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Self-consistent Modeling of Thin Conducting Wires and their Interaction with the Surrounding Electromagnetic Field

The need for thin conducting wires in FEM



*HVDC converter station
with virtual ground plane*



Connection cables



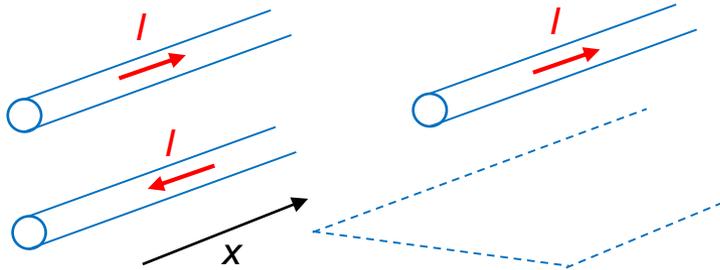
Communication mast



TV antenna

- Thin conducting wires, cables or rods occur in many important electromagnetic applications
- Wire currents can be induced due to external time-varying magnetic fields
- Wires can also act as radiating antennas when currents are driven along them
- In complex systems both effects have to be solved for simultaneously
- Modeling wires with cylinders would require an excessive amount of elements and computer power

Telegrapher's equation for transmission lines

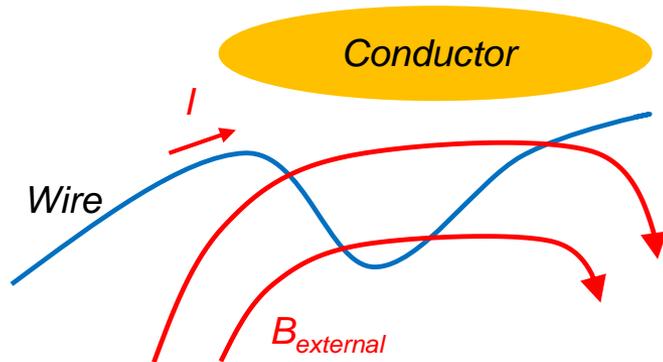


$$\frac{\partial}{\partial x} \left(\frac{1}{R + i\omega L} \frac{\partial V}{\partial x} \right) - (G + i\omega C)V = 0$$

$$I = \frac{-1}{R + i\omega L} \frac{\partial V}{\partial x}$$

- V is the voltage between the two conductors (or between the single conductor and the conducting plane)
- For a transmission line with two parallel conductors positioned very close to each other (or a single conductor near a conducting plane) one can use the 1D so-called Telegrapher's equation
- The line parameters L , C , G , and R can easily be calculated numerically or analytically
- This feature exists in the RF module
- I can be used as input when calculating \mathbf{E} but it does not allow the wire current to be influenced by \mathbf{E}

Modified telegrapher's equation coupled with field solver for a single wire



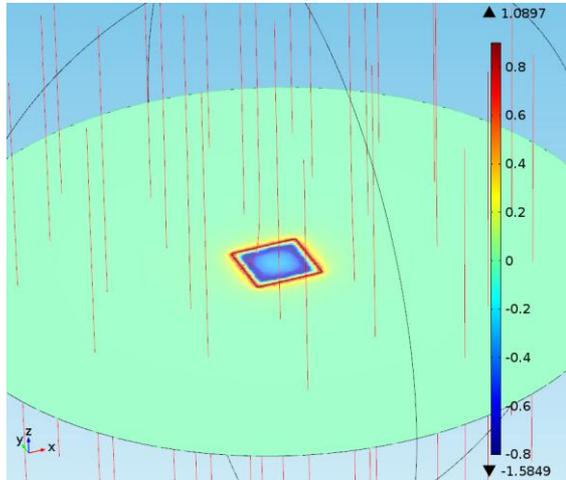
- For a single wire placed in a 3D geometry the interaction with the surrounding EM field becomes much more complex and the simple Telegrapher's equation with constant L , C , G , and R cannot be used
- Here, we try to implement a method that works well in FDTD (finite differences)
- L_w and C_w correspond to the magnetic and electric energies within one element away from the wire
- The global field E_x takes care about the interaction with sources further away
- V is now proportional to the charge density
- This provides the desired two-way coupling

$$\frac{\partial}{\partial x} \left(\frac{1}{R + i\omega L_w} \left(\frac{\partial V}{\partial x} - E_x \right) \right)$$

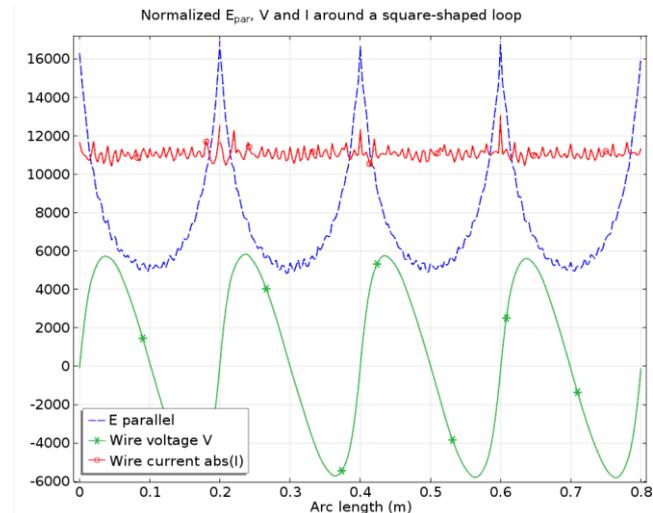
$$- (G + i\omega C_w) V = 0$$

$$I = \frac{-1}{R + i\omega L_w} \left(\frac{\partial V}{\partial x} - E_x \right)$$

Example 1: Closed wire loop in an applied magnetic field



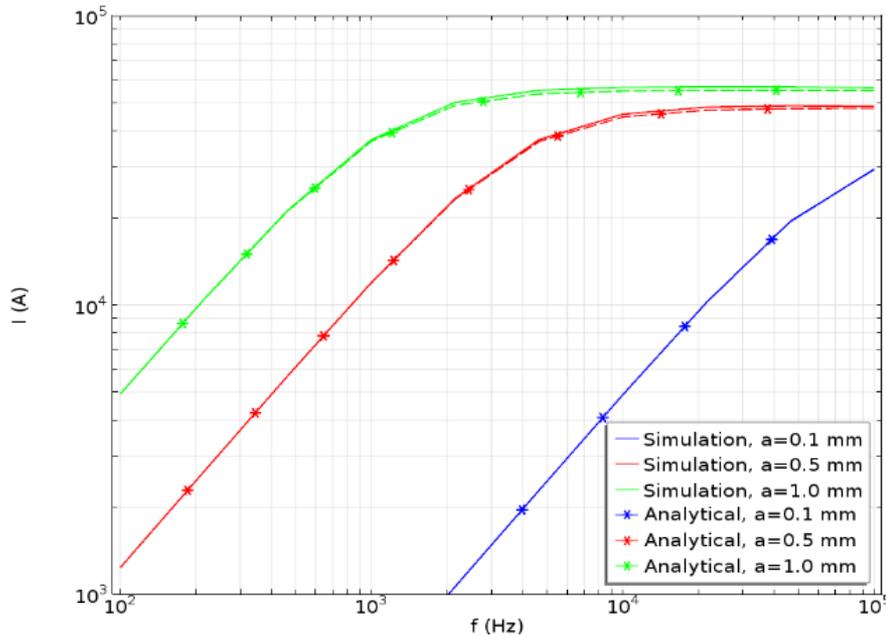
Magnetic field distribution



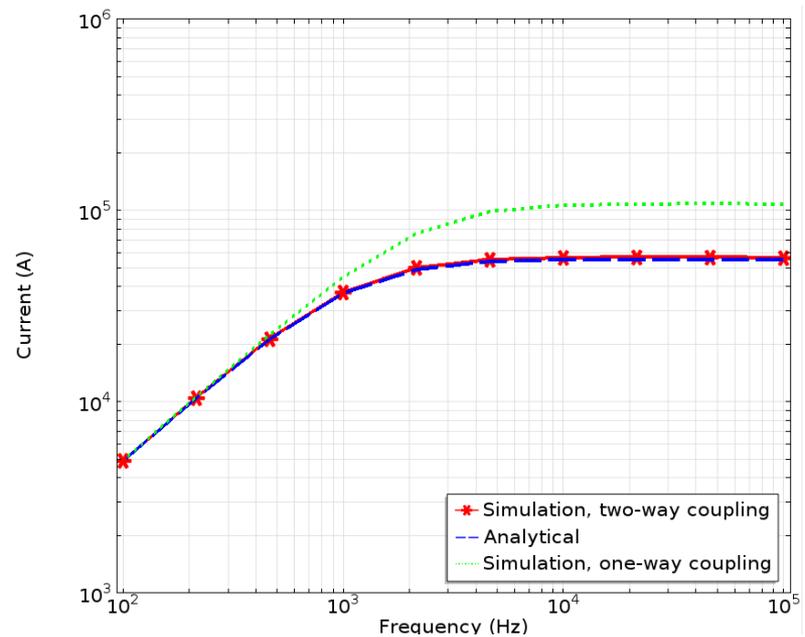
E_{par} , I, and V along the wire

- A closed square-shaped loop, made of a very thin wire, is placed in an oscillating vertical magnetic field
- The induced current generates a counteracting magnetic flux
- Small fluctuations caused by the FEM formulation
- Overall current and charge distributions are OK

Example 1: Closed wire loop in an applied magnetic field



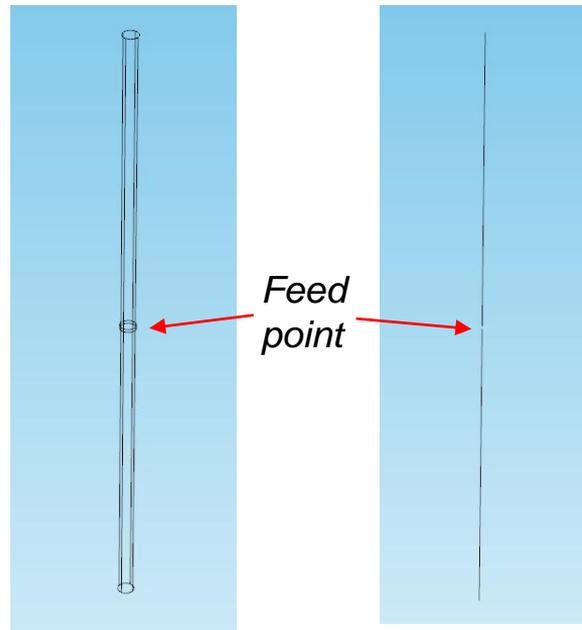
Induced loop current as function of frequency



Impact of leaving out the E_x term (dotted curve)

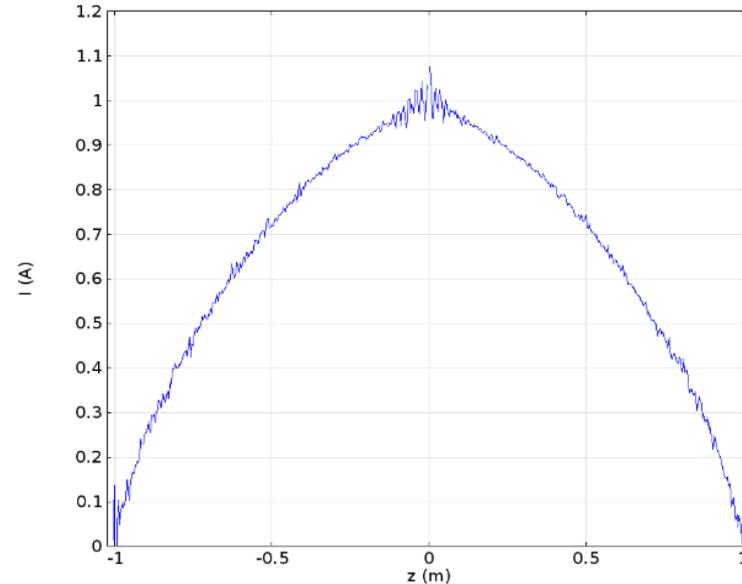
- The wire current agrees well with an analytical expression for several values of the wire radius a and within a wide frequency range
- Leaving out the E_x term in the modified equation leads to a much too high current

Example 2: A straight wire dipole antenna



Resolved model

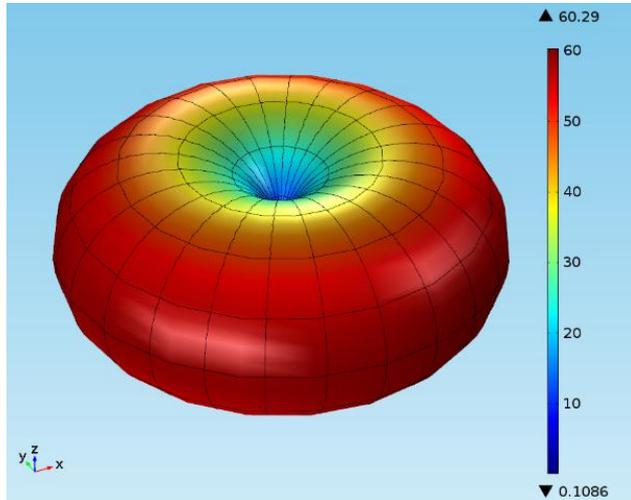
Wire model



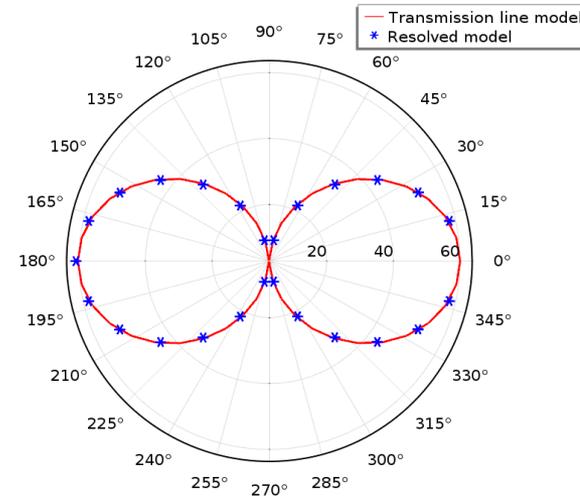
Current distribution along the dipole wire

- A straight dipole antenna is fed at a frequency corresponding to the half-wave resonance
- Comparison is made with a fully resolved model
- The current distribution is OK although there is a small ripple due to the FEM formulation

Example 2: A straight wire dipole antenna



3D far-field radiation pattern



Vertical radiation pattern from dipole

- Radiation patterns agree well
- Input impedance: $106 + 43i \Omega$ (resolved model) and $101 - 4i \Omega$ (wire model)
- Imaginary parts differ because feed points are modeled very differently and also because wire ends are not properly modeled

Conclusions

- A method adopted from FDTD has been applied to include wires having radii smaller than the element size
- By coupling the emw and transmission line physics in the RF module a self-consistent two-way coupling is provided
- Due to inherent properties of the FEM formulation the coupling is not exactly self-consistent, resulting in a slightly noisy solution (convergence criterion has to be relaxed)
- In principle, it should be possible to use the same method in the AC module