Lithic Hypar: New Frontiers in Structural Stone's Research

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Abstract: This research is based on the mechanical analysis of an innovative reinforced stone's structure, architecturally designed by Prof.Fallacara, University of Bari (Italy): the headquarters entrance portal of the French company SNBR (Sociète Nouvelle le Batiment Règional) located in Troyes (France), the realization of which is planned for October 2015. The shape characterizing the work is the hyperbolic paraboloid, well known for many structural applications related to reinforced concrete shells. The main idea of this lithic reinterpretation of the hypar is to replace reinforced concrete with pre-stressed stone through post-stressed steel bars: the stone is a symbol of these places, so it is a way to join tradition and structural experimentation.

Keywords: structural stone, hyperbolic paraboloid, post-stressed stone, parametric analysis, stereotomy.

Figure 1. Evocative render of the “Lithic Hypar” in front of the SNBR headquarters

1. Introduction

The “Lithic Hypar” introduces to the offices of the SNBR's company and it represents their great knowledge about stereotomic works. The position of the “Lithic Hypar” with respect to the building is absolutely central and it is located just a few meters from the main entrance, conducing visitors inside. Its geometry presents an aspect of considerable importance in terms of structural analysis: the hyperbolic paraboloid can be easily described in an analytic way as a parametric surface. The analytic description allows to directly calculating some geometric quantities that strongly characterize the structural behavior. The Comsol’s module of structural analysis directly provides for the possibility to model shells as parametric surfaces. The hypar is a very well known surface, used in the structural field for various reasons. It is a double curvature surface - in particular, an anticlastic surface - characterized by a negative Gaussian curvature: this means that, with two sectional planes mutually orthogonal, it has a positive curvature (upwards) and a negative one (downward).

2. Working Principles and Technical issues

The membrane solution of a hyperbolic paraboloid with self-weight load consists in the realization of elements having a compression behavior (arch) in the parabola’s direction with the concavity downwards, crossed with elements having a tension behavior (cable) in the parabola’s direction with the concavity upwards.

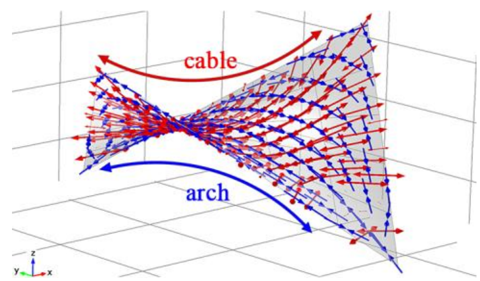

Figure 2. Mechanical behavior of a hyperbolic paraboloid subjected to uniform vertical load

This solutions is consistent only if on the entire boundary there is a shear stress ensuring the global balance. The part of the cables, considering a reinforced concrete work, is performed by post-stressed steel bar, while the arch's one is performed by concrete. The shear stress on the edge is absorbed (such as compression) by special edge beams. The main idea of this “lithic reinterpretation” of the hypar is to replace the reinforced concrete with pre-stressed stone. In this case, the stone is perfectly able to carry out the tasks of the “arches”, while the pre-compression has to eliminate the tensile stresses of the “cables”. The surface’s edge elements are made of steel beams with channel section, which also have the function of transferring the post-stress to the stone, such as evenly distributed compression.

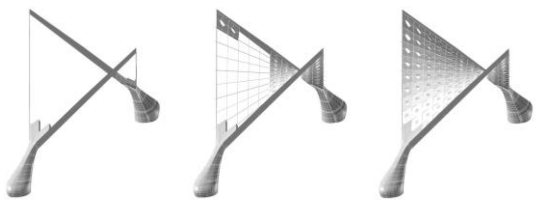

Figure 3. Main steps of construction

3. Linear Elastic Model Approach

Describing the structural analysis of lithic hyperbolic paraboloid it is important to note a few things:
1) The stone vault is a discontinuous structure
2) The ratio between thickness (0,16mt) and size of the surface (7x5mt) is greater than an usual reinforced concrete slab
3) The stone has a Young's modulus fairly high

The first aspect may lead to exclude calculations based on continuous models. However, if the pre-stress introduced is sufficient to eliminate the presence of tensile stresses in joints between an ashlar and the other, it is then reasonable to think the whole time as if it were a continuous shell. The second and the third aspects involve the presence of a flexural stiffness. Furthermore, the stiffness ratio between the elements are such as not to maintain the shape of the surface through thin steel edge beams, absorbing the shear stresses provided by the membrane solution. The maintenance of the shape will be quite assured by flexural stiffness. These considerations suggest, in order to obtain more realistic results, to replace the model of the membrane shell with a richer shell model, in which is also foreseen the presence of bending and torsional moments. In this case, the pre-stressing of the bars becomes even more important, because it must eliminate both the tensile stresses generated by the positive membrane resultant forces (cable behavior) and those generated by bending moments. These aspects allow to consider a linear elastic behavior if the residual tensile stresses are minimal along all the surface.

4. Use of Comsol

The hyperbolic paraboloid can be easily described in an analytic way, thanks to the Comsol software that allows to simply define a parametric surface. The analytic description permits to directly calculate some geometric quantities that strongly characterize the structural behavior and the determination of the stresses on the surface through the structural mechanical module, modeling the hypar as a shell.

The hyperbolic paraboloid is one of the most famous ruled surfaces: it may be generated, indeed, as a geometric place of two families of straights called "generatrices". This feature is particularly important in this case, because of it allows an easier division of stone into modules and, above all, it allows to realize a post-stresses conditions through rectilinear steel bars in order to eliminate residual tractions, arranged along the generatrices.

This connection among geometry and structure allows an easy modeling using the Comsol software.

In this research it were used the structural mechanics module and the sub-interface called shell to launch an analysis in stationary state.

4.1 Geometric Definition

In order to accurately describe the main geometry of the hypar, one of the most easily way in Comsol is represented by the "parametric surface" entity. Thanks to this tool, it is possible to set some important variables that quickly create a complex surface. In this specific case-study, the hypar can be graphically described in an analytic way through its mathematical equations (table 1, table 2). Through the variables substitution, it is possible to obtain the real geometric dimension of the lithic hypar (7x5mt) and the parametric equations are very important to optimize (changing values) the shape related to vertical and horizontal loads that determine the structure's mechanical behavior. With the aim to obtain more realistic response, especially near the constraints, it has been set a user-defined mesh size, extremely dense.

4.2 Parametric Definition

The pre-stressed stone has been modeled in an extremely simplified way, assigning a uniform load on the whole paraboloid's edge, in the direction of the generatrices. Comsol offers the possibility to insert some variables that allow to identify the generatrices in the geometry of the paraboloid. As shown in table 3, it has been defined some geometric parameters starting from the predefined edge/surface variables n, nx, ny, nz, nr: they represent the unit vectors nbx, nby and nbz, the vectors vec1x, vec2x, vec1y, vec2y, vec1z, vec2z and their modulus. These guidelines will also allow to correctly model the strength of post-stress by inducing the steel bars, which are arranged along the generatrices. Another important parameter is set directly from the options of the study to be taken: it was decided to select a study in stationary state and, in the extension panel of the study, it was set a parametric sweep called λ. This variable allows to vary the value of a load on the edge oriented along the generatrices by means of the vectors previously set up to remove possible tensile stresses present in all the surface.

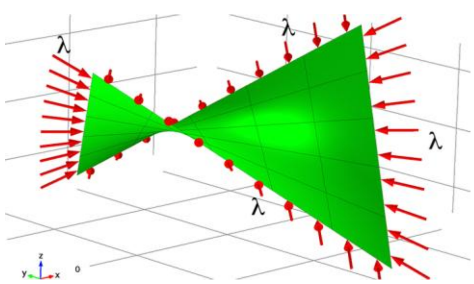

Figure 4. Variable edge load regulated by the parametric sweep, λ

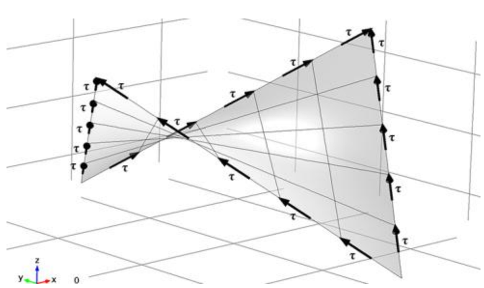

Figure 5. Edge's load effects, shear stresses along the edge

4.3 Boundary and Loads Conditions

The model, at the beginning in an unstressed configuration, has been hedged to the two points of support with fixed constraints, in a relevant way that simulate the real behavior of the reinforced concrete abutments. On the surface of the hypar, have been set the vertical loads, modeled as surface loads, which take into account the self-weight of the structure and the variable loads of snow. Horizontal loads represented by the wind have been modeled as concentrated loads, obtained in accordance with the technical Italian standards (NTC 2008) and applied in the center of mass of the area exposed to the wind. After processing the more demanding load conditions, has been set the force of pre-stress that acts on the surface, modeling it as an edge load which acts on the entire edge of the structure.

4.4 Material Properties Assignment

The used lithotype is the French "Semond Stone", a compact limestone featuring a lot of the buildings in the bordering architectural landscape, often used to restore stone constructions in this area. This material is not present in the program library, but Comsol allows to directly insert its material in the shell's properties menu, in this case characterized by a linear elastic mechanical behavior (in line with the previous hypothesis). Therefore, were set the physical properties of the stone, characterized by a Young's modulus of $3x10^6$ N/cm^2, by a Poisson ratio of 0,3 and a density of 2500 kg/cm^3.

4.5 Post-processing

The Comsol's post-processing tools allow to use simple panels in order to show graphics and useful information on the mechanical response of the model. The optimal value of λ is now easy to find through the two-dimensional plot. In Figure 8 is shown a 2D graph fixing the value of λ: as ordinates, there are the values of the three principal stresses and, in the abscissa, the value of the λ parameter. It is clear that the correct value of λ is defined by the point in which all the three stresses reach the line y=0.

The obtained value is about $0,8x10^5$ N/m^2. Considering that one side of paraboloid is 5mt long, it is possible to estimate that will be applied a normal force of 45kN to the steel bars. Using the three-dimensional plot after setting the value of λ $0,8x10^5$ N/m^2, it is possible to verify that the tensile stresses acting on the surface in the three main directions are negligible, as shown in Figure 7.

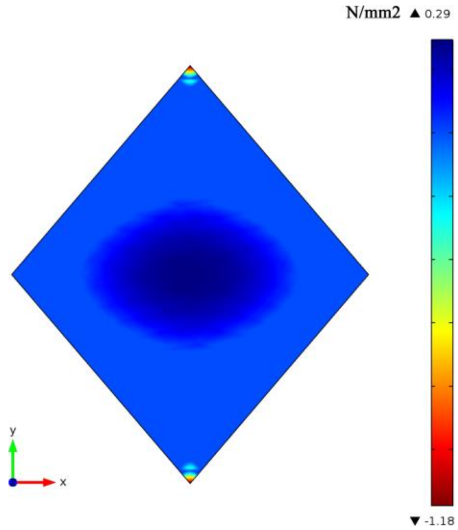

Figure 6. Second principal stress before the pre-stress load along the edge

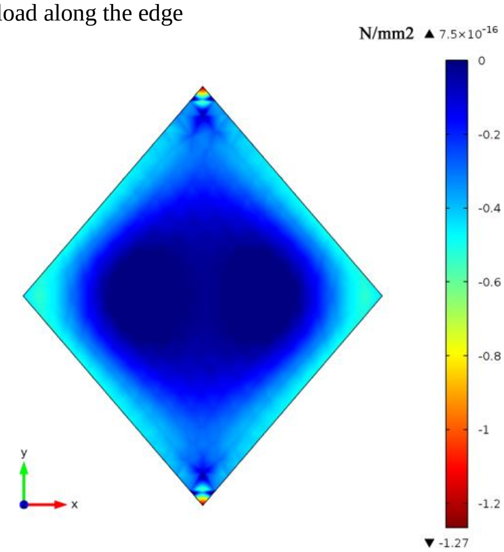

Figure 7. Second principal stress after the pre-stress load along the edge

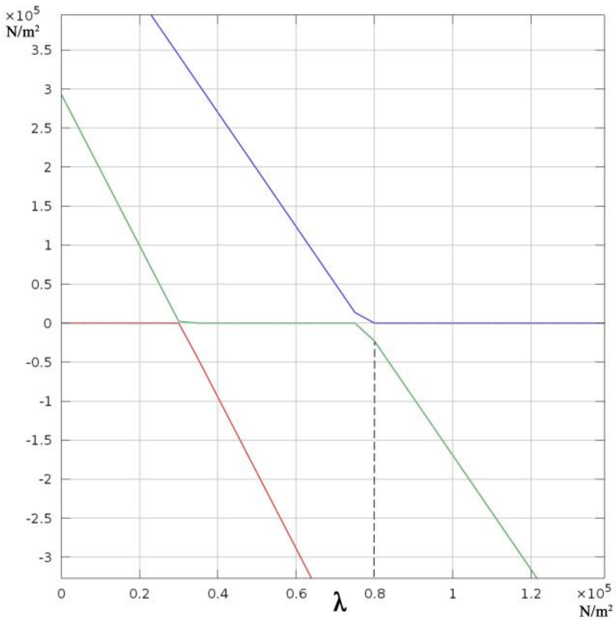

Figure 8. Determining λ, i.e. the steel bar's post-stress value

5. Numerical Results

This research shows a particular kind of interaction between shape and structure, analyzed by active reinforcing elements with the aim of eliminate the problems related to a not optimizing structural configuration at the beginning. It is characterized by the integration of this two approaches: the distinctive paraboloid's geometry - a ruled surface - "suggests" the designer a reinforcements layout along the generatrices, by setting an effective exchange relation between descriptive and mechanical field. The results obtained by this modeling allow a fast and reliable valuation of some important physical quantities: the parameter value, that indicates the necessary pre-compression level, is of about 8 kN/mt, i.e. 45 kN for each steel bar. Moreover, it has been possible to verify that the uniform compression of the stone is admissible and there are no relative slidings among the ashlars, being the normal stresses (multiplied by the static friction value) much more than the shear stresses.

6. Conclusions

This analysis showed accurate and immediately utilizable data on the structural behavior of the architectural work. The adopted modeling allows to in depth examine important technical issues: the possibility to describe the pre-compression in an analytic and geometric way and to foresee the behavior of stone allow the realization of a prototype based on advanced and reliable studies and the reduction of the number of mechanical test on physical models, influencing the work also in economic terms.

7. References

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8. Appendix

Table 1: Geometric values of the hypar

First Parameter			
Name	Minimum	Maximum	Variable
S1	-5	5	X=S1
Second Parameter			
Name	Minimum	Maximum	Variable
S2	-5	5	Y=S2

Table 2: Analytic description of the hypar

Equation	$[S1^2*A^2] - [S2^2*B^2]$	
Name	Expression	Value
A	0,4	0,4
B	A/(3,7*3)	0,32
Z1	$[1,52*A^2] - [3,7/2]2*B^2$	-
Z2	$-[1,52*A^2] - [3,7/2]2*B^2$	-

Table 3: Geometric and Parametric set-up

Name	Expression
nbx	nz*shell.tley-ny*shell.tlez
nby	-nz*shell.tlex+nx*shell.tlez
nbz	ny*shell.tlex-nx*shell.tley
vec1x	1/mod1
vec1y	-a/b/mod1
vec1z	2*a*(a*x+b*y)/mod1
mod1	sqrt(1+a^2/b^2+4*a^2*(a*x+b*y)^2)
vec2x	1/mod2
vec2y	a/b/mod2
vec2z	2*a*(a*x-b*y)/mod2
mod2	sqrt(1+a^2/b^2+4*a^2*(a*x-b*y)^2)