Finite Element Evaluation of *J*-integral in 3D for Nuclear Grade Graphite Using COMSOL-Multiphysics

Presented by

Awani Bhushan
S.R.F. (B.R.N.S, DAE, India)
Research Scholar
Department of Mechanical Engineering
IIT (BHU) Varanasi

COMSOL
CONFERENCE
2015 PUNE

Organization of presentation

- Introduction
- Finite Element model
- Bimodular formulation
- Results and discussion
- Conclusion

Introduction

Advantages of nuclear Energy

- **Capable of full-fill the energy crisis**
- **Less Polluting**
- **Reliable**
- **Economical**

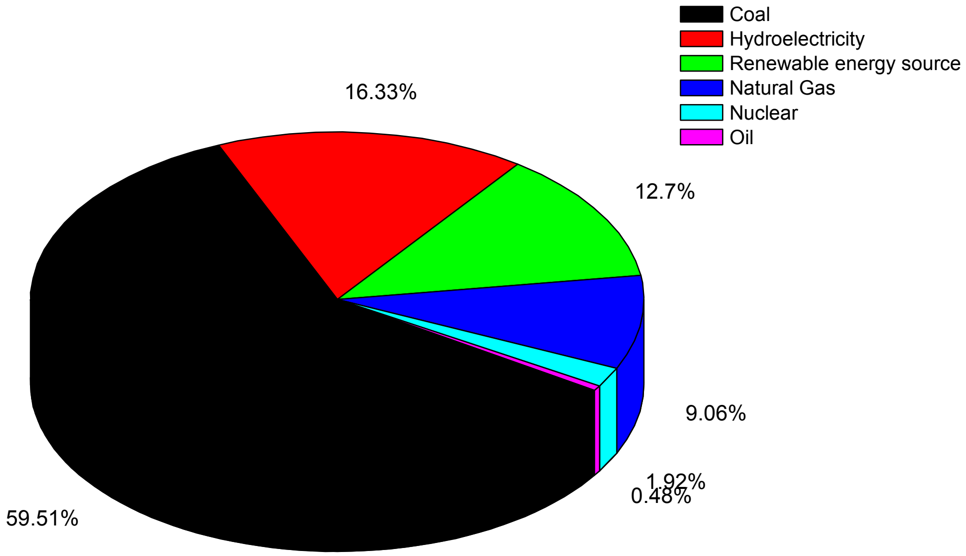
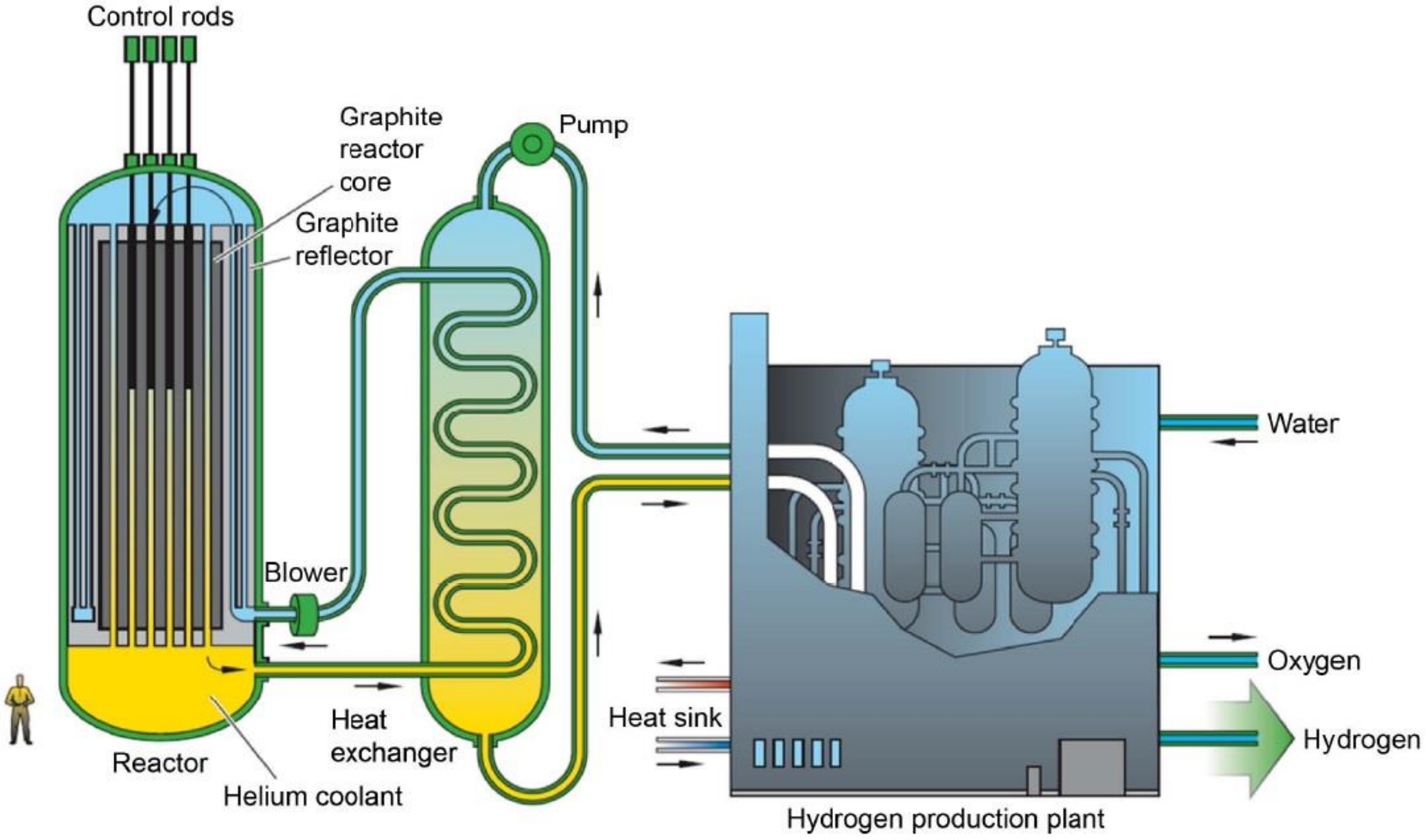


Figure: Distribution of energy in India (Wikipedia)

Evolution of Nuclear reactor

Generation IV Nuclear Reactor



Nuclear Grade Graphite

USE

- Moderator
- Built material
- Fuel Element

Properties

- Stable at high temperature
- Bimodular in nature
- Brittle

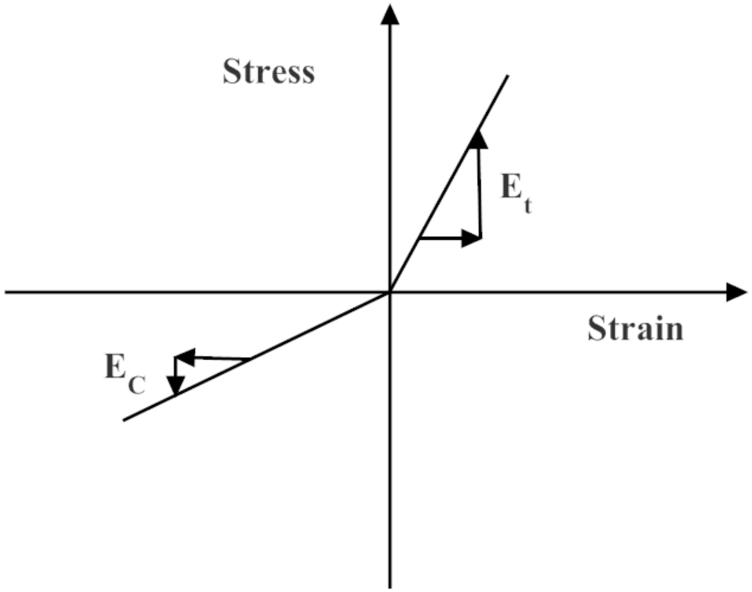


Figure 4: Bi-modulus material having different Young's Modulus in tension and compression

Nuclear Grade Graphite (contd..)

Modeling

- Bi-moderator Finite Element formulation
  - Cracked three point bend specimen

Objective

- To evaluate
  - J-integral for a range of E_t/E_c ratio.
  - The effect of bi-modularity on stress region belonging to tension and compression.

Finite Element Model

Problem Formulation

Crack length 5mm
Specimen
length=250mm
height=25mm
Load P= 500 N
E_t=7.14 757GPa
E_c=3.9145 GPa
(Graphite design
handbook)

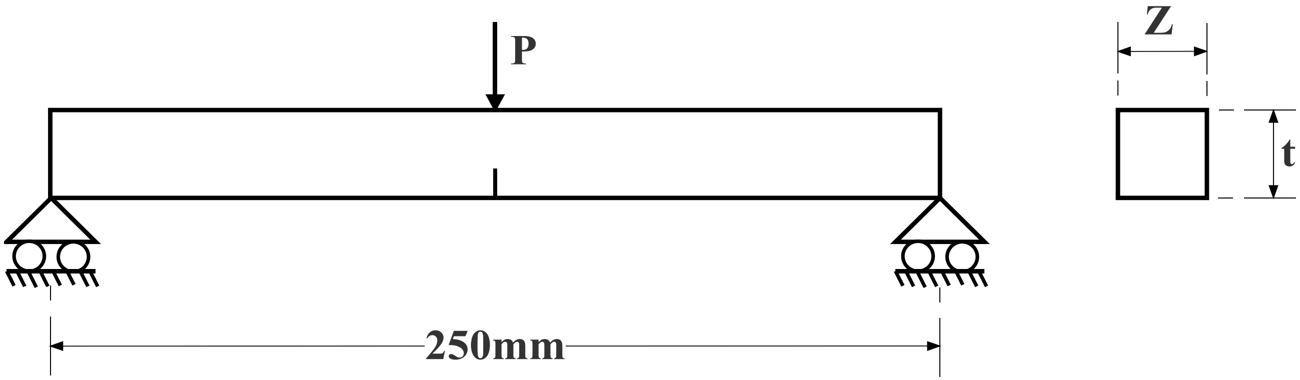
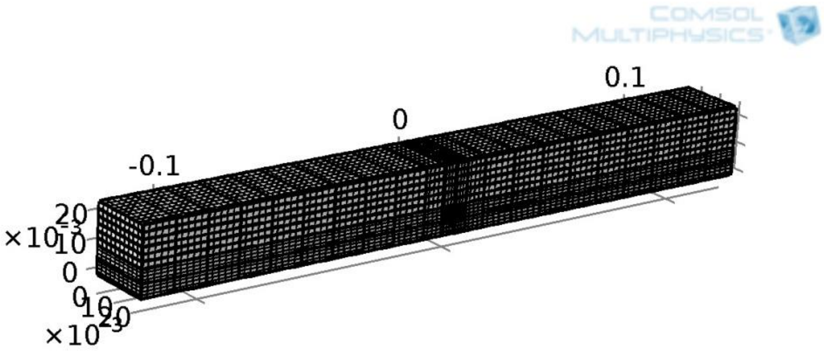
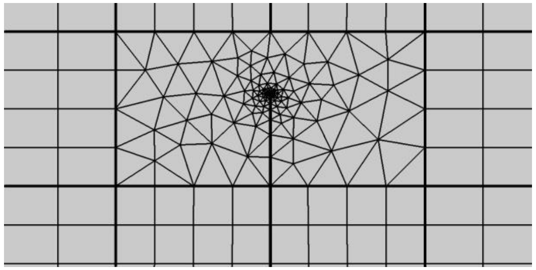


Figure : Single edge-crack bend specimen having point load at the middle

Mesh Distribution



3D mesh whole geometry



3D mesh near crack tip

Discretized into 14520 hexahedral elements present in 3D mesh

Bi-Modular formulation

- Young's Modulus of elasticity in tension /compression is controlled by a step function. Stress strain relation is defined by:

$$\varepsilon = \left(\frac{U(\sigma)}{E_T} + \frac{U(-\sigma)}{E_C} \right) \sigma$$

Where U(x) is a **step function,** and U(x)=1, when x>0, otherwise U(x)=0

COMSOL implementation

Steps:

- Define a step function for Young’s modulus of elasticity.
- Built the geometry.
- Define boundary condition.
- Define the weak contribution.

  test(p)*(p-solid.pm)

$$solid.pm = \left(-\frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} \right)$$

- Define auxiliary dependent variable p.
- Solve the model.
- Define path and area to evaluate the J-integral value in 3D.
- Define the integral and differential expression for J-integral evaluation.

J-integral for 3D (R.H. Dodds, 1987)

$$J_{C1}(s) = \int_{\Gamma} W^{e} n_{1} \, d\tau$$

$$J_{C2}(s) = \int_{\Gamma} W^{p} n_{1} \, d\tau$$

$$J_{C3}(s) = -\int_{\Gamma} u_{i,1} T_{i} \, d\tau$$ (i=1,2,3)

$$J_{A1}(s) = -\int_{A} W^{p}_{,1} \, dA$$

$$J_{A2}(s) = \int_{A} \left(\sigma_{ij} \varepsilon^{p}_{ij,1}\right) dA$$ (i=1,2,3)

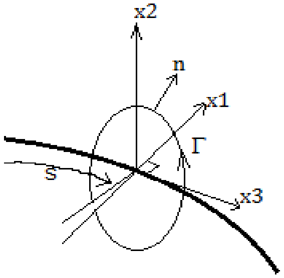
$$J_{A3}(s) = -\int_{A} (\sigma_{i3} u_{i,1})_{,3} \, dA$$ (i=1,2,3)

$$J(s) = JC_{1}(s) + JC_{2}(s) + JC_{3}(s) + JA_{1}(s) + JA_{2}(s) + JA_{3}(s)$$

For, Elastic region only

$$J(s) = J_{C1}(s) + J_{C3}(s) + J_{A3}(s)$$

$$J(s) = \int_{\Gamma} W^{e} n_{1} \, d\tau - \int_{\Gamma} u_{i,1} T_{i} \, d\tau - \int_{A} (\sigma_{i3} u_{i,1})_{,3} \, dA$$



Crack front

Results & Discussion

Contour for J-integral evaluated

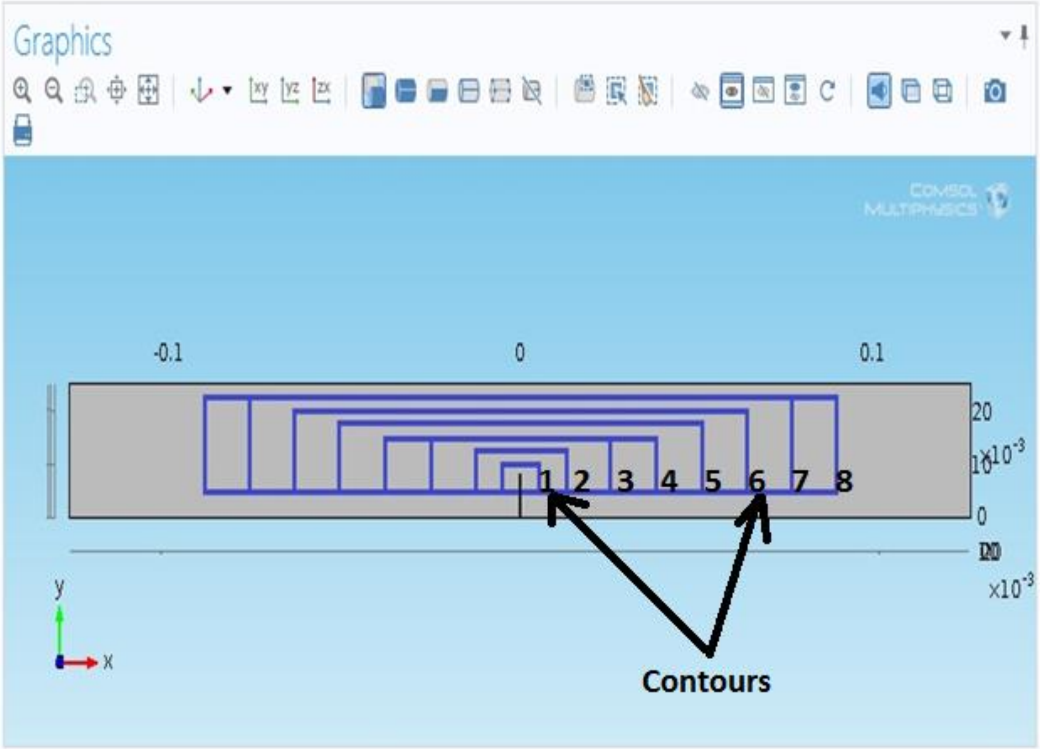
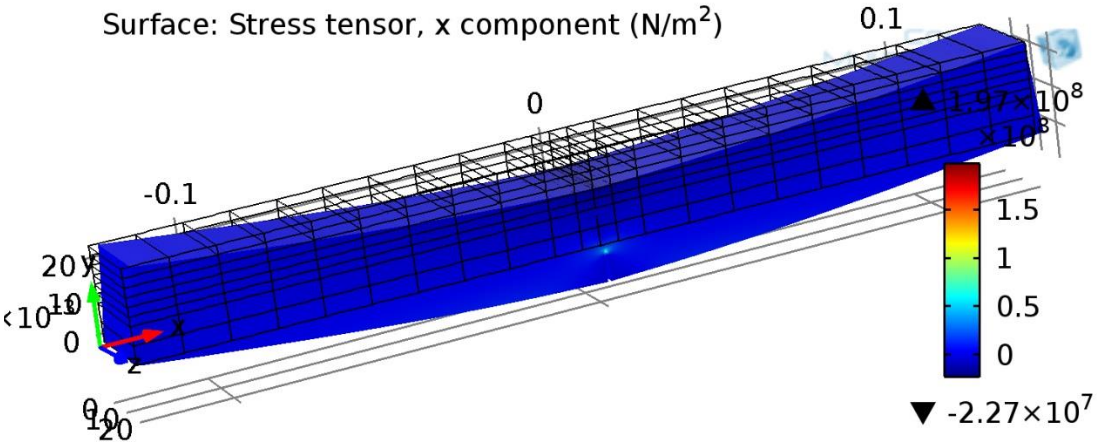
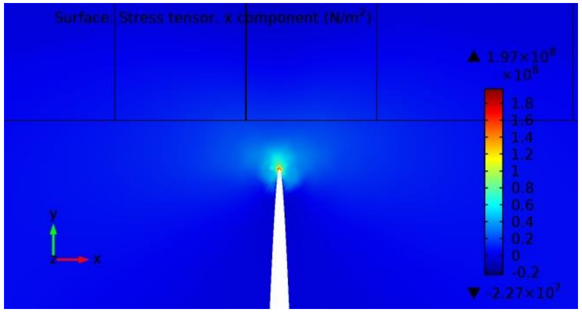


Fig: Contour for 3D J-integral

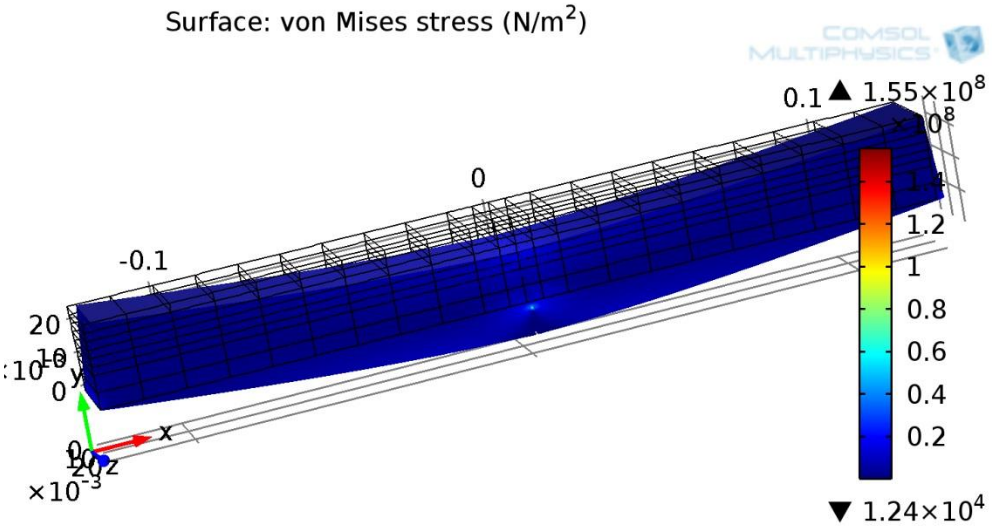
Normal stress distribution in X-direction

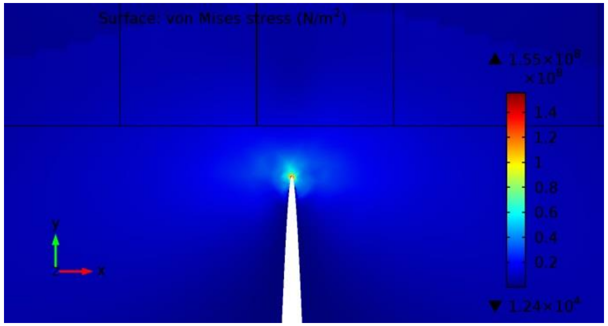




near crack tip

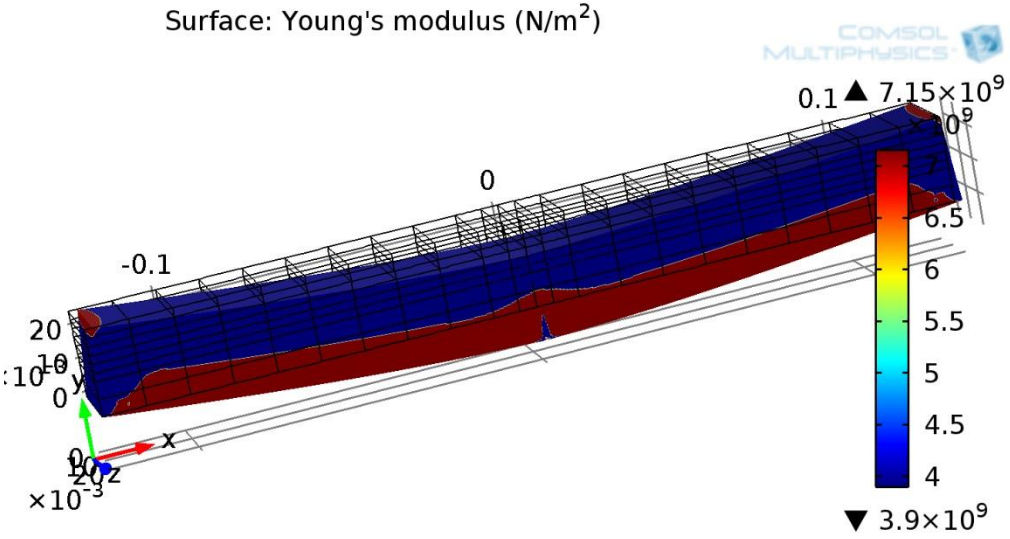
Von-Mises stress distribution

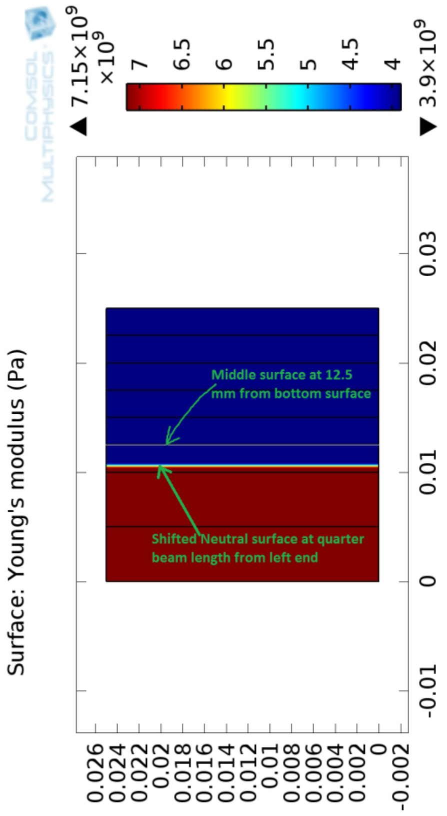




near crack tip

Young's Modulus plot





Middle surface at 12.5 mm from bottom surface

Shifted Neutral surface at quarter beam length from left end

Path-independent J-integral in 3D

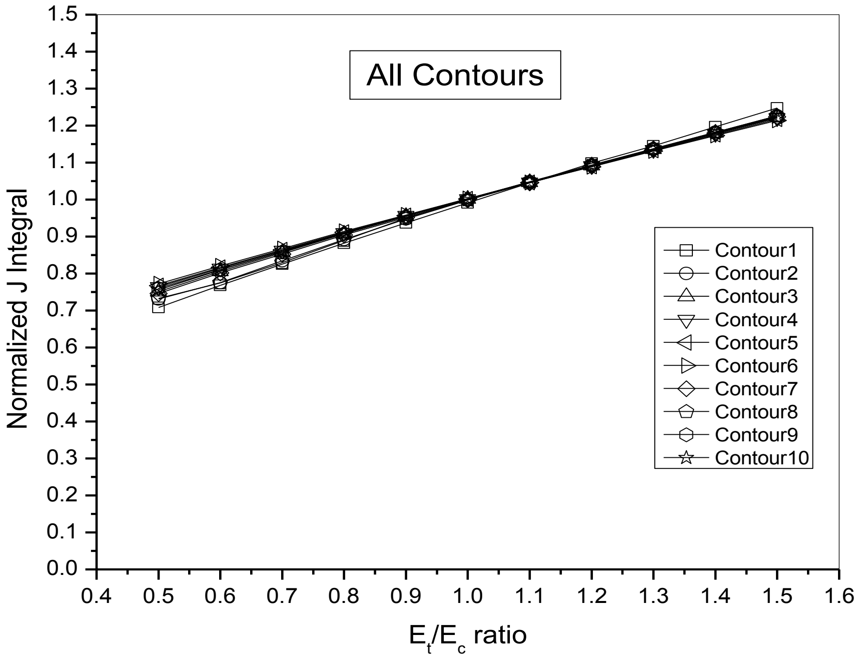


Fig.: Effect of E_t/E_c ratio on *J*-integral for three-point bend specimen in 3D

J-integral in 3D

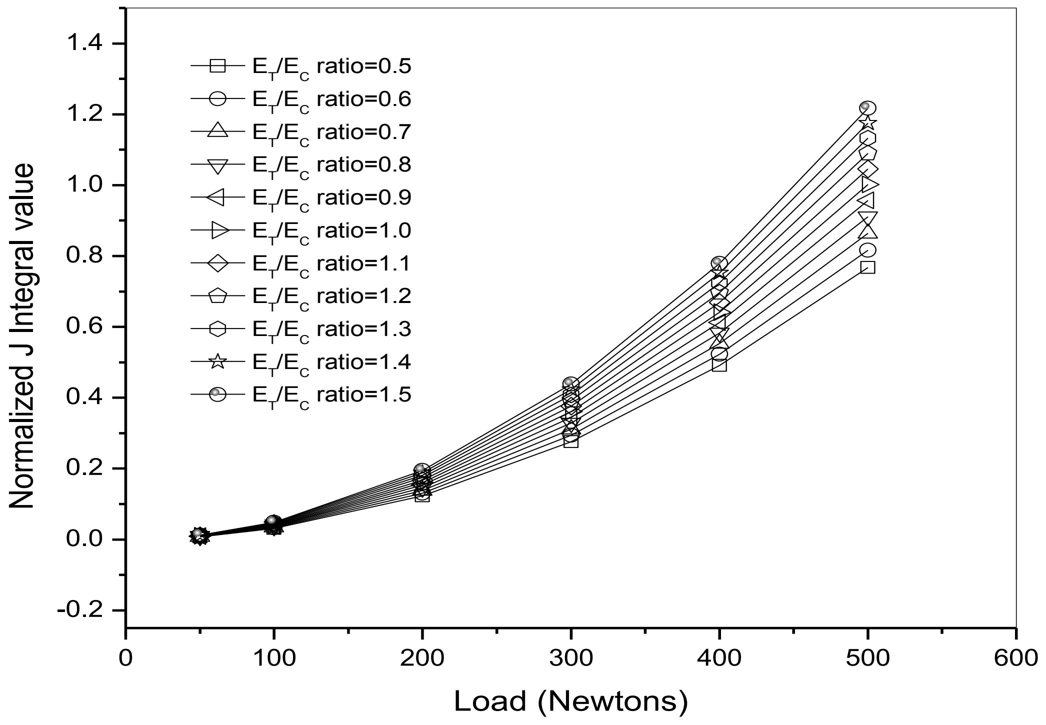


Fig.: Variation of J-integral versus different loading for different material having a range of E_t/E_c ratio.

Conclusions

- The effect of bimodularity on the stresses, and *J*-integral values for nuclear grade graphite beam has been studied in 2D and 3D.
- It was found that the ratio of E_t/E_c has a significant effect on the beam deflection, axial normal stresses, and the crack extension force on the crack tip.
- This suggests that the bimodularity effect on nuclear grade graphite is significant and should be taken into account in the design process.

References

- Nemeth N.N., Walker A., Baker E., Murthy P., Bratton R. (2013), “Large-Scale Weibull Analysis of H-451 Nuclear-Grade Graphite Rupture Strength,” Carbon, 58 , 208–225.
- Nemeth N.N., Walker A., Baker E., Murthy P., Bratton R. (2012), “Large-Scale Weibull Analysis of H – 451 Nuclear- Grade Graphite Specimen Rupture Data,” NASA/TM—2012-217409.
- Tabaddor F. (1981), “Two-Dimensional Finite Element Analysis of Bi-Modulus Materials,” Fibre Science and Technology, 14, 229–240.
- El-Tahan W.W., Staab G.H., Advani S.H., Lee J.K. (1989), " Structural analysis of bimodular materials," Journal of Engineering Mechanics, 115(5), 963–981.
- Saint-Venant B. (1864), "Notes to Navier's Resume des lecons dela resistance des corps solids," 3rd Ed., Paris, 175.
- Timoshenko S. (1941), "Strength of materials, Part 2." Advanced Theory and Problems, 2nd Ed., Van Nostrand, Princeton, N.J., 362–369.
- Marin J. (1962), "Mechanical behavior of engineering materials," Prentice- Hall, Englewood Cliffs, N.J., 86–88.
- Ambartsumyan S.A. (1965), "The axisymmetric problem of circular cylindrical shell made of materials with different stiffness in tension and compression." Izvestia Akademiya Nauk SSSR. Meckanika,. (4), 77-85; English Translation (1967), NTIS Report FTD-HT-23-1055-67, Nat. Tech. Info. Service, Springfield, Va.
- Ambartsumyan S.A. (1966), "Equations of the plane problem of the multimodulus theory of elasticity," Izvestiya Akademii Nauk Armanskoi SSR, Mekhanika, 19(2), 3-19. Translation available from the Aerospace Corp., El Segundo, Calif, as LRG-67-T-14.
- Ambartsumyan, S.A. (1969), "Basic equations and relations in the theory of elasticity of anisotropic bodies with different moduli in tension and compression." Inzhenemyi Zhurnal, Mekhanika Tverdogo Tela, 3, 51-61. Translation available from the Aerospace Corp., El Segundo, Calif, as LRG-70-T-1.
- Bert, C.W. (1977), "Models for fibrous composites with different properties in tension and compression," Eng. Mat. and Tech. Trans., ASME, 99H, Oct., 344- 349.
- Bert, C.W. (1978), "Recent advances in mathematical modeling of the mechanics of bimodulus, fiber-reinforced materials," Proc. 15th Annual Meeting, Society of Eng. Science, Gainesville, Fla., Dec, 101-106.
- Green, A.E., and Mkrtichian, J.Z. (1977), "Elastic solids with different moduli in tension and compression," J. Elasticity, 7(4), 369-386.
- Isabekian, N.G., Khachatryan, A.A. (1969), "On the multimodulus theory of elasticity of anisotropic bodies in plane stress state," Ivestiya Akademii Nauk Armianskoi SSR, Mekhanika, 22(5), 25-34. Translation available from R. M. Jones.
- Jones, R.M. (1971), "Buckling of stiffened multilayered circular cylindrical shells with different orthotropic moduli in tension and compression," AIAA Journal, 9(5), 917-923.
- Jones, R.M. (1977), "Stress strain relations for materials with different moduli in tension and compression," AIAA Journal, 15(1), 16-23.
- Iwase T., Hirashima K. (2000), "High-accuracy analysis of beams of bimodulus materials," Journal of Engineering Mechanics, (126)149-156.
- Rice, J.R. (1968), “A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks” Journal of Applied Mechanics, 35, 379–386.
- General Atomics (1988), "Graphite design handbook," DOE-HTGR-88111
- COMSOL-multiphysics, Version 4.4, (2013).

Acknowledgements

The authors wish to gratefully acknowledge the financial support for this research provided by BRNS under Grant No. 2011/36/62-BRNS with Indian Institute of Technology (Banaras Hindu University).

Thank You