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Flow-Induced Vibration Analysis of Supported Pipes with a Crack

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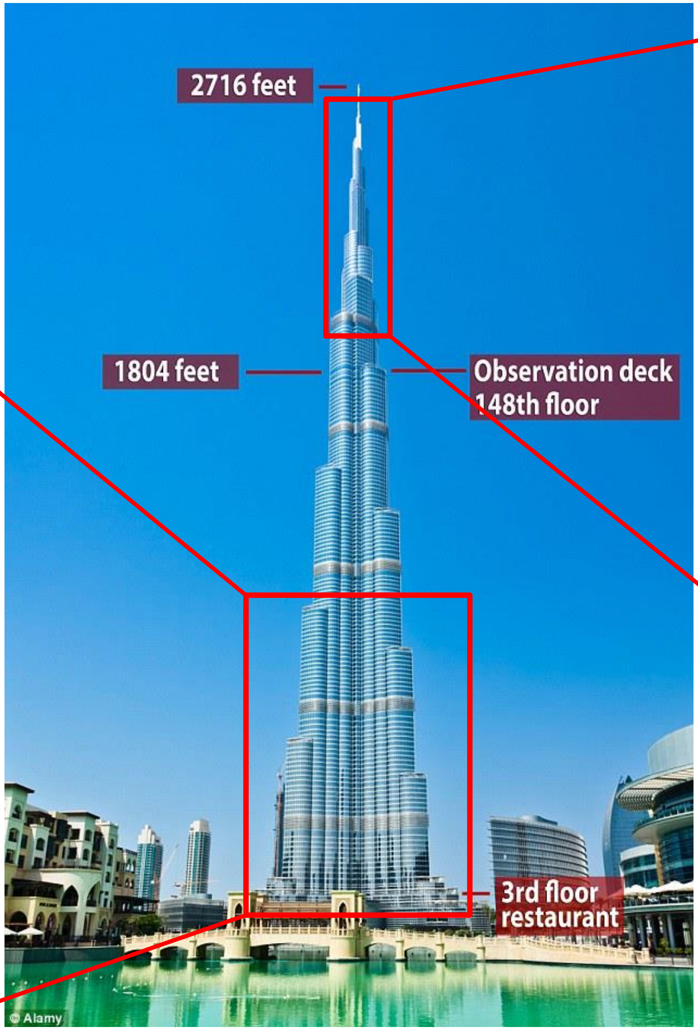
Introduction & Motivation



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https://www.youtube.com/watch?v=uUEffnV5YfY

Introduction & Motivation

The effect of a crack to the flow-induced vibration characteristics of supported pipes is investigated based on vibration method.

We need to utilize the variation of the difference between the natural frequencies of the pipe conveying fluid with and without a crack.

The pipe is fluid loaded via interaction with the fluid.

Fluid loading has two main effect on vibrating pipes:

1. Fluid mass loads the pipe, i.e., the pipe's natural frequencies are altered
2. Viscous loading is provided to the pipe near the inner wall due to shear force

COMSOL Multiphysics®

1. Aeroacoustics Module
2. Structural Moduel

Kinematic Energy of pipe:

$$T_{pipe} = \frac{1}{2} m \left\{ \int_0^{x_c} \left(\frac{\partial y_1(x,t)}{\partial t} \right)^2 dx + \int_{x_c}^{L} \left(\frac{\partial y_2(x,t)}{\partial t} \right)^2 dx \right\}$$

, where M = mass per unit length of the pipe

x_c = the distance to the crack location

y = the deflection of the pipe

L = total length of the pipe

Kinetic Energy due to fluid inside the pipe:

$$T_{fluid} = \frac{1}{2} M \left[\int_0^{x_c} \left\{ U^2 + 2U \left(\frac{\partial^2 y_1}{\partial x \partial t} \right) + \left(\frac{\partial y_1}{\partial t} \right)^2 \right\} dx + \int_{x_c}^{L} \left\{ U^2 + 2U \left(\frac{\partial^2 y_2}{\partial x \partial t} \right) + \left(\frac{\partial y_2}{\partial t} \right)^2 \right\} dx \right]$$

, where M = mass per unit length of the pipe

U = fluid velocity

Potential Energy of pipe due to strain energy:

$$V_{pipe} = \frac{1}{2} EI \left\{ \int_0^{x_c} \left(\frac{\partial^2 y_1}{\partial x^2} \right)^2 dx + \int_{x_c}^{L} \left(\frac{\partial^2 y_2}{\partial x^2} \right)^2 dx \right\} + \frac{1}{2} K_R \frac{\partial^2 y_2(x_c)}{\partial x^2}$$

, where E = modulus of elasticity
I = area moment of inertia
K_R = spring coefficient due to crack
y_k = transverse displacement (k = 1,2)

The transverse displacement:

$$y_k(x,t) = \sum_{i=1}^{n} \phi_{ki}(x) d_i(t)$$

, where ϕ = admissible function
d = generalized coordinate
k = number of divided pipes due to crack

Lagrange equation:

$$[\mathbf{M}]\ddot{\mathbf{q}} + [\mathbf{K}]\mathbf{q} = \mathbf{F}_{\mathbf{ext}}$$

Considering an external forcing term is assumed to be a viscous drag force due to shear stress inside the pipe wall, it can be replaced with

$$F_{viscous} = \eta A \frac{\Delta \upsilon}{\Delta y}$$

, where A = surface area

$\Delta \upsilon$ = average fluid velocity

Δy = separation distance between the wall and the center of the pipe

$$\eta = \frac{F/A}{\Delta \upsilon / \Delta y}$$

, where F = force required to maintain the motion

GEOMETIC MODEL AND BOUNDARY CONTIDIONS

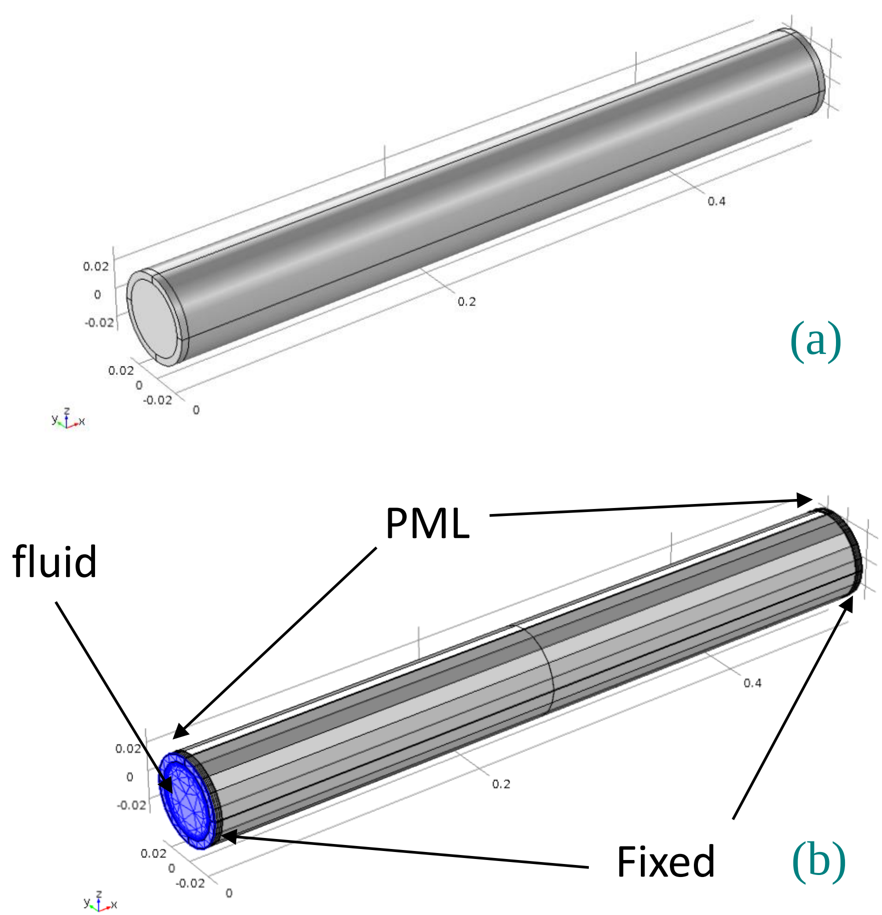

Table 1: Structure and material properties of the fluid flow conveying pipe.

	Pipe
Material	**Copper**
Outer Diameter	**6e-2m**
Thickness	**5e-3m**
Modulus of Elasticity	**110GPa**
Density	**8700 kg/m^3**
Poisson ratio	**0.35**
Pipe length	**0.5m**
Beam width	**1.2mm**

Figure 1. (a) Geometric pipe model, and (b) meshed pipe model without crack where blue colored section represents PML.

FSI Problem in Frequency Domain

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Linearized Navier Stokes (Aeroacoustic) + Structural Mechanics Module

The governing equations used to solve for the frequency analysis are the continuity, momentum, and energy equations + structural equations.

- *Infinite boundary* (*absorb boundary*):
  PML

- *Fluid-structure interaction* (FSI) *boundary:*

The Aeroacoustic-Structure Boundary coupling prescribes continuity in the displacement field between two different domains

$$u_{fluid} = i\omega u_{solid}$$

, where u_{fluid} = fluid velocity
u_{solid} = solid displacement

This results in the stress being continuous across the boundary between two different domains. This will play an important role investigating the effects of the fluid to the vibration mode of the pipe system.

The local flexibility in the presence of the crack can be defined as a function of the geometry of a crack.

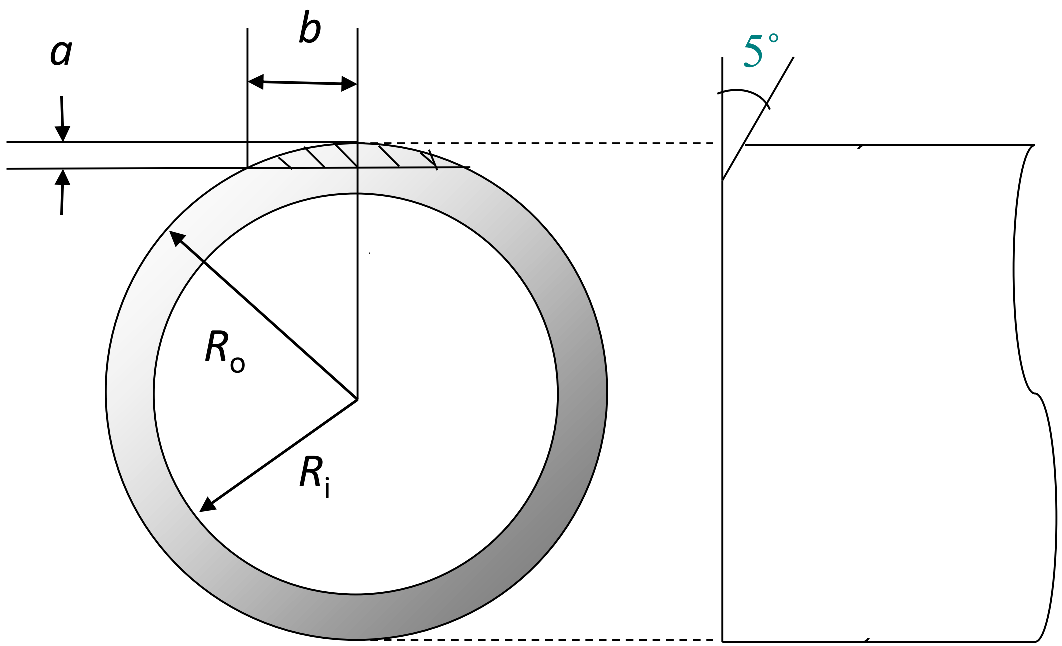

Figure 2. Cross section of the cutaway cracked pipe and the side view:
(a) Geometric pipe model, and (b) the meshed pipe model without crack where blue colored section represents PML.

Flow velocity profile

Assuming the steady, laminar ($Re \leq 2300$), imcompressible flow of fluid with constant properties, the fully developed velocity profile is chosen.

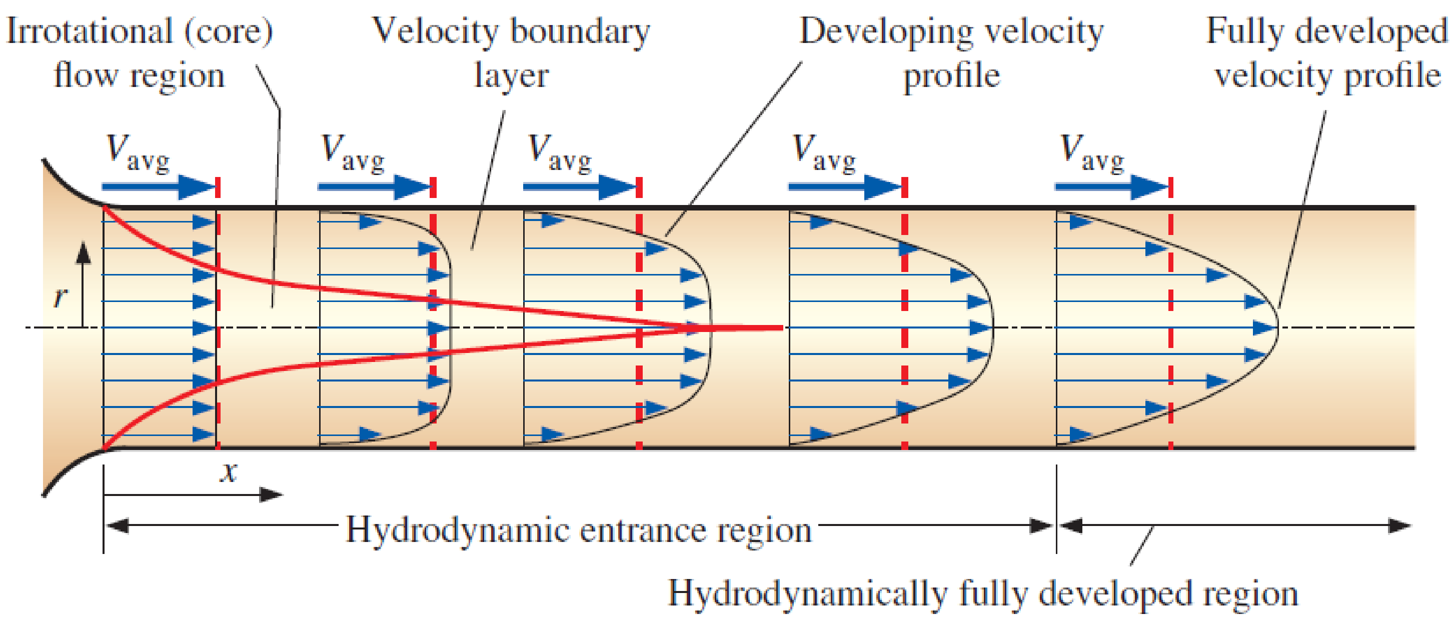

Figure 3. The development of the velocity boundary layer in a pipe [10].

$$u(r) = V_{\max}\left[1 - \left(\frac{y^2 + z^2}{R_o{}^2}\right)\right]$$

, where $\boldsymbol{V_{max}}$ is maximum velocity, $\boldsymbol{R_o}$ is the inner pipe radius, $\boldsymbol{y}$ and $\boldsymbol{z}$ are radial distance from the center to each axis

Crack Meshing and Flow Boundary Contidions

Fluid flow is assumed to be Newtonian fluid for laminar case, and the no slip condition for the flow on a hard wall inside the pipe.

The No slip condition is $u = 0$

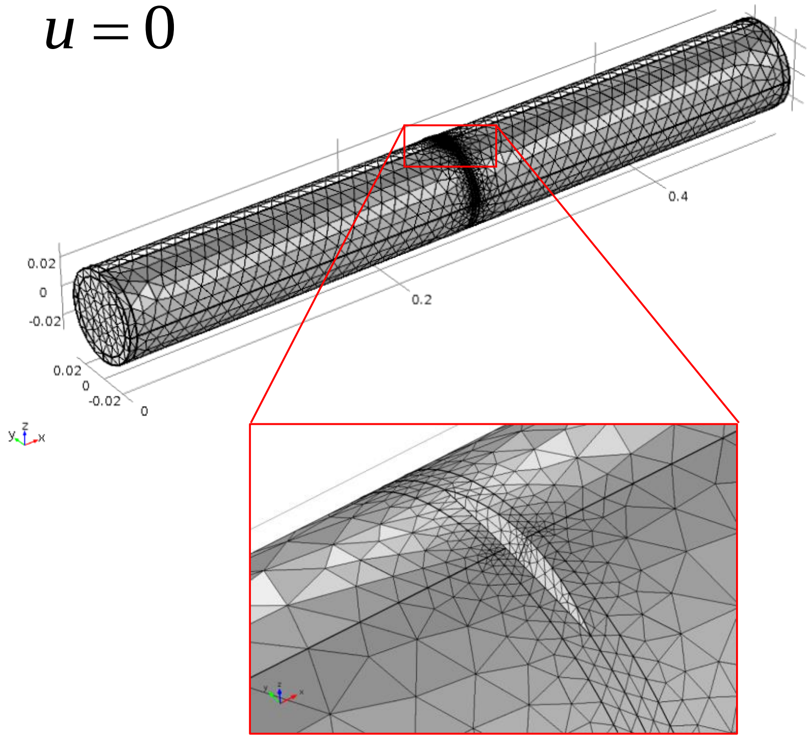

Figure 4. Meshed cracked pipe.

The maximum element size is $h_{max} = 0.2\lambda$, where λ is wavelegth.

Table 2: Comparison for *in vacuo* and the pipe filled with water for its eigenfrequency

	eigen-frequency	eigenfrequency w/ crack
In vacuo	1 843.56 Hz	2 836.96 Hz
Filled with water	3 749.63 Hz	4 747.58 Hz
Percentage decrease	11 %	10 %

The fluid added mass effect is estimated by calculating the frequency reduction ratio δ of each natural frequency defined as

$$\delta = \frac{(f_v - f_w)}{f_v}$$

, where f_v and f_w are the natural frequencies *in vacuo* and with fluid inside the pipe.

Simulation Results:
Fluid loading & Crack Effects (cont'd)

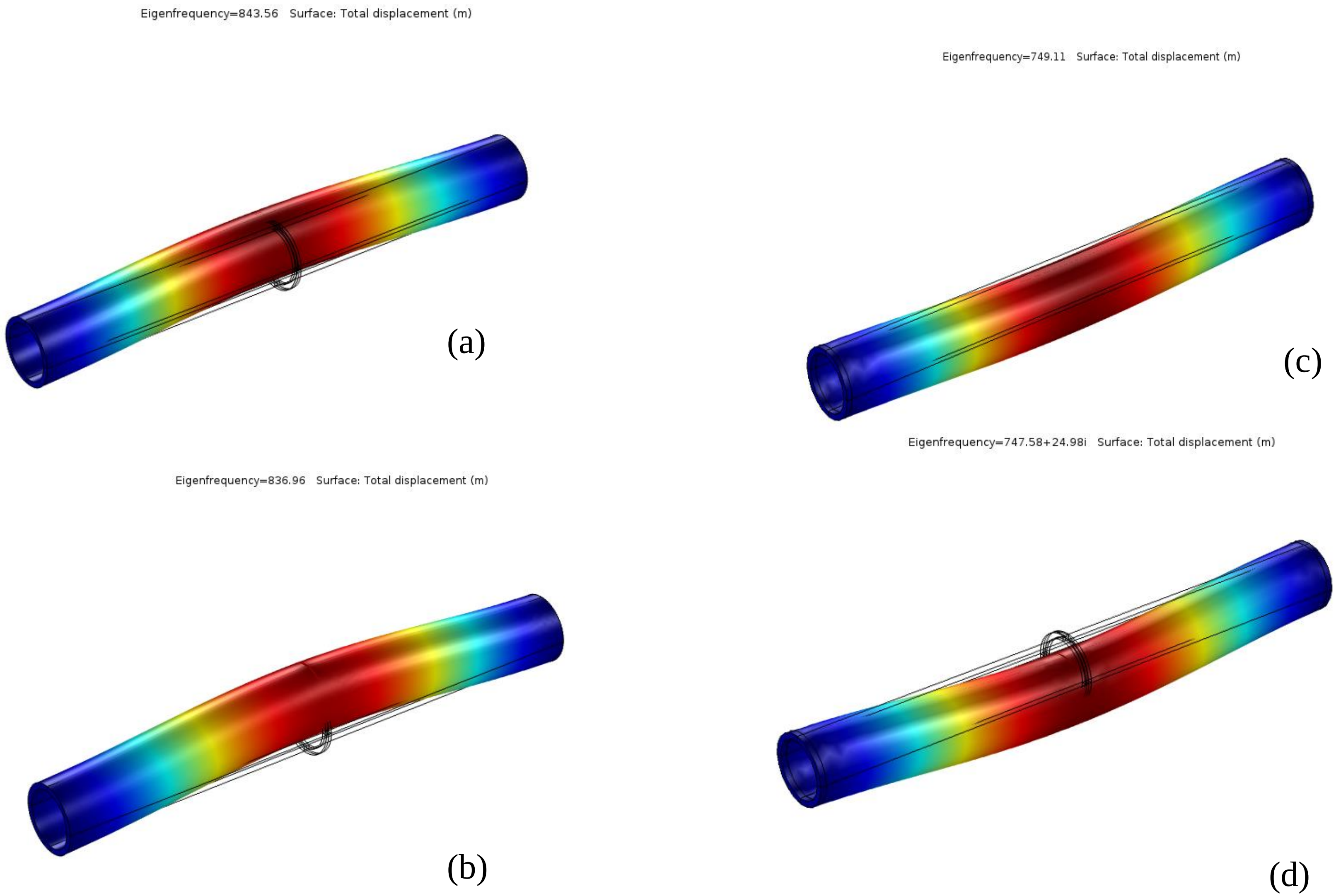

Figure 5. Eigenmodes for the cases in Table 2 where ①, ②, ③, and ④ from Table 2 denote (a), (b), (c), and (d) in Fig. 5, respectively..

Table 3: Comparison for different velocities of the fluid flow inside the pipe for its eigenfrequency

Max. velocity	eigen- frequency	eigenfequency w/ crack
1 m/s	749.11 Hz	747.58 Hz
10 m/s	702.14 Hz	742.44 Hz

Conclusion

- ❖ In this paper, a pipe conveying fluid flow with crack has been investigated through numerical simulation using COMSOL Multiphysics software.

- ❖ Vibrational behavior of the pipe system has been studied to show the effect of fluid within a pipe as well as that of crack.

- ❖ Due to the added mass effect induced by the fluid inside the pipe, a significant reduction in natural frequencies is observed (Table 2).

- ❖ The velocity of the fluid inside the pipe seems to affect the natural frequency of the pipe system such that as the velocity increases, its eigenfrequency decreases. However, there has not been found any specific correlation between them in this work.

Future Work

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The work to be done in the future would be as follows;

(1) investigation of further study of the velocity of the fluid flow in greater detail,

(2) investigation of the crack location,

(3) study of dual crack effect rather than single crack, and

(4) derivation of mathematical model that corresponds with the simulation study.

Q & A

Thank you!