Sensibility Analysis of Inductance Involving an E-core Magnetic Circuit for Non Homogeneous Material

K. Z. Gomes[*1], T. A. G. Tolosa[1], E. V. S. Pouzada[1]
[1]Mauá Institute of Technology, São Caetano do Sul, SP, Brazil
*kevin.zezel@hotmail.com

Abstract: In this work, a methodology is developed, based on the application of finite element method in the frequency domain, aiming the sensibility analysis of inductance calculation involving some configurations of an E-core magnetic circuit. Such important analysis, are made from several geometries and considering different frequencies for the source current applied, providing enough information to study magnetic phenomena, intrinsic problems, like: Foucault losses, frequency response, magnetic reluctance variation, skin effect and temperature distribution. A detailed description of this study is presented in this paper with an example of application involving an inductive sensor that can be used in the sense of providing parameters for a control system in a magnetic bearing.

Keywords: Inductive sensor, magnetic circuit, finite-element method, frequency domain analysis.

1. Introduction

Many problems concerning inductance calculation and consequently involving analysis of current distribution in conductive media refer to proximity effects and eddy currents. Those phenomenas are responsible for changing the profile of current distribution and in general can lead to increase losses and heating of the conductive material [1].

Time varying electromagnetic fields in conductive media can be analyzed in terms of the current density vector, usually defined by an integral equation, which analytical solution is possible only in simple configurations. For arbitrary complex structures dealing with non-homogeneous media, numerical procedures are developed either in time domain or in frequency domain and, among them, the Finite Element Method (FEM) can be applied.

In the case of presence of nonlinear materials, a time domain analysis is indicated to obtain accurate solutions [2, 3, 4].

A frequency domain approach is also possible and the solution is obtained successively for different frequencies that represent the spectrum of the excitation function. The limitation of this approach is related to Gibbs phenomenon when dealing with transient analysis which leads to difficulties to describe a transient excitation, especially in the case of waveforms with small rise and fall times, as a square wave, for example. The frequency domain study can be used to compute response of linear or linearized models subjected to harmonic excitation, therefore the magnetic non linearity of the material is neglected [5, 6]. The study of many parameters that are inherent of inductance calculation, like frequency response and proximity effects, could be taken into account to develop sensibility studies in association with a proximity inductive sensor [7]. The problem consists, fundamentally, by an oscillator circuit that is responsible for generating a high frequency electromagnetic field over the inductor. The electromagnetic principle for detection is related to an interaction to the field lines with one metallic object positioned near to the inductor. The proximity of the object modifies the magnetic reluctance of the system which could be detected with one extern circuit.

FEM analysis requires a convenient mesh grid study to obtain a correct and optimized solution about the model [8]. The optimization of the mesh is related to some specific physics characteristics, like skin depth, which determines the numbers of elements over the wave length. These characteristics only exist in specifics areas, but it is enough to create a high computational solution process.

2. EM fields in conductors

The problem of electromagnetic analysis at a macroscopic level is that of solving Maxwell's equations subject to certain boundary conditions. Maxwell's equations are a set of equations, written in differential or integral form, stating the relationships between the fundamental electromagnetic quantities.

The equations can be formulated in differential form or integral form. The differential form is presented here because it leads to differential equations that the finite element method can handle.

The use of auxiliary vector potentials helps to obtain solutions for the electric and magnetic fields is a common practice in the analysis of electromagnetic boundary-value problems. Considering a linear, homogeneous and isotropic medium, and by the definition of the magnetic vector potential $\vec{A}(t)$, a basic equation can be written:

$$\vec{\nabla}\times\vec{E}+\frac{\partial\left(\vec{\nabla}\times\vec{A}\right)}{\partial t}=\vec{\nabla}\times\left[\vec{E}+\frac{\partial\vec{A}}{\partial t}\right]=0\,. \qquad (1)$$

Then, assuming a scalar potential Φ,

$$\vec{E}+\frac{\partial\vec{A}}{\partial t}=-\vec{\nabla}\Phi\,. \qquad (2)$$

From equation (2), the current density $\vec{J}(t)$ in a conductor region with conductivity σ, excited by a given electric field $\vec{E}_o(t)=-\vec{\nabla}\Phi$, can be calculated by

$$\vec{J}(t)=-\sigma\cdot\frac{\partial\vec{A}(t)}{\partial t}+\vec{J}_o(t)\,, \qquad (3)$$

where $\vec{J}_o(t)=\sigma\cdot\vec{E}_o(t)$.

In the case of conductive media, where the displacement current can be neglected, the magnetic vector potential, imposing $\vec{\nabla}\cdot\vec{A}=0$, relates to the current density by

$$\nabla^2\vec{A}=-\mu\vec{J}\,. \qquad (4)$$

Equation (4) shows that $\vec{A}(t)$ is defined by an integral of the current distribution, so that equation (3) takes the form of an integral equation on $\vec{J}(t)$.

3. Numerical Analysis

In the study of an electromagnetic device some information about electrical, mechanical and magnetic aspects must be considered in the model design. Such aspects are interrelated by physical phenomena and the equations that describe them allow temporal and spatial analyzes of all the phenomena in question.

Therefore, practical problems can be involved by solutions for multiphysics systems with complex structures considering arbitrary geometries dealing with the non-linearity of materials. In this case, the solutions are obtained only by application of numerical methods, so that in this work, COMSOL Multiphysics® software is used to make the sensibility analysis of inductance calculation in an E-core magnetic circuit.

3.1 Geometry of the problem and materials

Figure 1 shows the inductor geometry with the measure units in cm. The source current is imposed to the coil in the central part of the core ensuring larger contact area between the core and the target. It provides more efficiency in the sensibility analysis of the inductance.

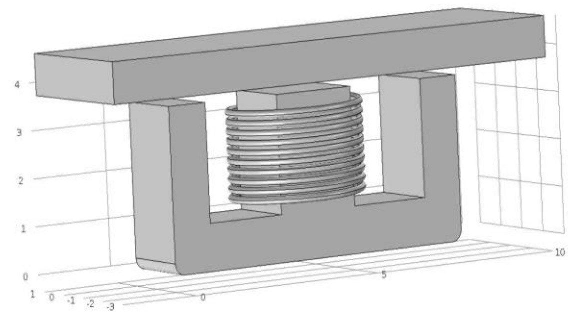

Figure 1. Geometry of the inductive sensor.

Some electrical properties like relative permeability, relative permittivity and conductivity of the materials used in the computational simulations are shown in table I.

Table I – Electrical properties of the materials.

	(μ_r)	(ε_r)	(σ) S/m
Copper	1	1	5.998x10^{7}
Ferrite Thornton IP12r (core)	4000	1	1.000x10^{-6}
Low Carbon Steel 1020 (target)	750	1	8.410x10^{6}

As stated earlier the magnetic non linearity of the materials is neglected due to the frequency domain analysis used to solve the problem so it is important to notice that hysteresis is neglected in the present work.

Even so the Foucault losses are taken into account but it depends on the frequency and materials parameters. The energy loss appears as heat which increases the temperature of the materials until a steady state is established. In this way the heat that transfer simulations will be used into stationary study.

Table II shows the thermal properties of the materials: heat capacity C_p, density ρ and thermal conductivity k.

Table II – Thermal properties of the materials.

	C_p J/(kg.K)	ρ kg/m³	k W/(m.K)
Copper	385.0	8700	400.0
Ferrite Thornton IP12r	33.1	4800	2.9
Low Carbon Steel 1020	450.0	7870	51.9

The first physics interface used in COMSOL Multiphysics® was the *Magnetic Fields* using the *multi-turn coil domain* for applying a source sinusoidal signal with amplitude 15 V to the coil with 50 turns and wire thickness AWG 25. The voltage was applied through an external circuit defined by the physics interface *Electric Circuit*. The materials parameters were predefined and the boundary conditions defined as *default* were used for the simulations.

It was applied the physics interface *HeatTtransfer in Solids* responsible to establish boundary conditions, as well as the properties of the materials used in the simulations. It was also established an external convection condition, which corresponds to a convection coefficient of 20 W/(m².K). Thus, through all the physics used it was possible to link them from the multiphysics node *Electromagnetic Heat Source*.

3.2 Simulations results

The Foucault losses are evident in time dependent problems. The effect of these losses worsens with increasing frequency, because the induced voltage in a system susceptible to time-varying fields is proportional to the derivative of the magnetic flux. Therefore, materials with high electrical conductivity reduce the apparent resistance of the target conductor. In order to verify the distribution of Foucault losses, it was imposed to the proposed system a frequency of 10 kHz and a distance gap that is fixed near to zero.

Induced currents cause the Joule effect and may worsen with the appearance of the skin effect. So that the effective electrical resistance of the conductor increases in relation to the resistance measured at constant current. Consequently there is more loss by heat, per unit length, and can heat the system significantly.

Figure 2 shows the spatial distribution of temperature at an initial time of simulation using the time domain analysis.

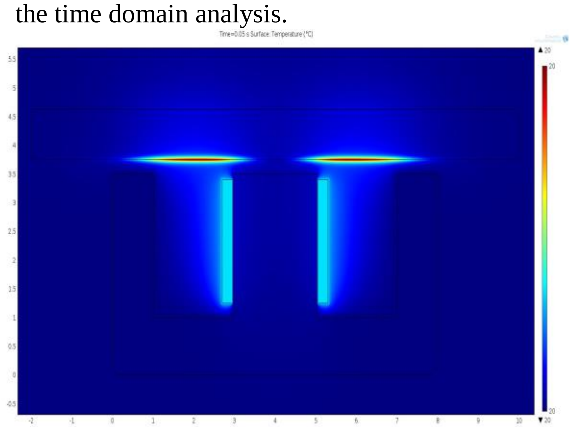

Figure 2. Temperature distribution imposed by induced currents and the source current in the coil.

Another characteristic to consider is that the target is a magnetic medium, with μ_r =750. As a result, the skin effect also influences the magnetic distribution because the induced currents prevent the magnetic flux penetrate the material completely, reducing the effective permeability, as can be seen in figure 3.

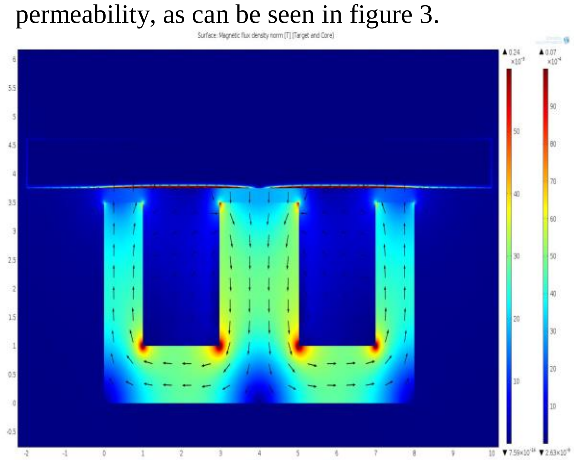

Figure 3. Surface magnetic flux density.

Another important characteristic observed is the high concentration of magnetic flux density in the core edges, increasing the relative permeability in these regions.

The principle of inductive sensor detection is based on the variation of inductance due to the gap variations between the core and the target.

Figure 4 shows some results for different gaps about the frequency response of coil inductance. It can be noticed that inductance decreases with the increasing frequency since the losses are directly proportional to the frequency. It occurs because the greater the induced current in the target generates greater magnetic flux from the eddy currents, which have an opposite direction from the source flux.

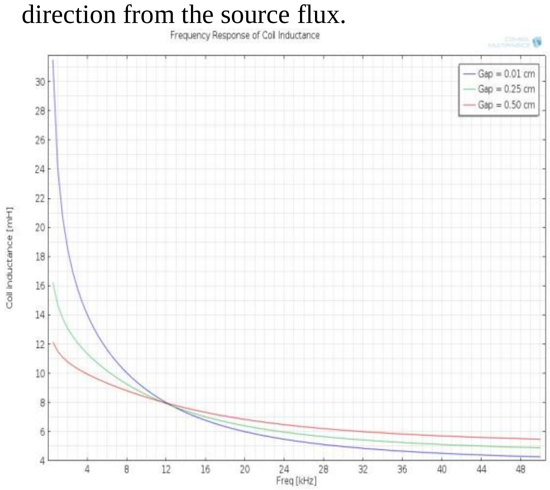

Figure 4. Coil inductance versus frequency for some gaps applied in the study.

The results obtained show that the frequency response curves have an intersection at a certain cutoff frequency of approximately 12 kHz. It can be noticed that for frequencies below the cutoff frequency, the inductance falls with increasing gap, such that the lower the frequency the higher is their sensibility to gap variations. Whereas for frequencies above the cutoff point the inductance is directly proportional to the gap with linear variation, but with significant lower sensibility when compared to frequencies below the cutoff point.

The first analysis deals with the inductance values below the cut-off region, because it has the largest inductive values of sensibility related to the variations of the gap. This characteristic provides the best applicable frequency range for the development of inductive sensor and it is important to define the operation frequency. For low frequencies, at 50 Hz or 60 Hz for example, the inductance variations are high, reaching around 80 mH with the gap near to zero, but all the variations are concentrated by 0.20 cm, making it impossible to measure beyond this distance. From this analysis, it was determined an operating frequency of 1.0 KHz to satisfy sensibility conditions, range and linearity.

Figure 5 shows another region of interest. It is set to frequencies above the cutoff point where the behavior of the inductance can be considered atypical. However, even this region having linear characteristics it will not be used in inductive sensing due to lack of sensibility.

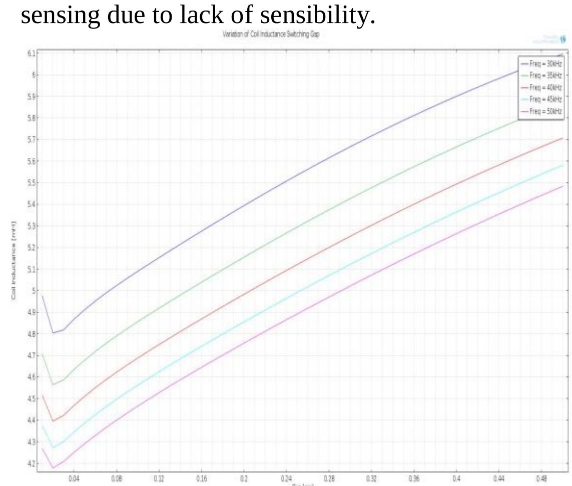

Figure 5. Coil inductance versus gap for some frequencies of study.

From the previous curves, it is possible to verify the existence of two distinct effects present in the variability of inductance depending on the distance of air gap. The first is the magnetic reluctance that increases with the increasing of the gap. The second consists on the contribution of the skin effect and eddy currents that increases with the frequency. These effects will be studied in a future work.

The sensibility analysis of inductance now can be made considering an external circuit that encodes the inductance and turn into an electrical signal depending on the air gap distance . Direct changes in inductive sensibility require changes of physical parameters of the system or change in operating frequency. However, as stated previously, it is not feasible to use different frequencies of 1.0 kHz, below this level, despite the increased sensibility in this case, the range is compromised. Above this level, the sensibility decreases and there is the undesirable values close to the cutoff point where the behavior is highly non-linear. Thus optimization of the sensibility that is obtained through an external circuit.

The RLC circuit is a simple analog band pass filter of the first order with the inductor the sensor itself. Through variations of the inductance caused by the increase of the air gap, it is possible to modify the point of maximum gain of the filter to yield specific gain for each gap. Such that for a fixed operating frequency the excursion is obtained under the asymptotes of the filter, allowing more sensibility.

Figure 6 shows the bode diagram of the filter for various gaps and figure 7 shows the relation between output voltage of the filter and gap for a given frequency of 1.0 kHz.

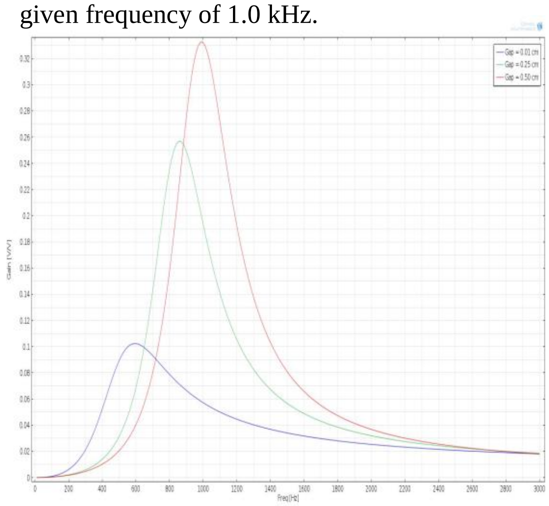

Figure 6. Bode diagram of the band pass filter for some gaps of study.

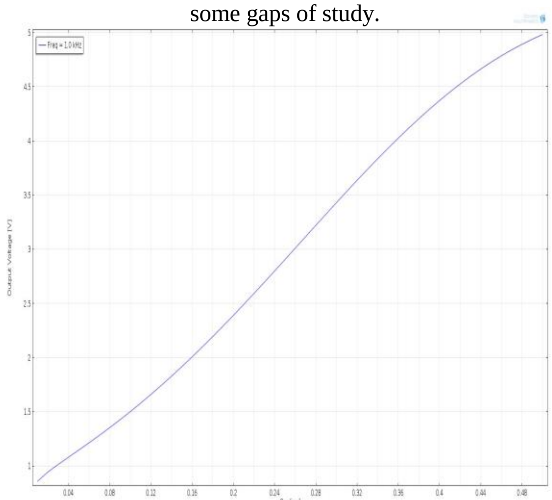

Figure 7. Output voltage of band pass filter versus gap for a frequency of 1.0 kHz.

In order to compare the numerical method solution with a theoretical method it was implemented the electric circuit of figure 8 and the analysis in sinusoidal steady state was done.

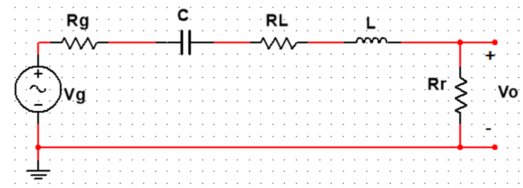

Figure 8. Electric Circuit used for solutions comparison.

Considering the sinusoidal steady state, the frequency response is obtained (5):

$$\frac{\dot{V}_o(j\omega)}{\dot{V}_g(j\omega)} = \frac{jR_r C\omega}{\left(1-\omega^2 LC\right) + j\left[\omega C\left(R_g + R_r + R_L\right)\right]} \quad (5).$$

By neglecting the internal generator resistance, the modulus of (5), yields (6):

$$\left|\frac{\dot{V}_o(j\omega)}{\dot{V}_g(j\omega)}\right| = \frac{R_r C\omega}{\sqrt{\left(1-\omega^2 LC\right)^2 + \left[\omega C\left(R_r + R_L\right)\right]^2}} \quad (6).$$

The sensor coil was modeled as an ideal inductor complemented with a resistance of loss provided numerically by COMSOL Multiphysics®. It was obtained the best-fitted curve for comparison in figure 9.

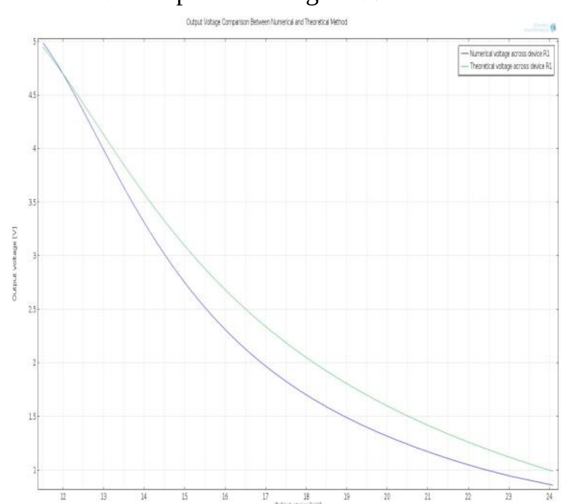

Figure 9. Numerical and theoretical solutions for the output voltage versus coil inductance.

It can be noticed the proximity in the solutions given by the numerical method and the theoretical approach. The band pass filter provides optimal output variation in gap function, increasing its apparent sensibility.

4. Conclusions

The paper has presented an efficient procedure for frequency domain analysis of inductance calculation and such sensibility analysis in the case of non-homogeneous media through the application of FEM.

Some particular configurations of an E-core magnetic circuit were analyzed and the results for the inductance calculation have shown the effects of the problem geometry, material parameters and the way currents flow on the coil.

For the application of the method it is important to take into account the corresponding stability condition that was not analyzed in this paper and can be shown in a future work.

The proposed method can be improved by application of techniques for better performance of the Method and tools for reducing the computational effort by scaling the values of the elements generated in the mesh.

5. References

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