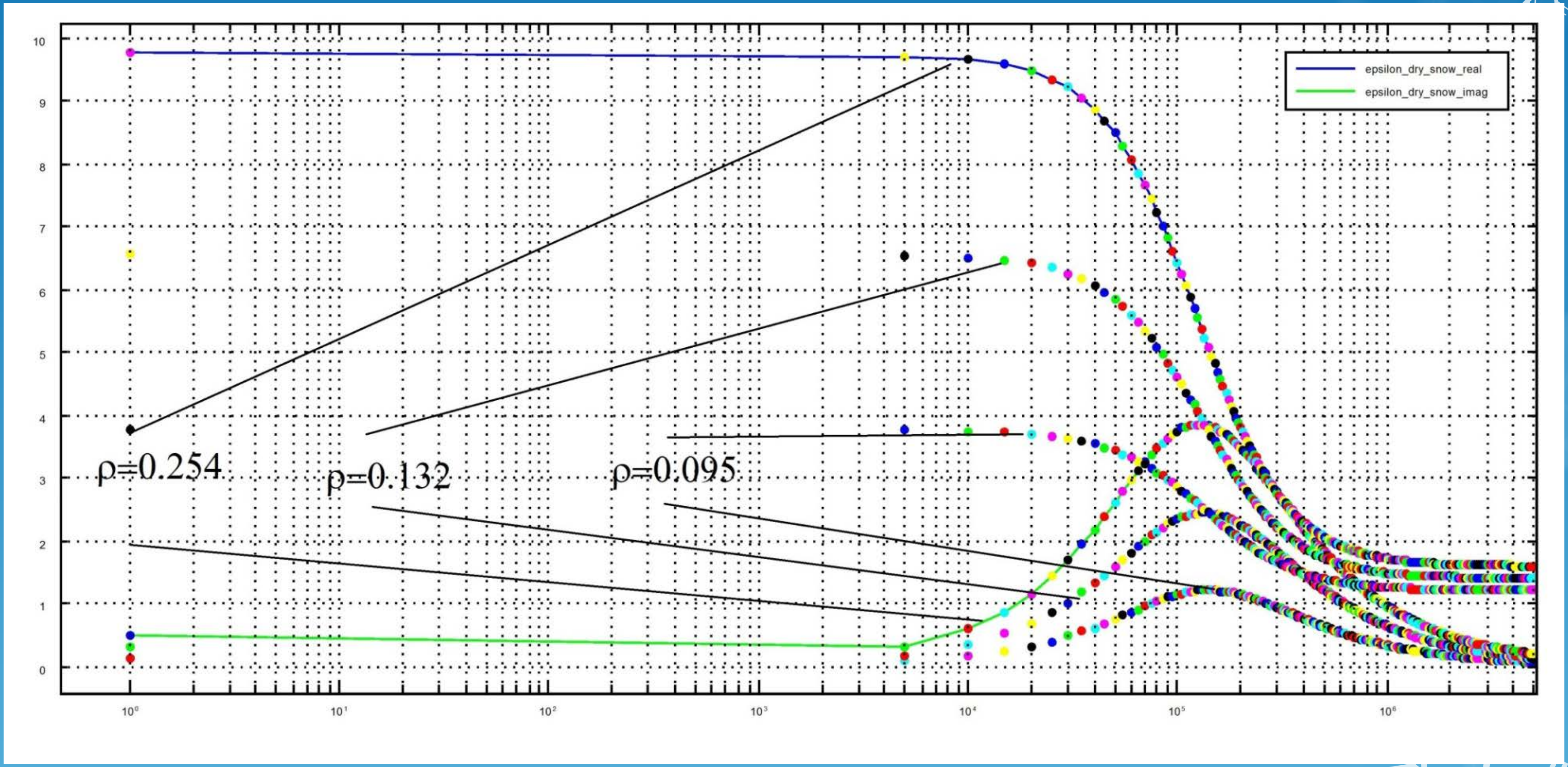
ARGAND DIAGRAM AT DIFFERENT ρ



ε_r''

ε_r'

ε_r'
&
ε_r''



Log(ω)

Ongoing research activities,

1. Finding a conductivity equation as a function of temperature and frequency,

2. Measuring the dielectric properties of wet snow using Comsol,

3. Using conductivity function in ice, dry snow and wet snow permittivity functions,

4. Tabulating the capacitance data with respect to the thickness of ice, dry snow and wet snow,

5. Tabulating the rate of change of capacitance with respect to rate of change of atmospheric ice thickness.

Conclusion

- Debye equations forms a basis for the dielectric based sensing technique for atmospheric icing at different conditions.
- 2D Experimental results closely matches with the numerical results for pure ice and dry snow at different temperatures and at different density ratios respectively.
- Modelling snow is little tricky and dielectric mixing relation needs modification
- More materials as like pure ice, dry snow and wet snow need to be added in Comsol Cold Climate Research Technologies. Also there are many mathematical relations (I am already working on some) for these materials which can be utilized effectively.
- We are working on dielectric measurement technique to develop a sensor to effectively measure the icing rate, ice thickness, icing type and ice load and theses results forms a basis for our sensor.

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I appreciate your attention

I am now open for all questions

ACKNOWLEDGMENT

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