A Three Dimensional (3D) Thermo-Hydro-Mechanical Model for Microwave Drying

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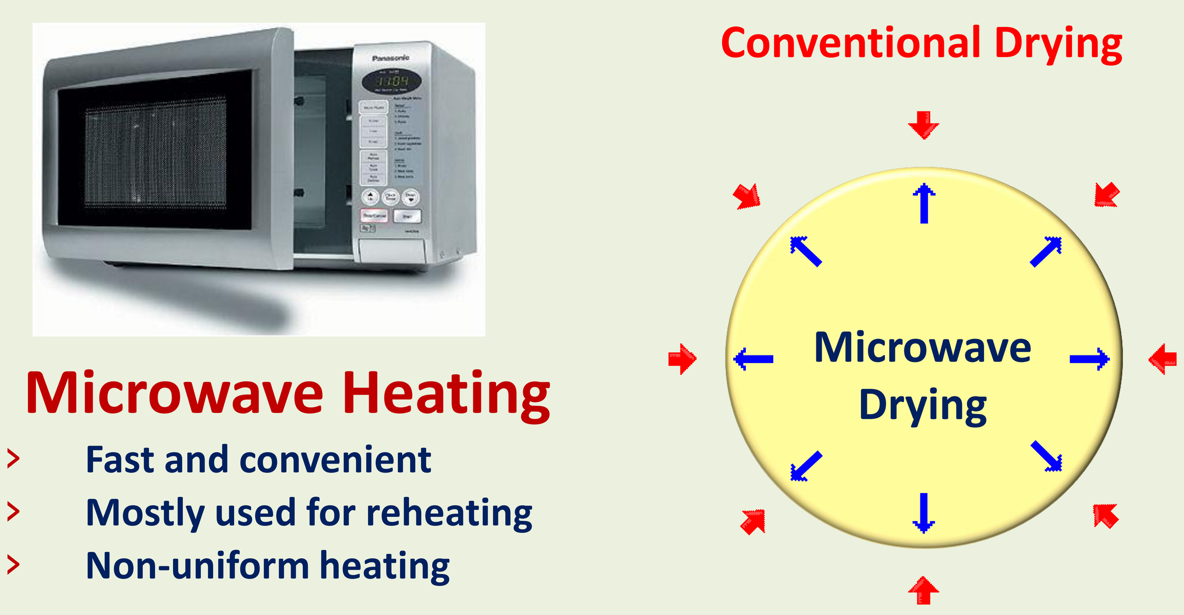
Overview

- Microwave drying of foods have been found to result in better quality products having higher rehydration capabilities, higher porosity and minimal case hardening effects compared with conventional drying
- Microwave drying is a complex interplay of mass, momentum and energy transport coupled with large deformation of the solid
- For the present study, microwave drying of a potato cube in a domestic microwave oven operating at 10% power level is taken as an example
- A fundamentals-based three dimensional (3D) multiphase porous media based model is developed to simulate the drying process. Deformation is included as part of the framework to be able to predict key textural attributes
- An elaborate experimental is built to validate surface temperature profiles, average moisture content and volume changes of the potato sample at different times during the drying process
- The model developed could aid in predicting key quality attributes associated with microwave drying such as porosity, case hardening, volume fixation, stress cracking conditions

Microwave Drying vs. Conventional Drying



Microwave Heating

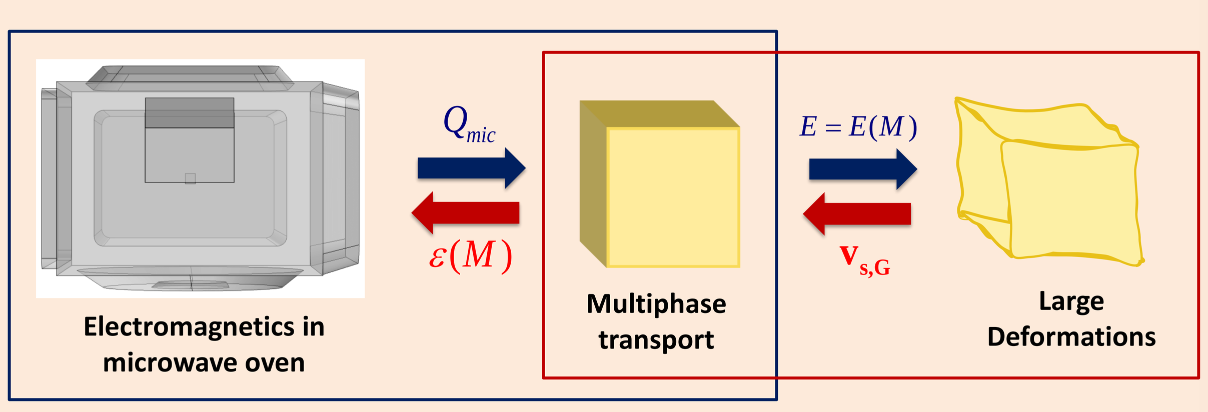
- Fast and convenient
- Mostly used for reheating
- Non-uniform heating

Microwave Drying vs Conventional Drying

- Volumetric heating, rapid moisture removal
- Case hardening effects are minimized
- Better rehydration and higher porosity

Modeling Framework

Microwave drying of potatoes carried out at 10% Power Level for 10 min.



- **Properties**– *Dielectric and Mechanical properties are functions of moisture content*
- **Shrinkage and puffing cause deformation** - *Moisture loss results in the shrinkage and gas pressure generation lead to expansion*

Transport Model

Mass conservation of different species:

Liquid Water:

$$\frac{\partial c_w}{\partial t} + \nabla\cdot(\rho_w \mathbf{v_w}) = \nabla\cdot(D_w \nabla c_w) - \dot{I}$$

(bulk flow; capillary flow; phase change)

Water Vapor:

$$\frac{\partial c_v}{\partial t} + \nabla\cdot(\rho_g \omega_v \mathbf{v_g}) = \nabla\cdot\left(\varphi S_g \frac{C^2}{\rho_g} M_a M_v D_{eff,g} \nabla x_v\right) + \dot{I}$$

(bulk flow; binary diffusion)

Air:

$$\omega_v + \omega_a = 1$$

(Mass Fraction)

Energy balance:

$$\rho_{eff} C_{p,eff} \frac{\partial T}{\partial t} + \sum_{i=w,v,a}\left(c_{p,i} T \nabla\cdot(c_i \mathbf{v_i})\right) - c_{p,w} T \nabla\cdot(D_c \nabla c_w) = \nabla(k_{eff} \nabla T) - \lambda \dot{I} + Q_{mic}$$

(convection; conduction; Evaporative Cooling; microwave source)

Momentum balance:

$$\frac{\partial c_g}{\partial t} + \nabla\cdot(\mathbf{n_{g,G}}) = \dot{I} \qquad \mathbf{n_{g,s}} = -\rho_g \frac{k_g k_{r,g}}{\mu_g} \nabla P$$

(gas pressure gradient)

Darcy (convective) Velocity

$$\mathbf{v_i} = -\frac{k_i k_{r,i}}{\mu_i} \nabla P$$

Phase change:

$$\dot{I} = K \frac{M_v}{RT}\left(p_{v,eq} - p_v\right)$$

(Non-Equilibrium Formulation)

Flux due to material deformation:

$$\mathbf{n_{i,G}} = \mathbf{n_{i,s}} + c_i \mathbf{v_{s,G}}$$

(Solid Velocity)

Electromagnetics

Maxwell's Equation

Faraday's Law of Induction: $\nabla \times \mathbf{E} = j\omega\mu\mathbf{H}$

Ampere's Law: $\nabla \times \mathbf{H} = -j\omega\varepsilon_0\varepsilon^* \mathbf{E}$

Gauss Law for electricity: $\nabla\cdot(\varepsilon\mathbf{E}) = 0$

Gauss Law for magnetism: $\nabla\cdot\mathbf{H} = 0$

(Time Harmonic Electric Field; Complex Permittivity; Time Harmonic Magnetic Field)

Power Dissipation obtained from *Poynting Theorm*:

$$P(\mathbf{x},T) = \frac{1}{2}\omega\varepsilon_0\varepsilon''(\mathbf{E}\cdot\mathbf{E}^*)$$

Solid Mechanics Model

Solid Momentum Balance:

$$\nabla.\sigma = \nabla P$$

(Gas pressure gradient causes puffing)

Stresses:

$$\sigma = J^{-1}\mathbf{F}.\mathbf{S}.\mathbf{F}^T, \quad \mathbf{F} = \mathbf{F_{el}}\mathbf{F_m} \quad \mathbf{S} = \frac{\partial W_s}{\partial \mathbf{E}}$$

(Cauchy Stress; Deformation Gradient; 2[nd] PK Stress)

Large Strains:

$$\mathbf{E} = \frac{1}{2}\left[(\nabla\mathbf{u})^T + \nabla\mathbf{u} + (\nabla\mathbf{u})^T \nabla\mathbf{u}\right]$$

(Solid displacement)

Constitutive Law:

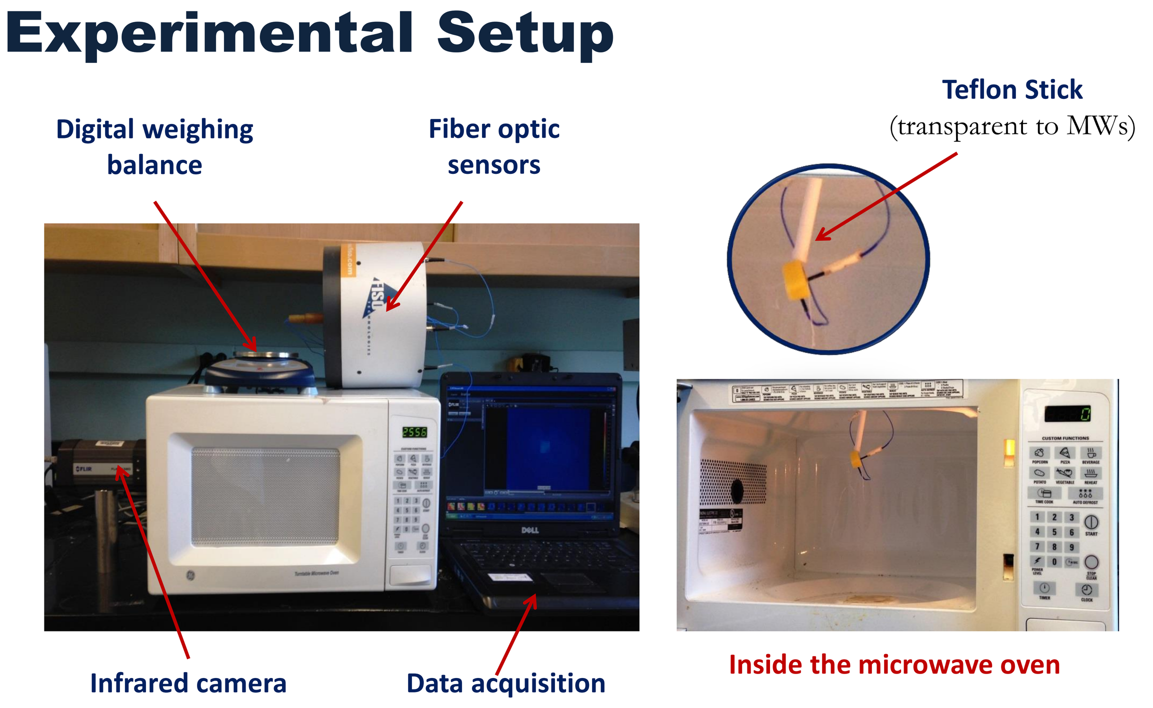
$$W_S = \frac{1}{2}\mu(I_1 - 3) - \mu\ln(J) + \frac{1}{2}\lambda\left[\ln(J)\right]^2$$

Shrinkage due to moisture loss:

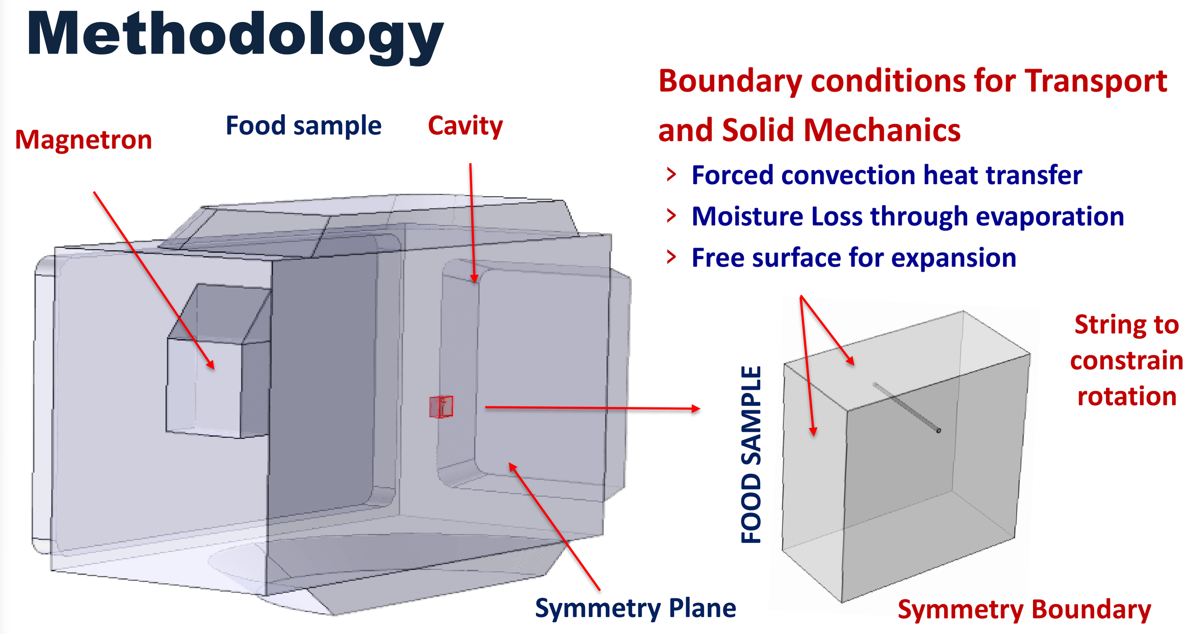
$$J_M = \frac{1-\varepsilon_{W,0}}{1-\varepsilon_W}, \quad J_{el} = \frac{J}{J_M}$$

(Volume fraction of water)

Experimental Setup



Methodology



Boundary conditions for Transport and Solid Mechanics
- Forced convection heat transfer
- Moisture Loss through evaporation
- Free surface for expansion

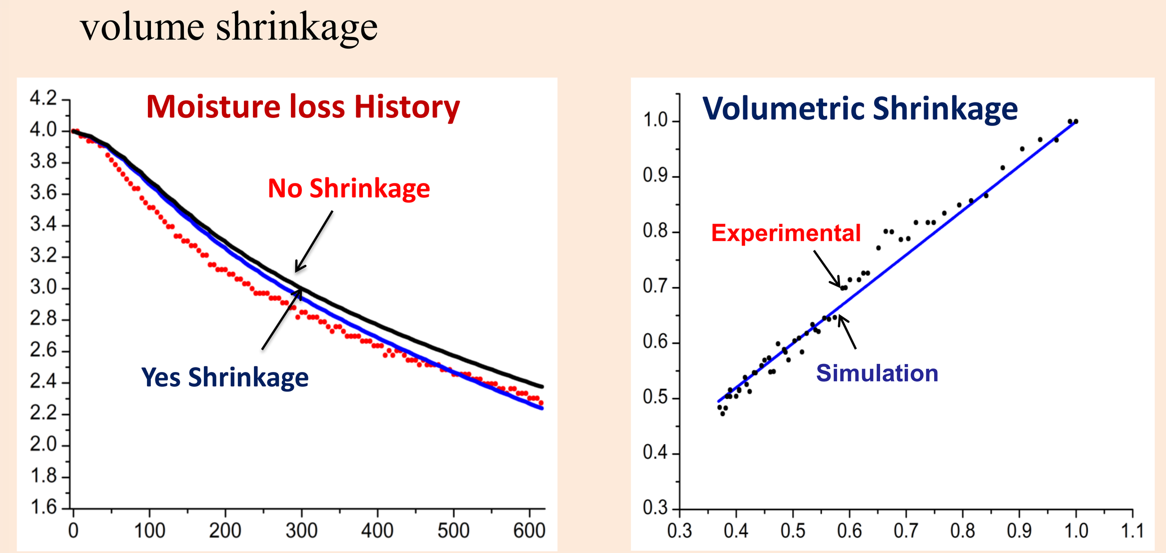
Boundary conditions for Electromagnetics
- Perfect Electric Conductor for Cavity walls
- Perfect Magnetic Conductor for symmetry plane

- No displacement
- Insulated for energy and moisture transfer

Results

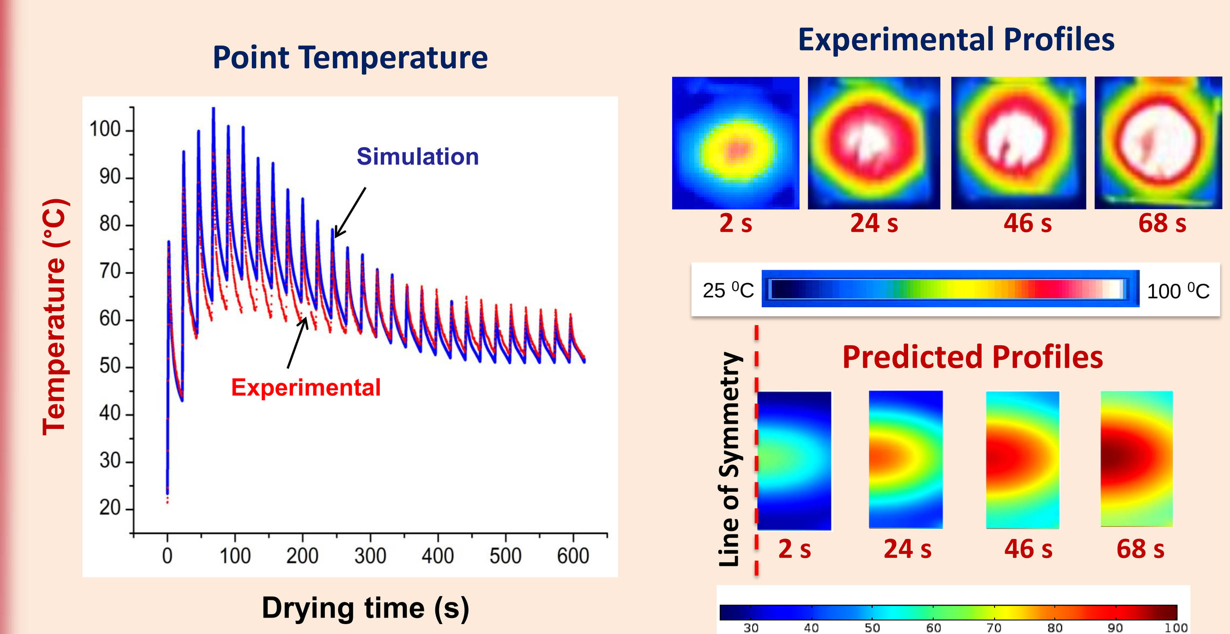
Moisture and Shrinkage Histories

- The drying model was validated for moisture history and volume shrinkage

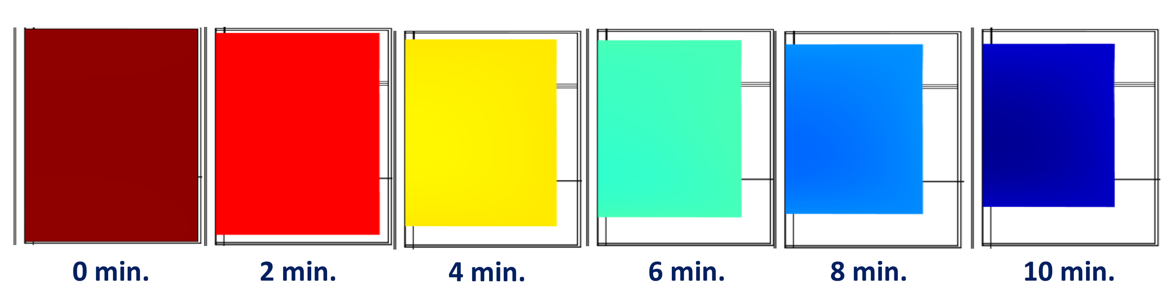


Temperature Profiles

- Point temperature and temperature profiles were compared with simulated results

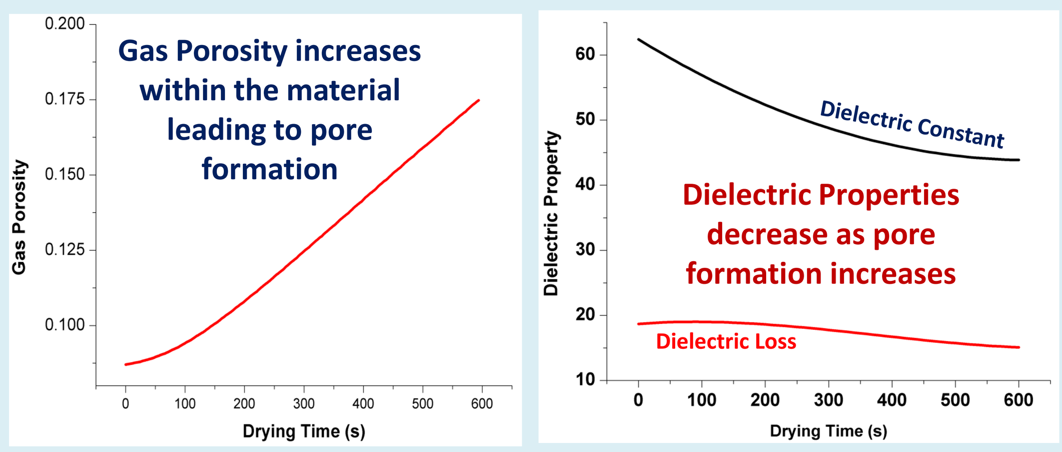


Simulated Shrinkage with drying time



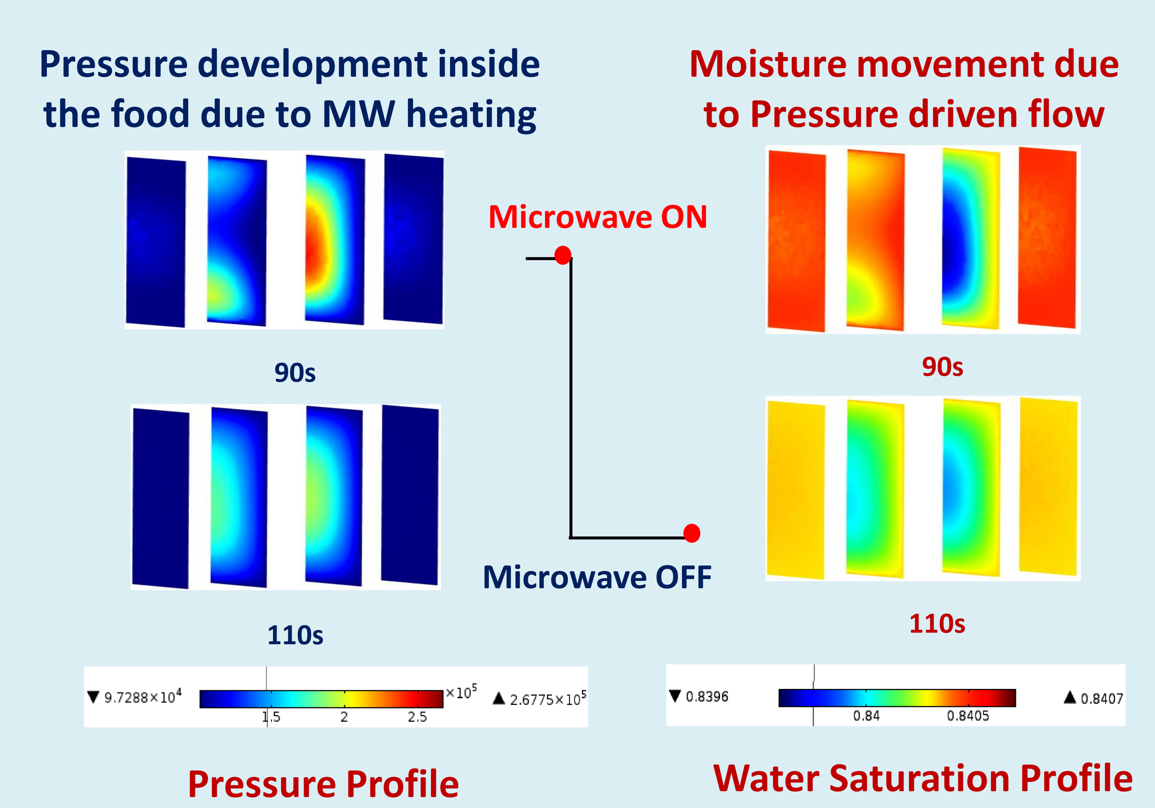
Shrinkage is more from the base as compared with the top where the material is constrained by the string

Gas Pressures and Porosity Development



Effects of Power Cycling

10% Power Level = Microwave ON for 2s and OFF for 20s



Summary

- A fundamentals-based 3D electromagnetics-transport-poromechanics model for microwave drying is presented
- The model developed successfully predicts temperature moisture and shrinkage histories during the drying process
- Evolution of gas phase and porosity development affect dielectric properties significantly
- A mechanistic approach to understanding microwave drying of foodstuffs is developed that could aid in predicting key quality attributes associated with microwave drying.

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