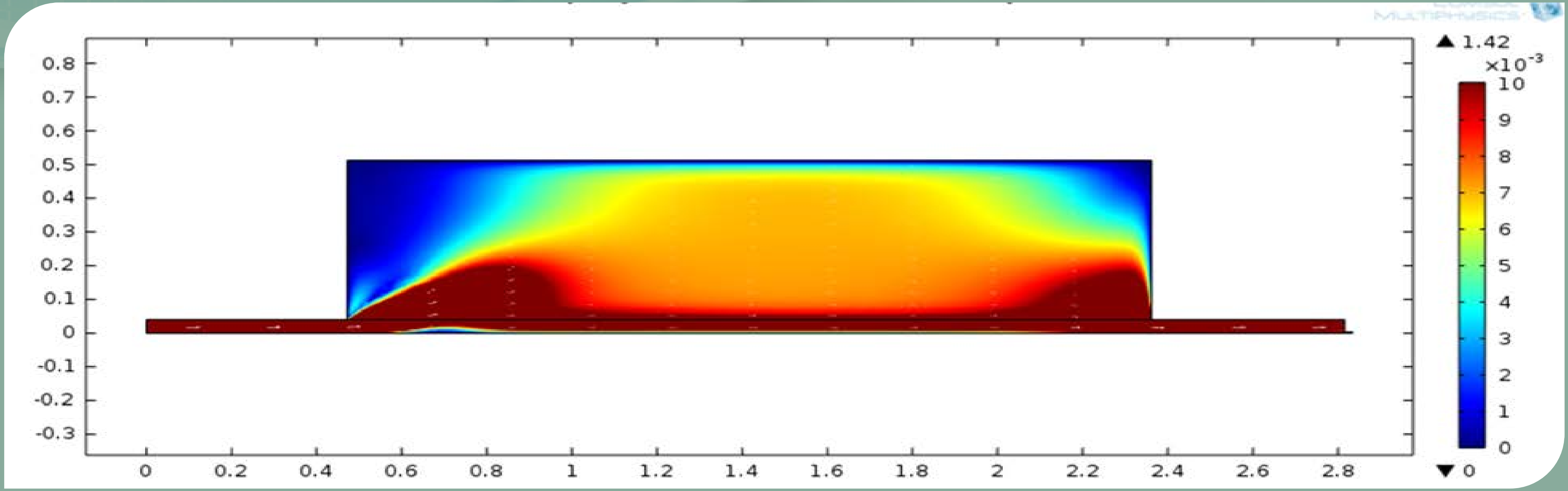
A Dual Continuum Model for Groundwater Flow in Karst Aquifers

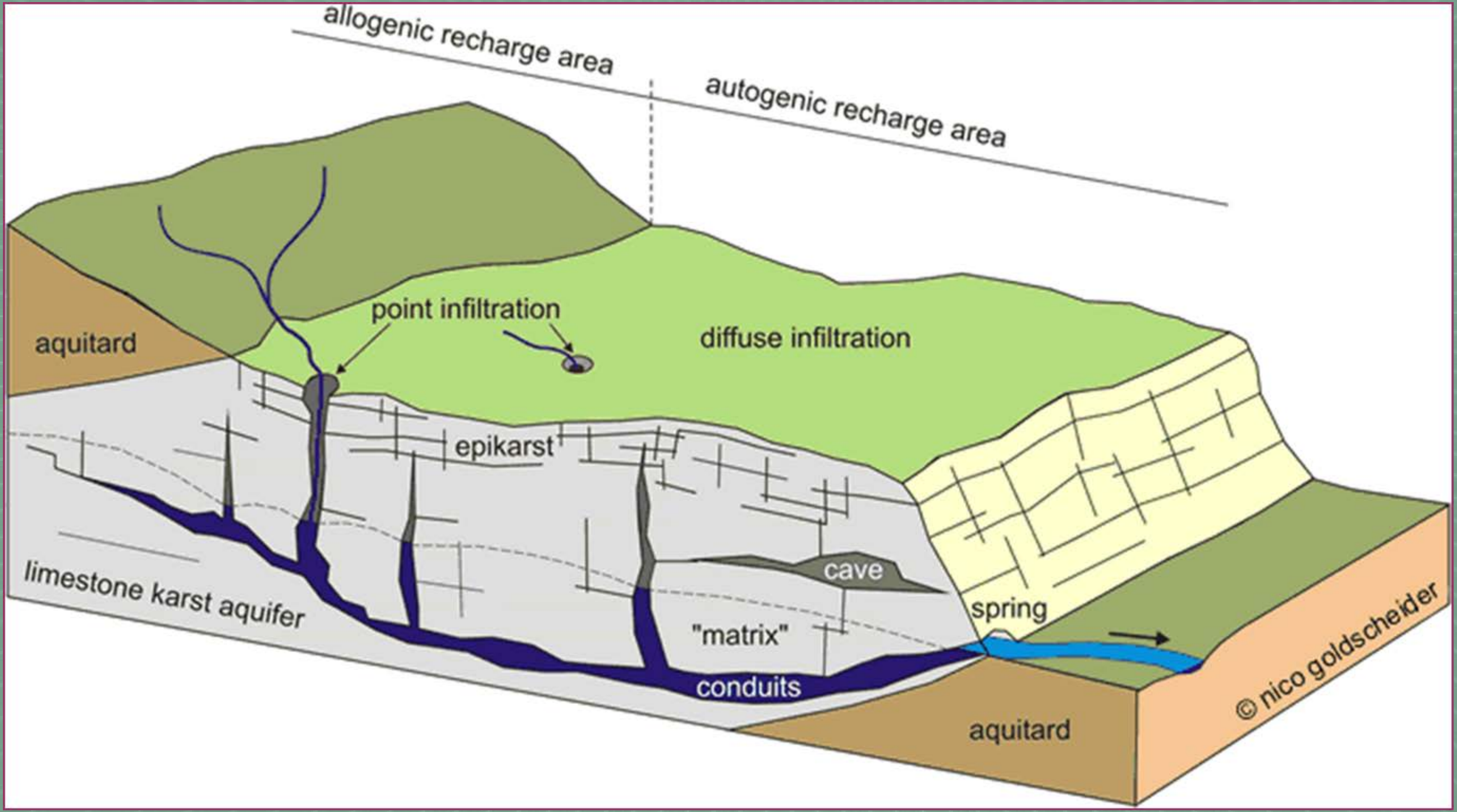
By Roger Painter, Justin Harris, Tom Byl, Lonnie Sharpe

Tennessee State University, College of Engr., 3500 John A Merritt Blvd., Nashville, TN 37209



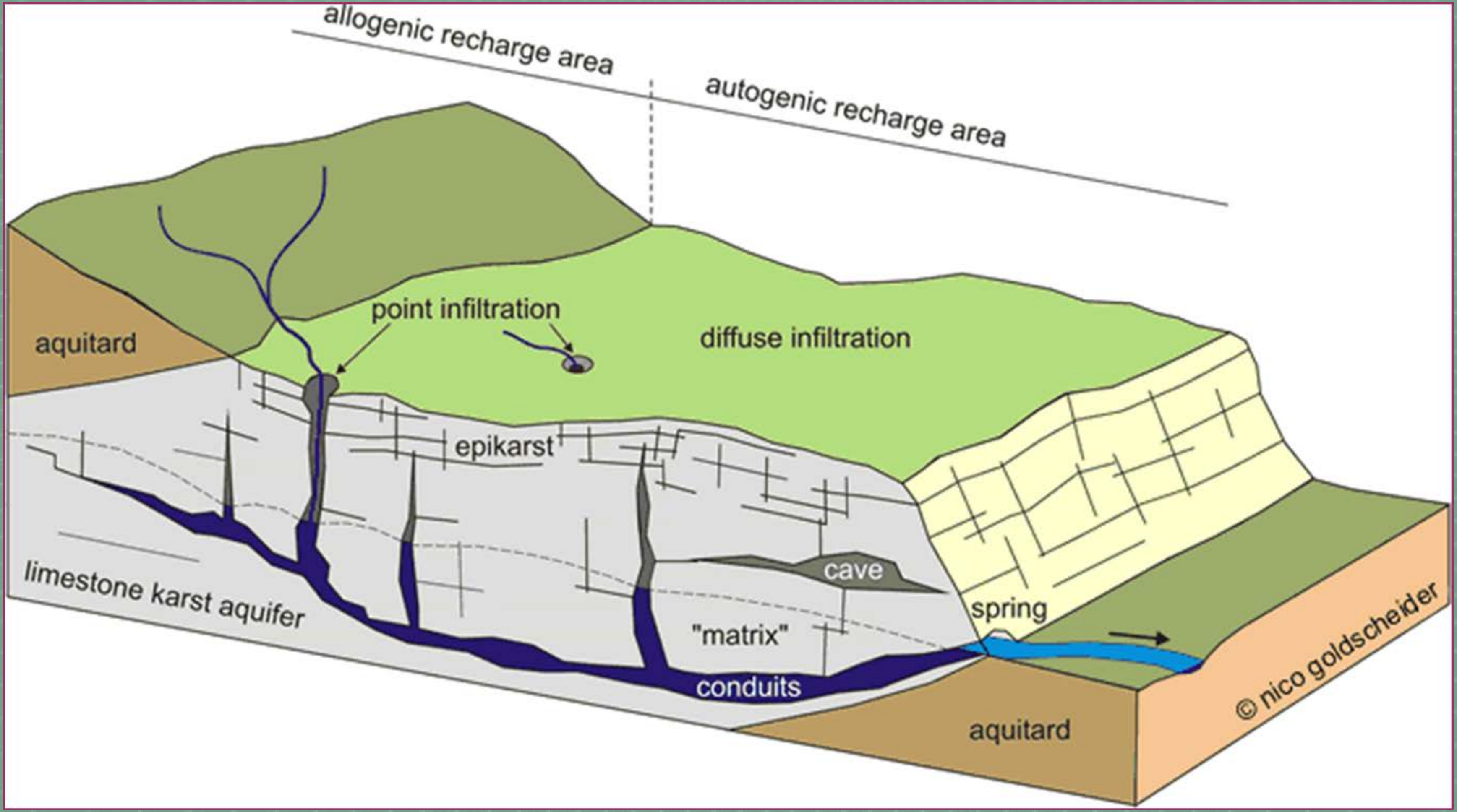
COMSOL CONFERENCE 2014 BOSTON

Diagram of Karst Attributes



Conventional Approach to Modeling Karst

Diagram of Karst Attributes



Conventional Approach to Modeling Karst

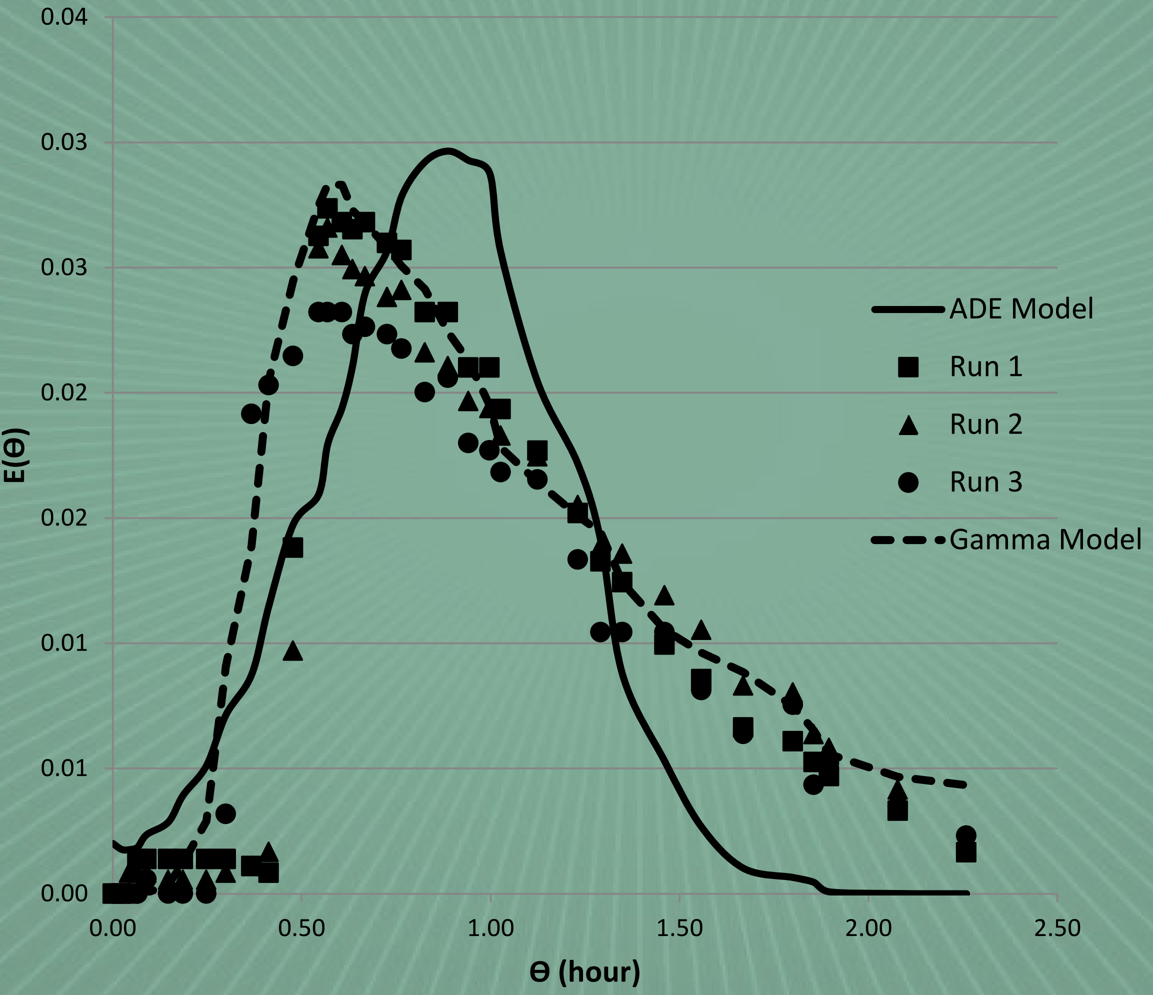
Karst has traditionally been modeled as plug flow with axial dispersion in terms of the advection dispersion equation with constant groundwater velocity.

$$\frac{\partial C(t)}{\partial t} = D_a \frac{\partial^2 C(t)}{\partial z^2} - U \frac{\partial C(t)}{\partial z}$$

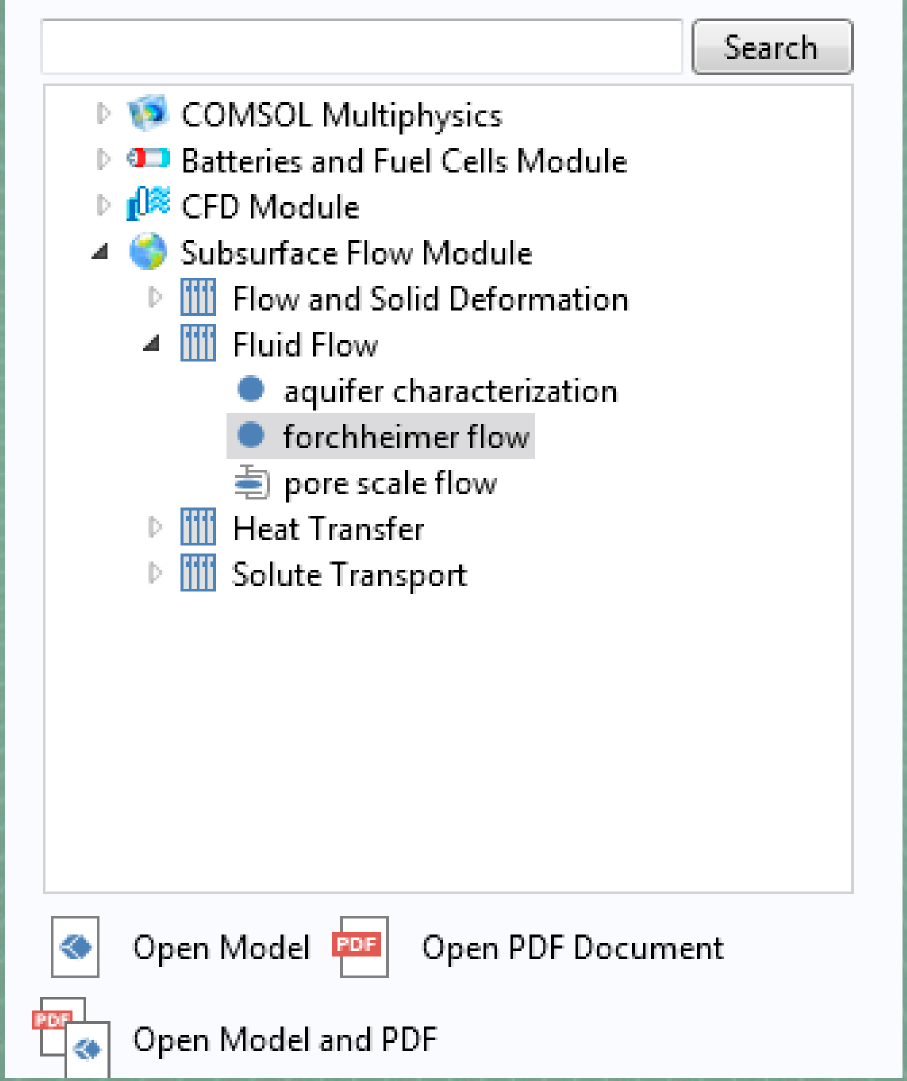
The classic solution to this equation has shortcomings with regard to its ability to predict tracer response residence times and regarding the lack of inference it makes about karst flow.

$$\frac{C}{C_0} = \frac{1}{2\sqrt{\frac{\pi\theta}{Pe}}} e^{\frac{-(1-\theta)^2}{\frac{4\pi}{Pe}}}$$

Conventional Approach to Modeling Karst



COMSOL Model Theory



COMSOL Model Theory

The model is based on the finite element solution of a rigorous mathematical model in terms of the Navier-Stokes and continuity equations describing conduit flow,

$$\nabla . \rho(u . \nabla)u = -\nabla . [-pI + \mu(\nabla u + (\nabla u)\rho(\nabla \, . \, u)]$$

$$\nabla . u = 0$$

and Forchheimer-corrected Brinkman equations describing the diffuse phase flow,

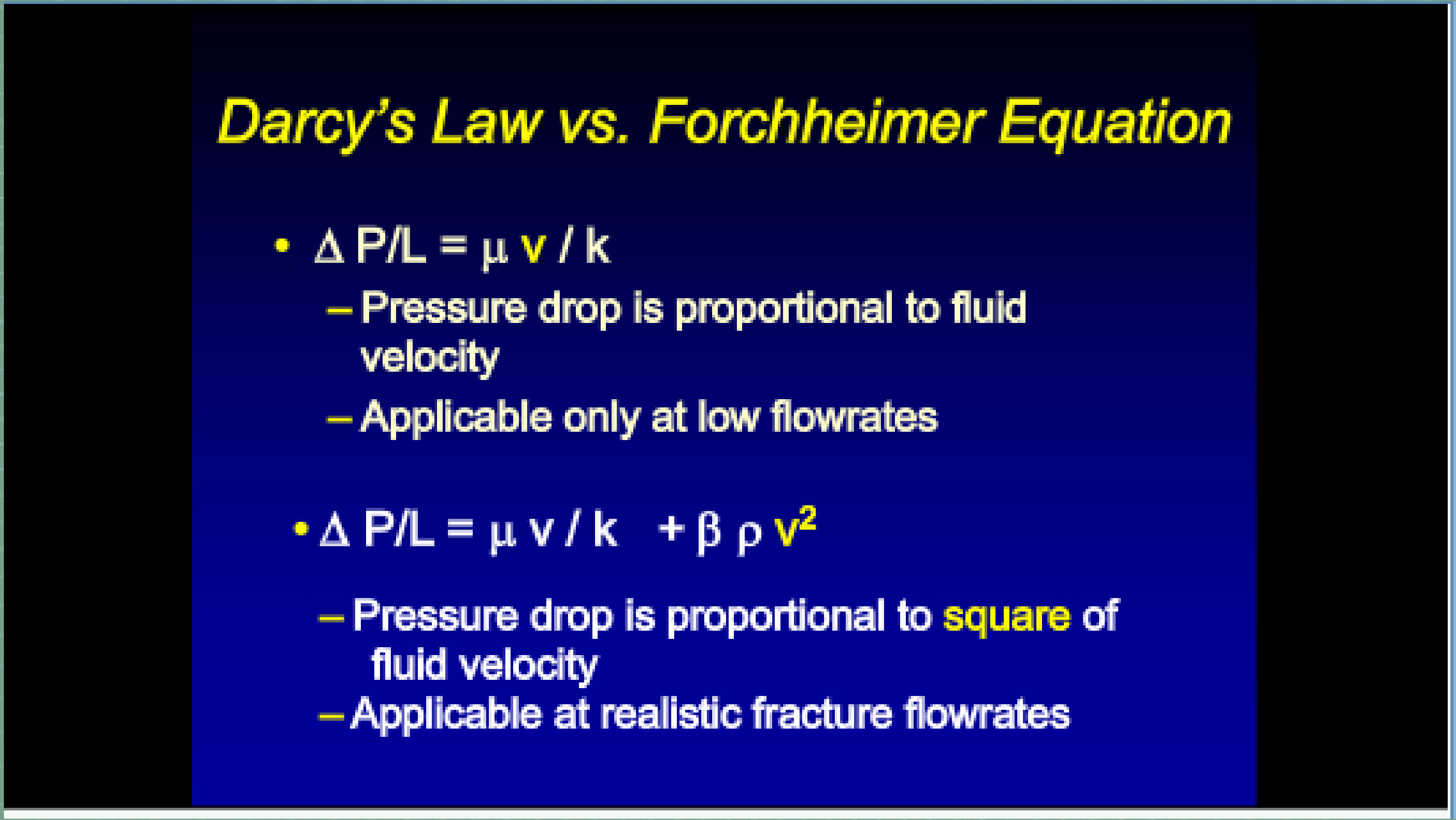
$$\frac{\mu}{k} = \nabla . \, [-pI + \mu\left(\nabla u + (\nabla u)\rho(\nabla \, . \, u)\right] - \frac{\rho \, \varepsilon p \, Cf}{\sqrt{k}} u \; |u \; |$$

COMSOL Model Theory

And finally, the transient ADE describing solute concentration is coupled to the flow equations.

$$\frac{\partial c}{\partial t} = u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} - \kappa\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}\right)$$

COMSOL Model Theory

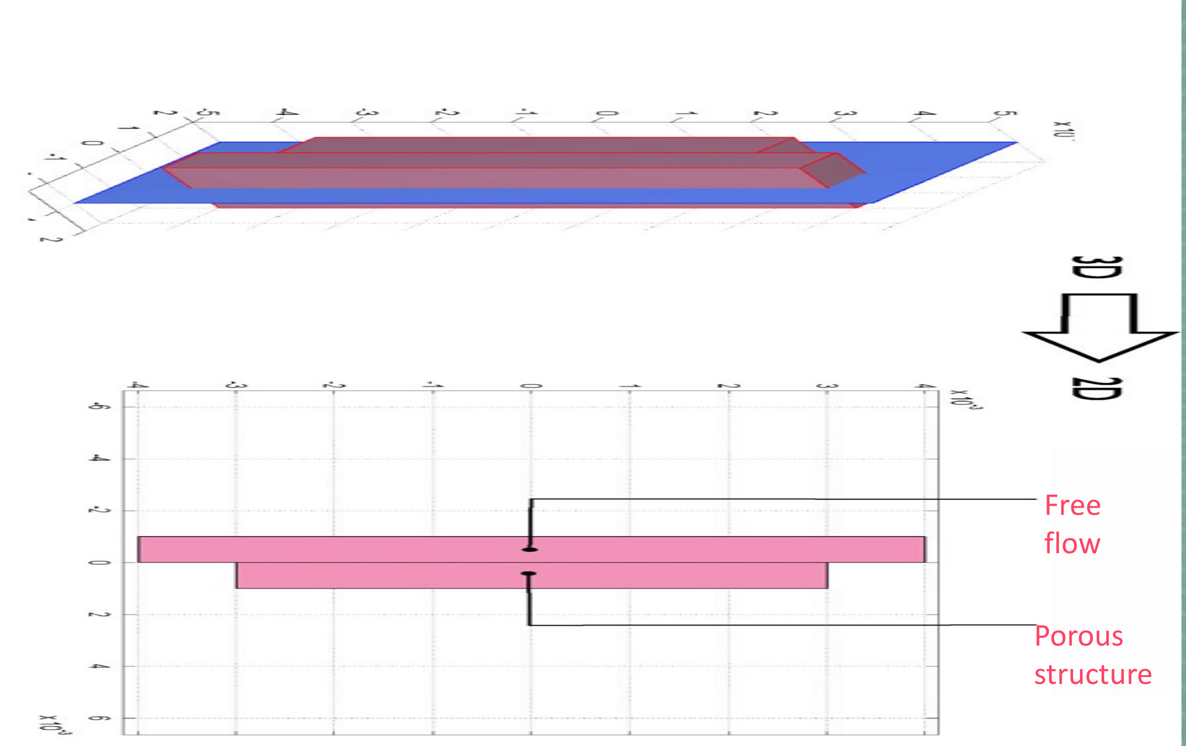


Darcy's Law vs. Forchheimer Equation

- $\Delta P/L = \mu v / k$
  - Pressure drop is proportional to fluid velocity
  - Applicable only at low flowrates

- $\Delta P/L = \mu v / k + \beta \rho v^2$
  - Pressure drop is proportional to square of fluid velocity
  - Applicable at realistic fracture flowrates

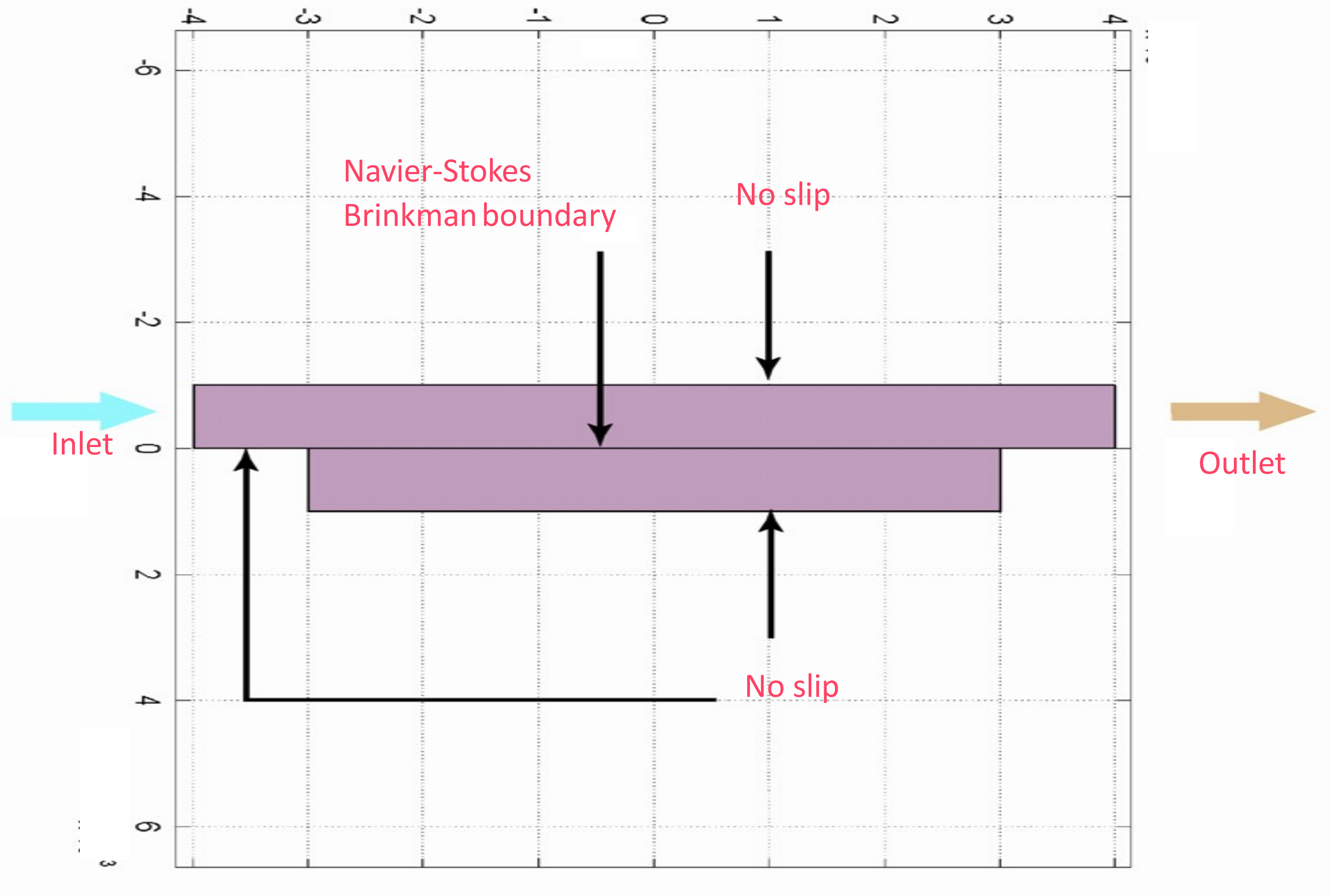
COMSOL Model Geometry



Free flow

Porous structure

COMSOL Model Geometry



Navier-Stokes
Brinkman boundary

No slip

Inlet

Outlet

No slip

Dual Continuum Model for RCA NPL Site

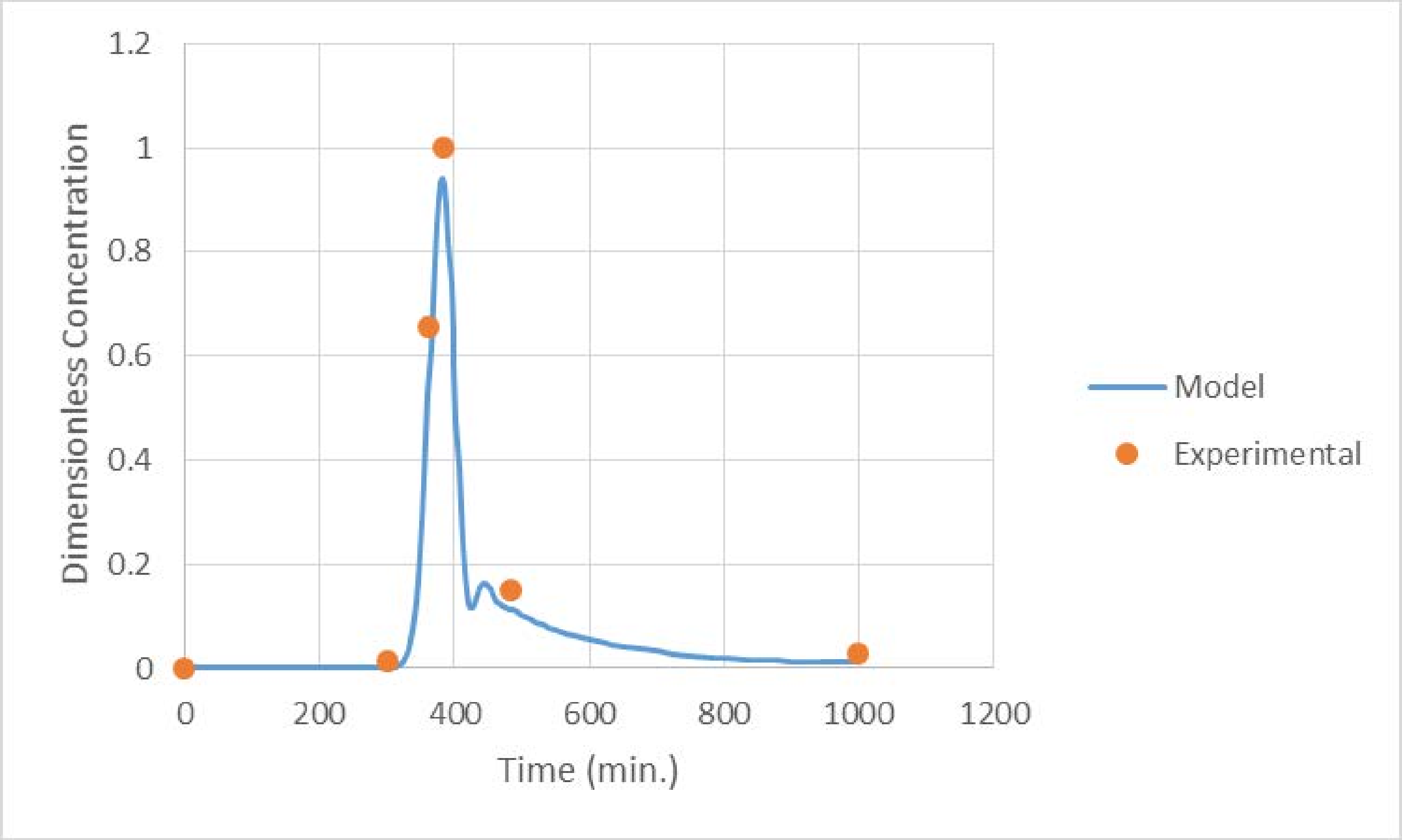
Table 4. Spring discharge values and tracer recovery values at specific times.

t (h)	Q (m^3 s^{-1})	C (mg m^{-3})	$C \times Q$ (mg s^{-1})	$t \times C \times Q$ (mg)
0.00×10^0	3.79×10^{-4}	0.00×10^0	0.00×10^0	0.00×10^4
1.00×10^0	3.79×10^{-4}	0.00×10^0	0.00×10^0	0.00×10^6
2.00×10^0	3.79×10^{-4}	0.00×10^0	0.00×10^0	0.00×10^6
3.00×10^0	3.79×10^{-4}	0.00×10^0	0.00×10^0	0.00×10^6
4.00×10^0	3.79×10^{-4}	0.00×10^0	0.00×10^0	0.00×10^6
5.00×10^0	3.79×10^{-4}	5.00×10^3	1.90×10^0	3.42×10^6
6.00×10^0	3.79×10^{-4}	2.50×10^5	9.48×10^1	2.05×10^5
7.00×10^0	3.79×10^{-4}	3.80×10^5	1.44×10^2	3.63×10^5
8.00×10^0	3.79×10^{-4}	2.00×10^5	7.58×10^1	2.18×10^5
9.00×10^0	3.79×10^{-4}	1.25×10^5	4.74×10^1	1.54×10^5
10.00×10^0	3.79×10^{-4}	7.50×10^4	2.84×10^1	1.02×10^5
11.00×10^0	3.79×10^{-4}	5.50×10^4	2.09×10^1	8.28×10^5
12.00×10^0	3.79×10^{-4}	4.00×10^4	1.52×10^1	6.57×10^5
13.00×10^0	3.79×10^{-4}	2.50×10^4	9.48×10^0	4.44×10^5
14.00×10^0	3.79×10^{-4}	2.00×10^4	7.58×10^0	3.82×10^5
15.00×10^0	3.79×10^{-4}	1.50×10^4	5.69×10^0	3.07×10^5
16.00×10^0	3.79×10^{-4}	1.40×10^4	5.31×10^0	3.06×10^5
17.00×10^0	3.79×10^{-4}	1.30×10^4	4.93×10^0	3.02×10^5
18.00×10^0	3.79×10^{-4}	1.20×10^4	4.55×10^0	2.95×10^5
19.00×10^0	3.79×10^{-4}	1.10×10^4	4.17×10^0	2.85×10^5
20.00×10^0	3.79×10^{-4}	1.00×10^4	3.79×10^0	2.73×10^5
21.00×10^0	3.79×10^{-4}	9.00×10^3	3.41×10^0	2.58×10^5
22.00×10^0	3.79×10^{-4}	8.00×10^3	3.03×10^0	2.40×10^5
23.00×10^0	3.79×10^{-4}	7.00×10^3	2.65×10^0	2.19×10^5
24.00×10^0	3.79×10^{-4}	6.00×10^3	2.27×10^0	1.96×10^5
		$\sum_{i=1}^{n}$	4.85×10^2	1.55×10^7

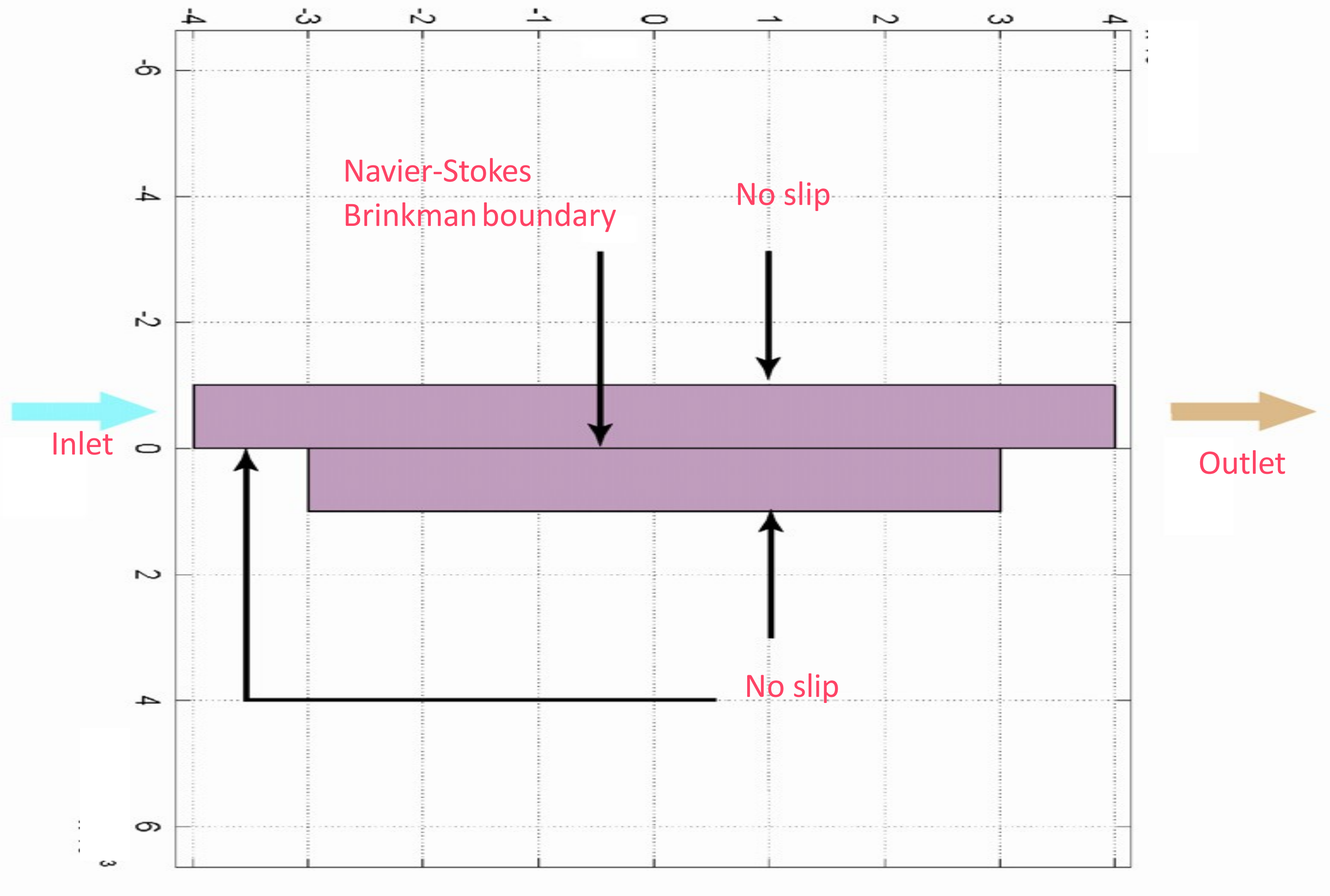
(source: RCA, 1992)

Dual Continuum Model for RCA NPL Site

The bimodal response of the dual continuum model accurately reflects the long upper tail and lag time of the experimental tracer response curve.



COMSOL Model Geometry



Navier-Stokes
Brinkman boundary

No slip

Inlet

Outlet

No slip

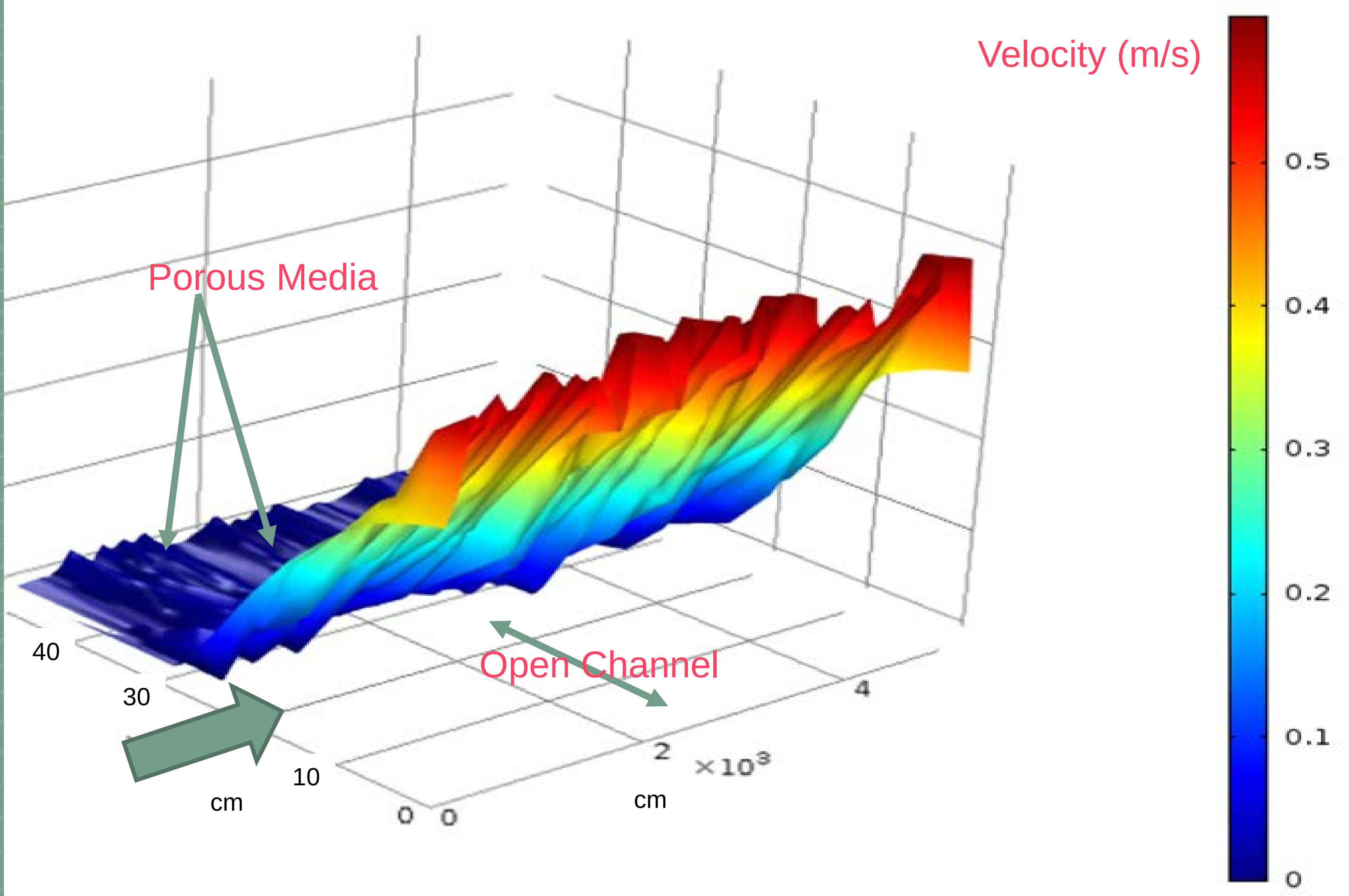
Dual Continuum Model for RCA NPL Site

The dual continuum model also provides additional inference about the nature of the karst groundwater flow.

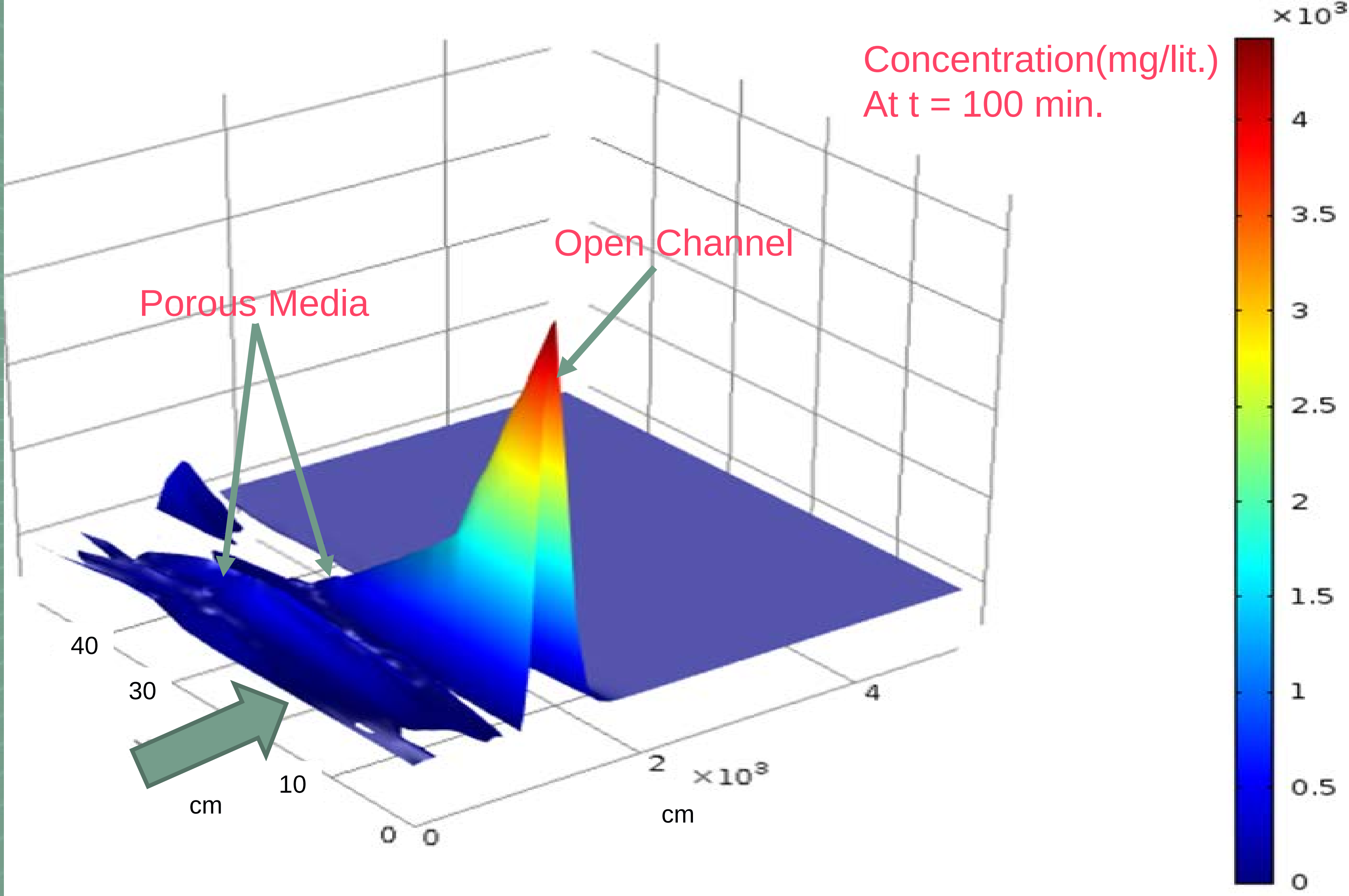
The ADE solution models the aquifer as a 50 meter long tube with a cross-sectional area of 0.24 m^2 and volume of 12.0 m^3.

The dual continuum model models the aquifer as a 50 meter long tube with cross sectional area of 0.24 m^2 and a volume of 12.0 m^3. The model also infers that the 58% of the aquifer volume is open conduit and 42% of the volume consists of a porous matrix with porosity of 0.4 and hydraulic conductivity of 1.0E-3 cm^2

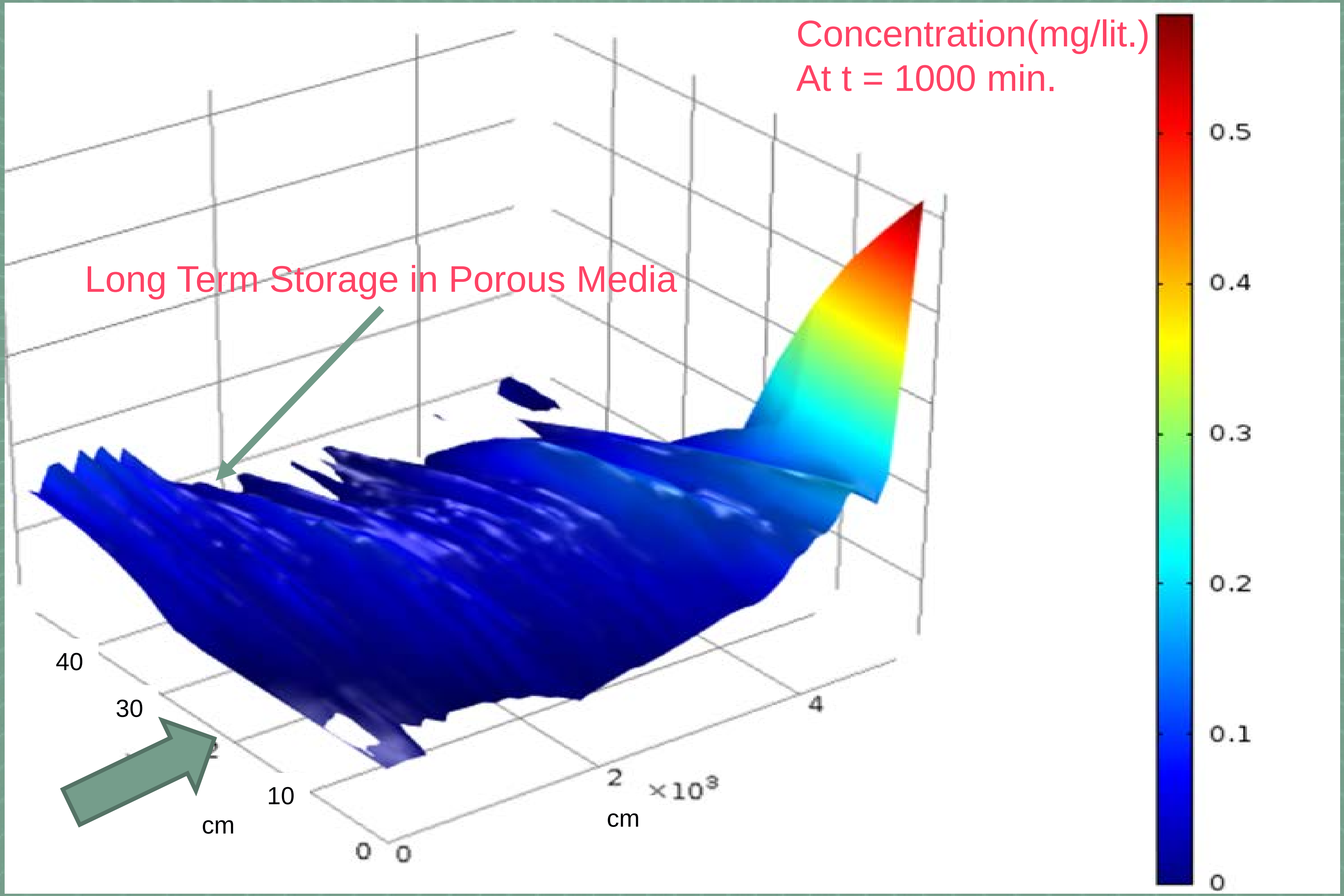
Flow and Mass Transport Visualization



Flow and Mass Transport Visualization



Flow and Mass Transport Visualization



Concentration(mg/lit.)
At t = 1000 min.

Long Term Storage in Porous Media

THANK YOU

QUESTIONS?