Numerical Study on the Acoustic Field of a Deviated Borehole with 2.5D Method

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Abstract: The theoretical model in this paper is a wave-guide structure: a cylindrical borehole filled with fluid with infinite length penetrating a transversely isotropic (TI) solid formation, which also extends to infinity. It is the basic model of acoustic well logging. We use a 2.5D frequency wave-number domain method to simulate the acoustic field with the PDE interface of COMSOL Multiphysics. A convolutional perfectly matched layer (C-PML) is implemented to simulate the infinite solid area. There are three steps to solve such a problem. First, derive the 2.5D wave equation from the 3D form. Second, compare the 2.5D equation with the general formulas provided by COMSOL and obtain the corresponding coefficients. Third, conduct post processing to analyze the mode distribution, the dispersion and the wave form in time domain. This method can be used to analyze other similar model like elliptic borehole or logging-while-drilling borehole with very little modification.

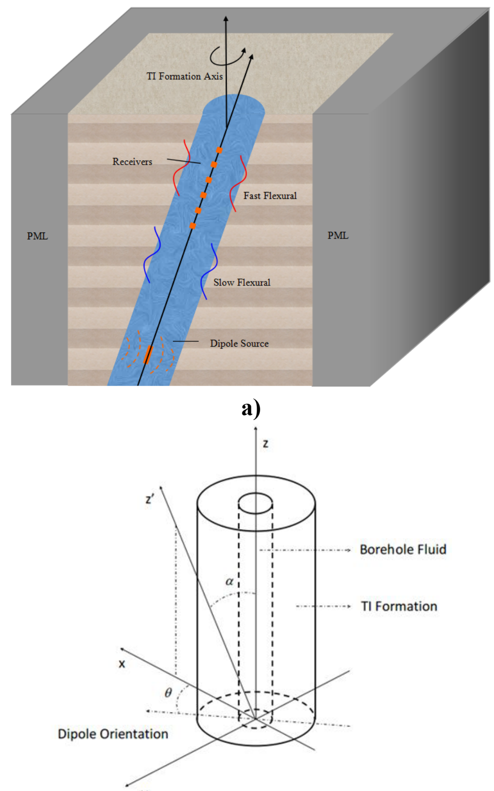
Keywords: 2.5D, PML, borehole acoustics, guided-wave, PDE

1. Introduction

Acoustic well logging is important in oil exploration and development industry. Its simplified theoretical model is a wave-guide structure as a cylindrical borehole filled with fluid penetrating a solid formation (Figure 1.a). Nevertheless, unlike the traditional wave-guide structure, the solid formation extends to infinity, which complicates the problem.

In recent years, significant hydrocarbon reservoirs have been discovered in deep water environments. Offshore development adopts high angle wells to reduce drilling cost. Besides, these offshore reservoir formations often exhibit strong anisotropy [1]. Thus, we need to analyze the wave propagation in such a deviated borehole model.

In this paper, we use the PDE interface of COMSOL to implement the 2.5D frequency wave-number domain method to investigate the wave propagation in a deviated borehole penetrating a TI formation. A C-PML is realized to eliminate the reflections from the artificial truncation boundary, also using the PDE interface.



b)

Figure 1 A deviated borehole model

2. Model and theory

Figure 1.b shows the borehole structure, we assume that the borehole axis is vertical while the inclined angle of the formation symmetric axis is α. The acoustic source is located at the origin of coordinates. The borehole structure keeps invariant in z direction. Using the separation of variables technique, the wave propagation along z direction may be described by $\exp(ikz)$, where k is the wave-number in z direction [2]. Then we have:

$$p(x,y,z,t)= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y,\omega,k)e^{ikz-i\omega t}d\omega dk . \tag{1}$$

Now we just need to compute $p(x,y)$ for different ω and k instead of $p(x,y,z)$, which reduce computing scale greatly.

3. Equation-based modeling

3.1 C-PML and solid area

Figure 2 shows the schematic diagram of the computational model. There exist the circular fluid area, the TI solid formation and the C-PML from inside out.

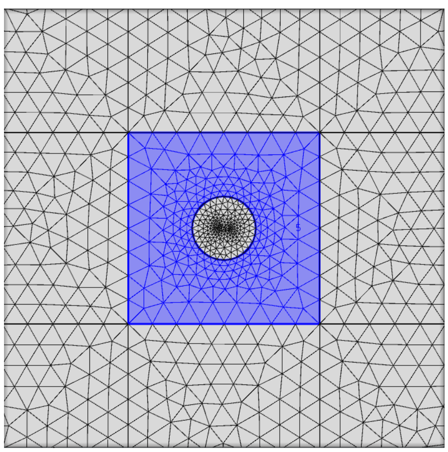


Figure 2 Schematic diagram of computational model

To apply the C-PML, we use the following complex coordinate transformation (take x for instance here, similar operations are carried out on y and z) [3]:

$$\tilde{x} = \int_0^x s_x(x')dx' . \tag{2}$$

So we have

$$\frac{\partial}{\partial \tilde{x}} = \frac{1}{s_x}\frac{\partial}{\partial x} . \tag{3}$$

where s_x is the complex frequency shifted stretched coordinate metrics:

$$s_x = \kappa_x(x) + \frac{\sigma_x(x)}{\alpha_x(x)+j\omega} . \tag{4}$$

Consider the propagation of waves in an elastic solid medium, the equation of motion in frequency domain is:

$$\frac{\partial \tau_{ij}}{\partial \tilde{x}_j} + \rho_s \omega^2 u_i = 0 . \tag{5}$$

And we also have the constitutive relation:

$$\tau_{ij} = C_{ijkl}\varepsilon_{kl} . \tag{6}$$

where

$$\varepsilon_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial \tilde{x}_l} + \frac{\partial u_l}{\partial \tilde{x}_k}\right) . \tag{7}$$

By combining the equation 5~7, we obtain 3D wave equations about displacement. To obatin the 2.5D form, we use $\partial/\partial z = ik$ and $s_z = 1$ since the borehole extends to infinity in the z direction [4]. Compare the results with equation 8.a, the general formula provided by COMSOL:

$$\nabla\cdot(-c\nabla u - \alpha u + \gamma) + \beta\cdot\nabla u + au = f . \tag{8.a}$$

$$n\cdot(c\nabla u + \alpha u - \gamma) + qu = g - h^T\mu . \tag{8.b}$$

$$hu = r . \tag{8.c}$$

we can obtain the corresponding coefficients.

What calls for special attention is that the elastic constants matrix should be rewritten in the new Cartesian coordinate system, using the Bond Transform [5]:

$$C = MC_0M^T . \tag{9}$$

where M is a 6*6 matrix about α . C_0 is the matrix in the $x'y'z'$ Cartesian coordinate system:

$$C_0 = \begin{bmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & 0 & 0 & 0 \\ c_{11}-2c_{66} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} . \tag{10}$$

3.2 Fluid area and boundary

The variable in the equation of fluid area is the displacement potential G, it fulfills the 2.5D frequency domain wave equation:

$$\frac{\partial}{\partial x}\left(-\frac{\partial G}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial G}{\partial y}\right) + \left(k^2 - \frac{\omega^2}{v_f^2}\right)G = 0 \cdot \tag{11}$$

Comparing it with the general formula equation 8.a as before, we can obtain the coefficients.

The physical description of the fluid-solid boundary condition has three items: the continuation of normal displacement and normal stress, the tangential stress equals zero, that is:

$$\begin{cases} \nabla G = \vec{n} \cdot \vec{u} \\ -\rho_f \omega^2 G = \sigma_{rr} \\ \sigma_{rz} = 0 \end{cases} . \qquad (12)$$

Comparing the equations with equation 8.b and 8.c, we can also get the corresponding coefficients.

4. Numerical results

The parameters of TI formation are listed in Table 1, other parameters are $v_f = 1500 m/s$, $\rho_f = 1000 kg/m^3$ and the borehole radius $R = 0.1m$. We use a dipole source to obtain the flexural mode wave. To guarantee the accuracy, the size of the mesh is smaller than $\frac{1}{6}$ of the minimum of the wave-length.

	c_{11} GPa	c_{33} GPa	c_{13} GPa	c_{44} GPa	c_{66} GPa	ρ_s kg/m^3
Austin Chalk	22.0	14.0	12.0	2.4	3.1	2200
Cotton Valley shale	74.7	58.8	25.3	22.0	30.0	2640

Table 1 Parameters of the TI formations

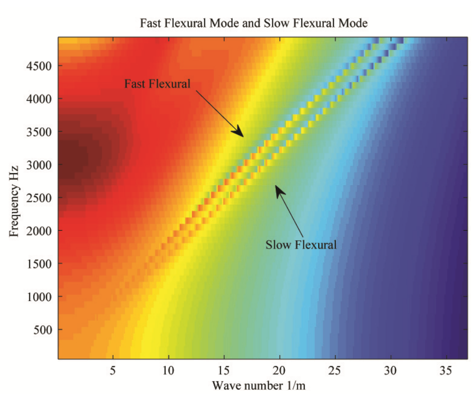


Figure 3 Flexural mode in the frequency wave-number domain

Figure 3 shows the frequency wave-number domain results at $\alpha = 90^\circ$, using soft formation Austin Chalk. It can be clearly seen that the flexural mode splits into fast and slow flexural mode wave corresponding to different dipole orientation.

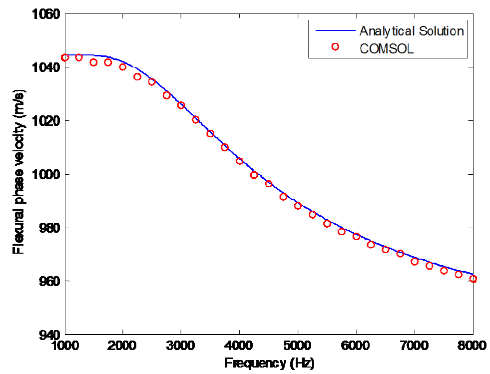


Figure 4 Comparison of flexural phase velocity by COMSOL with the analytic solution

Figure 4 shows the phase velocity dispersion curve which gives a validation of our method. The TI formation is Austin Chalk and $\alpha = 0^\circ$. The blue line represents the analytic result while the red circles are obtained by $v_{phase} = \omega / k$ from the direct frequency wave-number domain result like that in Figure 3. The high accuracy in this figure also indicates the good performance of the C-PML.

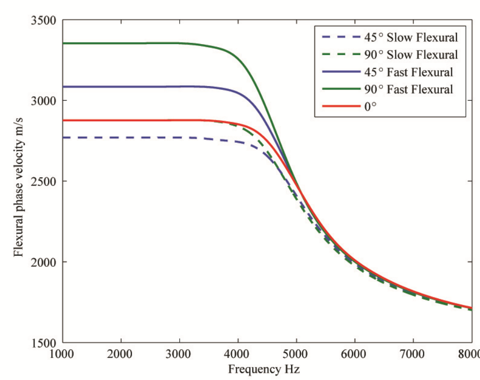


Figure 5 Flexural phase velocity dispersion curves, $\alpha = 0^\circ, 45^\circ, 90^\circ$, hard formation

Figure 5 gives the dispersion curves for different inclined angles $\alpha = 0^\circ, 45^\circ, 90^\circ$, and the TI formation is a hard formation Cotton Valley shale. Results like this is important in the borehole acoustic analysis. It is also the advantage of our 2.5D frequency wave-number method that we can obtain the mode distribution and dispersion clearly and accurately without considering the specific form of source wave form.

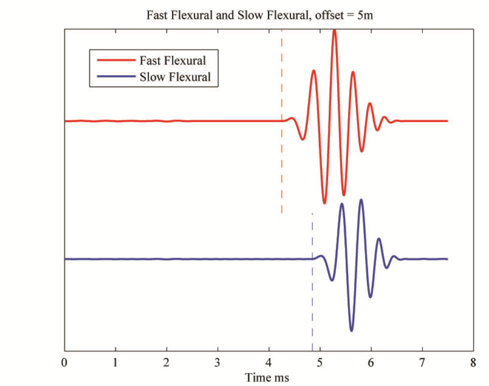


Figure 6 Wave form for fast and slow flexural wave

Although the above are the frequency domain results, we can also obtain the wave form in time domain using equation 1. Figure 6 shows the fast and slow wave form at offset=5m, $\alpha = 90^{\circ}$, the formation is Austin Chalk. From this figure we can see the split flexural in time domain.

5. Conclusions

The PDE interface of COMSOL is a strong tool to conduct equation based modeling and analysis. In this paper, we use this module to implement a 2.5D frequency wave-number domain method to investigate the mode wave propagation in a deviated borehole. There are several advantages of this method:

1. With this 2.5D frequency wave-number method, we can obtain the mode distribution, the phase velocity dispersion curve and also wave from in time domain.

2. The method can be used to analyze other borehole structure such as elliptical borehole and logging-while-drilling acoustic logging with a little modification.

3. As modeling and solving are based on finite element method, it is more accurate than traditional finite difference method, especially when the structure is irregular.

6. Acknowledgements

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7. References

[1] Tang X M, Cheng C H A. Quantitative borehole acoustic methods[M]. Gulf Professional Publishing, 2004.

[2] Ellefsen K J, Cheng C H, Toksöz M N. Effects of anisotropy upon the normal modes in a borehole[J]. The Journal of the Acoustical Society of America, 1991, 89(6): 2597-2616.

[3] Li Y F, Matar O B. Convolutional perfectly matched layer for elastic second-order wave equation[J]. The Journal of the Acoustical Society of America, 2010, 127(3): 1318-1327.

[4] Zhang H, Lin W, Wang X. Numerical Studies on the Acoustic Field Generated by a Dipole Source in Noncircular Pipes[C]. AIP Publishing, 2011, 1335(1): 1424-1431.

[5] Auld B A. Acoustic fields and waves in solids[M]. 1973.