Modelling the temperature-dependent dynamic behaviour of a timber bridge with asphalt pavement

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Abstract: The dynamic behaviour of a timber bridge with asphalt pavement is investigated. Measurements showed that the fundamental frequency can vary from 3.2 Hz to 4.1 Hz under the seasonal temperature variation. At the same time, damping is largest around 25°C and drops down for lower or higher temperatures. These surprising facts are due to the strong dependency of asphalt stiffness and damping on temperature. A numerical model is presented that is able to reproduce and explain the observed behaviour.

Keywords: Dynamic analysis, pedestrian bridge, timber, asphalt, temperature dependence.

1. Introduction

The fundamental frequency and the corresponding damping value are the main design parameters for footbridges against excessive vibrations induced by pedestrians. Since pedestrians typically walk at a pace of 1.6–2.4 Hz, this frequency range as well as the range of the second harmonic, namely 3.5–4.5 Hz, should be avoided. However it has been observed that the fundamental frequency of a bridge with asphalt pavement is not a constant but can vary considerably with temperature. This effect is mainly due to the temperature-dependent material behaviour of asphalt.

To better understand the influence of the temperature on the dynamic behaviour, a timber bridge was monitored over a long period, and frequencies and damping values were measured for a broad temperature range. These values have been compared with a numerical model. The bridge is shown in Figure 1.

Figure 1. Photo of timber bridge.

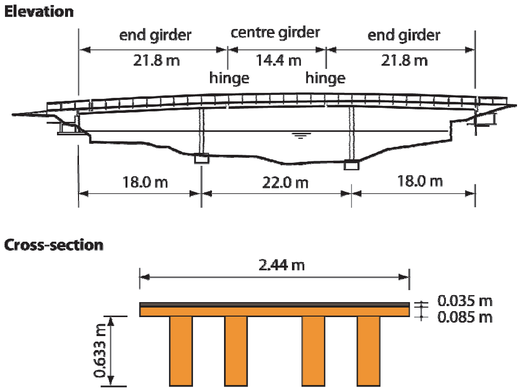

Figure 2. Elevation and cross-section of bridge.

2. Bridge model

The bridge has three spans with a main span of 22 m and two side spans of 18 m. It consists of three girders, which are connected by hinges. The hinges are placed in such a way that the bridge acts like a continuous beam for static loading (Gerber beam). An elevation view is shown in Figure 2 at the top. The girders consist of four glue-laminated timber beams with a cross-laminated timber plate on top as shown in Figure 2 at the bottom. The plate is covered by a layer of mastic-asphalt of 3.5 cm thickness.

2.1 Timber

Timber is an orthotropic material. For bending, the important moduli are the Young's modulus in the longitudinal direction $E_{\parallel}$ and the shear modulus $G_{\parallel\perp}$, where the indices denote the direction parallel and orthogonal to the fibre direction. These parameters have been determined previously for a similar material [1]. Their values for the two types of timber material are shown in Table 1. The glue-laminated beams have all fibres in the longitudinal direction, whereas the cross-laminated plate has some layers with fibres in the transverse direction, which explains the different numbers.

Table 1: Elastic material properties

Property	Beams	Plate
$E_{\parallel} = E_1$	14'000 [MPa]	10'000 [MPa]
$G_{\parallel\perp} = G_{12} = G_{13}$	600 [MPa]	700 [MPa]
$\nu_{\parallel\perp} = \nu_{12} = \nu_{13}$	0.4	0.4
$E_{\perp} = E_2 = E_3$	250 [MPa]	2'500 [MPa]
$G_{\perp\perp} = G_{23}$	89.3 [MPa]	893 [MPa]
$\nu_{\perp\perp} = \nu_{23}$	0.4	0.4

Even though the other parameters are less important, they have to be selected in a consistent way. It is, for instance, not possible to use an isotropic material, since Poisson's ratio corresponding to $E_{\parallel}$ and $G_{\parallel\perp}$ shown in Table 1 would be 10.7, meaning a negative compressibility. However, we can use a transverse isotropic material, neglecting the difference in the two transverse directions for the cross-laminated plate. Table 1 shows the properties used for the calculations.

Damping of the timber structure can be introduced as Rayleigh damping or a structural damping. For Rayleigh damping, one has to define two coefficients to match two damping values at two corresponding frequencies. However, when the eigenfrequency of the bridge changes due to the influence of asphalt, so would the damping of the timber structure, which is not feasible. It is thus more appropriate to use structural damping, described by complex elastic moduli. The loss factor η modifies the elasticity matrix by the factor $(1+\mathrm{i}\eta)$. We selected a loss factor for the timber structure of $\eta = 0.04$, which corresponds to a damping ratio of $\zeta = 0.02$ for all modes. The overall damping is the result from this timber damping and the temperature-dependent asphalt properties.

2.2 Asphalt

Asphalt is a viscos material with properties that strongly depend on temperature. The material can be characterized experimentally by measuring the so-called storage modulus and the loss modulus for different frequencies and temperatures. These moduli can be combined to a complex shear modulus:

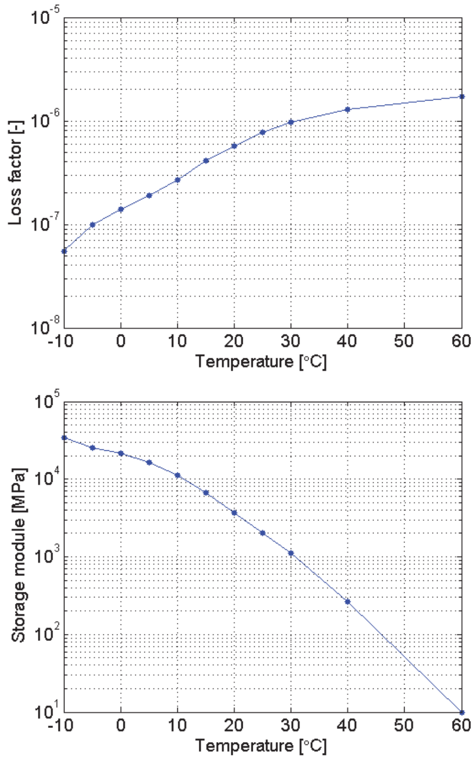

Figure 3. Asphalt storage modulus and loss factor.

$$G^* = G' + \mathrm{i}\,G'' = G'(1+\mathrm{i}\eta) \qquad (2)$$

where G' is the storage modulus and G'' is the loss modulus. Alternatively, the loss modulus can directly be expressed by the structural loss factor η.

A linear viscoelastic material is available in COMSOL, which implements the dependency on frequency and temperature by a generalized Maxwell model (series of spring-dashpot pairs). However asphalt would require a large number of terms in the series [2]. As the frequency band of interest is relatively narrow, the dependency on the frequency can be neglected. Parameters pertaining to 4 Hz have been used for the whole range of 3–8 Hz. The temperature dependency, on the other hand, is essential. For the observed asphalt temperatures from –10°C to +60°C, the elastic moduli vary strongly as shown in Figure 4. The storage modulus changes by more than three orders of magnitude in this temperature range.

Young's modulus can be calculated via Poisson's ratio. However, this value presumably also changes with temperature. Alternatively, we can use the bulk modulus. For a Poisson's ratio of $\nu = 0.2$ at –10°C, the bulk modulus becomes $K = 46000$ MPa. Leaving this value constant, Poisson's ratio becomes $\nu = 0.5$ at 60°C, corresponding to an incompressible material.

2.3 Interface between asphalt and timber deck

It has been observed already previously [1] that the influence of asphalt is overestimated by the model at low temperatures. This is believed to be due to slip in the interface between the asphalt and the timber deck. Although the exact mechanism is still under investigation, an elastic interface improves frequencies and damping values with respect to measurements at the same time. The elastic interface can be modelled by the "thin elastic layer" implemented in the solid mechanics module. A coefficient of $k_A = 60$ MPa/m was used, determined by calibration with measured frequencies.

2.4 Boundary conditions and constraints

At the two abutments, the beams are pinned at the lower edges.

More challenging are the internal hinges. This kind of hinge is a typical construction detail for timber beams. They are positioned at the points of zero bending moment for a continuous beam. In this way the structure acts like a continuous beam without the need for moment resisting connections, which are difficult to realize with this material. In the model, these connections allow a rotation in bending (around the transverse axis) but prevent vertical and transverse relative displacements as well as relative torsional rotation (axial twist). We can either use rigid connectors with corresponding constraints or couple some degrees of freedom in the 3D-model.

Preventing relative displacements of rigid connectors can be achieved by global constraints. Preventing relative torsion is more difficult, since the rotational degrees of freedom are defined by the four Euler parameters, which are difficult to visualize. The same is true if we want to introduce rotational springs. Therefore we preferred to directly simulate the actual connections in the joints. Good results were obtained by constraining the relative displacements of the beams in vertical and transverse direction and by welding the plates in all directions. Welding the plates provided the necessary rotational bending stiffness. The asphalt was left disconnected in the joints.

3. Use of COMSOL Multiphysics

Only the solid mechanics module is used for this investigation. Nevertheless, some advanced features were employed. All modelling aspects have been discussed already. Here we only give a summary of the features employed in this investigation.

An elastic orthotropic material is considered for timber. Damping of timber is introduced by an isotropic loss factor corresponding to a viscous damping of 2% for all modes. Asphalt is considered elastic with a temperature-dependent shear modulus and loss factor. A generalized Maxwell model could be used but is more complicated and not necessary here.

Internal hinges are modelled by enforcing relative displacements between adjacent beams. An implementation with rigid connectors proved possible but more complicated, due to the difficult geometric interpretation of Euler parameters.

Slip between the asphalt layer and the timber plate was considered with a thin elastic layer. Disabling the layer switches the model to full bond.

Using a loss factor for asphalt and for timber leads to a complex stiffness matrix. COMSOL solves the corresponding complex eigenvalue problem. Eigenfrequencies and damping values are extracted from the complex eigenvalues:

$$\lambda = \zeta\omega_n - \mathrm{i}\,\omega_n\sqrt{1-\zeta^2} \qquad (3)$$

A parametric sweep was used to cover the relevant temperatures.

4. Results

The main purpose of this study was to simulate the influence of the asphalt temperature on eigenfrequencies and damping values and to relate them to measured quantities. To this end a certain calibration of the model was needed. For high temperatures, the asphalt has only a small stiffness and only influences the frequencies through its mass. Therefore it was possible to calibrate the timber structure independently of the asphalt. The mode shapes at 60°C asphalt temperature are shown in Figure 4.

The first three bending modes are each sensitive to a different parameter. These parameters can thus be calibrated without affecting the other modes too much. The first mode is a bending mode, with the nodes close to the internal hinges. The first mode is thus primarily affected by Young's modulus of timber in the longitudinal direction. The second mode shows some kink at the internal hinges. This mode is susceptible to the rotational stiffness in these hinges. The third mode has the largest curvature and is thus affected most by the shear modulus of the timber structure.

Computed eigenfrequencies are compared with measured values in Figure 5. The standard deviation from 10 measurements at a given temperature is indicated by error bars and is relatively low. Remarkable is the large variation of the eigenfrequencies due to changes in the asphalt temperature. For high temperatures, the computed frequencies are not influenced by the interface between asphalt and the timber deck. For low temperatures, full bonding gives too high frequencies. Introducing an elastic layer improves the results substantially. Note that this improvement is achieved by calibrating only one parameter.

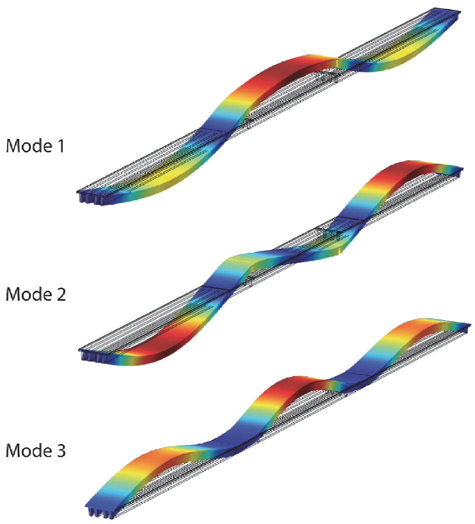

Figure 4. Mode shapes for 60°C asphalt temperature.

An analogous comparison of damping values is shown in Figure 6. Note that damping values can be measured with much less reliably than frequencies, as indicated by the larger standard deviations. Also here, the variation with changing asphalt temperature is notable. Somewhat surprising is the increase of damping at intermediate temperatures, whereas at low and high temperatures, the damping falls back to the value of the damping of the timber structure alone.

The elastic interface between asphalt and timber also gives a substantial improvement for the damping. Note that only the frequencies have been used for calibration of the elastic constant of the interface.

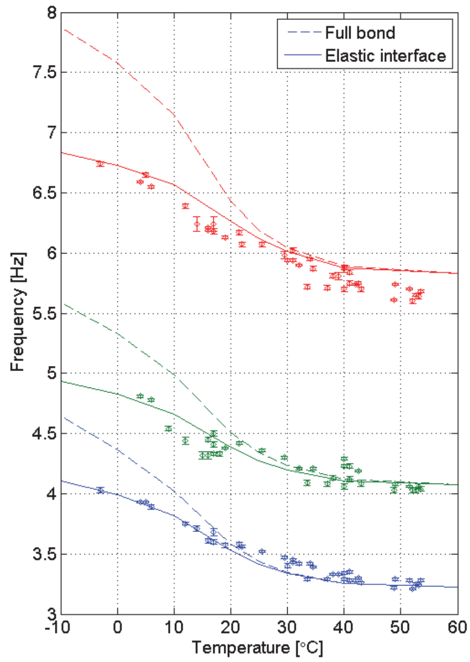

Figure 5. Comparison of calculated eigenfrequencies with measured values at different temperatures.

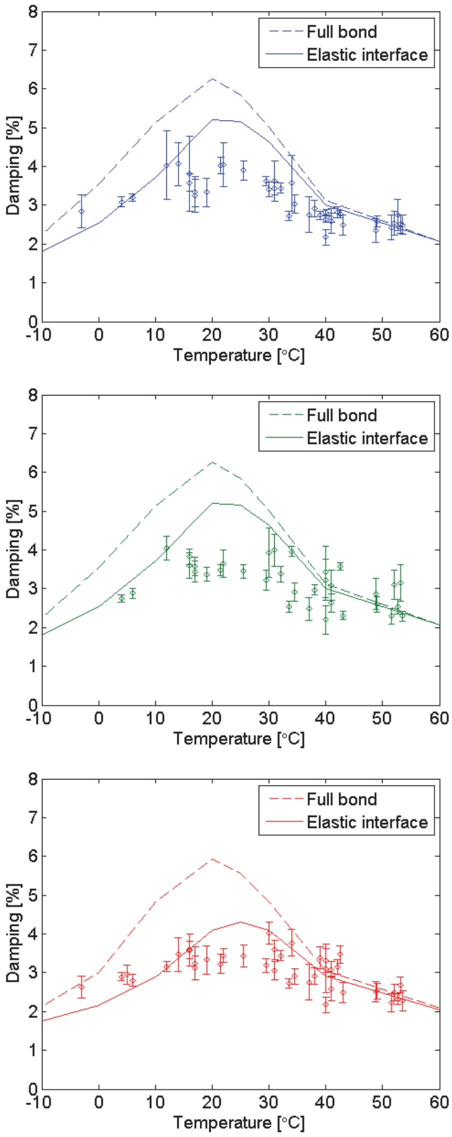

Figure 6. Comparison of calculated damping values with measured values at different temperatures.

5. Conclusions

A modal analysis of a timber bridge with asphalt pavement has been performed for a broad range of asphalt temperatures and compared to measured values. The large variation of the modal parameters observed in the measurements can be explained by the numerical model.

Although the model is pure mechanical, a number of advanced modelling techniques have been used: orthotropic elastic material, viscoelastic material, elastic interface, complex eigenvalues, and parameter sweep over temperatures.

6. References

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