UNIVERSITY OF LOUISVILLE

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# Simulating Spiking Neurons Using a Simple Mathematical Model

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# Introduction


There are more than 1.2 million Americans with Spinal cord injuries (SCI) [1]

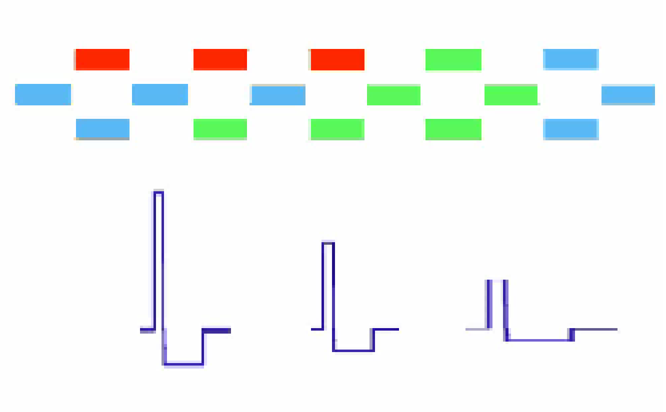
Patients with SCI can loose the ability to walk, stand, move their arms and SCI often leads to secondary complications in bladder and bowel functions, etc. Basic life activities become extremely difficult

Recent studies [2] have shown that spinal cord stimulation can enable voluntary motor function in patients with spinal cord injury

Since 2011, four individuals with SCI received epidural stimulation of the spinal cord. All regained the ability to independently stand, and acquire some voluntary control of the toes, ankles, knees and hips [3]

# Introduction

Trial and error methods are frequently used to determine the stimulus parameters needed to activate specific motor neurons.



Simulations are useful in predicting the response elicited from particular stimulus parameters.

# Hodgkin-Huxley model

In 1952, Alan Hodgkin and Andrew Huxley published a series of papers [4] describing mathematics behind propagation of action potentials in a squid axon

$$C_m \frac{\partial V_m}{\partial t} = \frac{r}{2R}\frac{\partial^2 V_m}{\partial x^2} + g_{Na} m^3 h (V_{Na} - V_m) + g_K n^4 (V_K - V_m) + g_L (V_L - V_m) \quad (1)$$

$$\frac{dw}{dt} = \alpha_w (1 - w) - \beta_w w \quad (2)$$

$$w\ is\ m, n\ or\ h$$

$$\alpha_m = \frac{2.5 - 0.1V}{e^{2.5 - 0.1V} - 1} \quad (3a)$$

$$\alpha_n = \frac{1 - 0.1V}{10(e^{1 - 0.1V} - 1)} \quad (3b)$$

$$\alpha_h = 0.07 e^{-\frac{v}{20}} \quad (3c)$$

$$\beta_m = 4 e^{-\frac{v}{18}} \quad (3d)$$

$$\beta_n = 0.125 e^{-\frac{v}{80}} \quad (3e)$$

$$\beta_h = \frac{1}{e^{3 - 0.1V} + 1} \quad (3f)$$

<u>Computationally challenging to use this model to simulate a complex system</u>

# Izhikevich Model

The Izhikevich model is a recently (2003) published simple mathematical model that is both computationally more efficient than the Hodgkin-Huxley model and is also capable of simulating multiple spiking and bursting patterns [5].

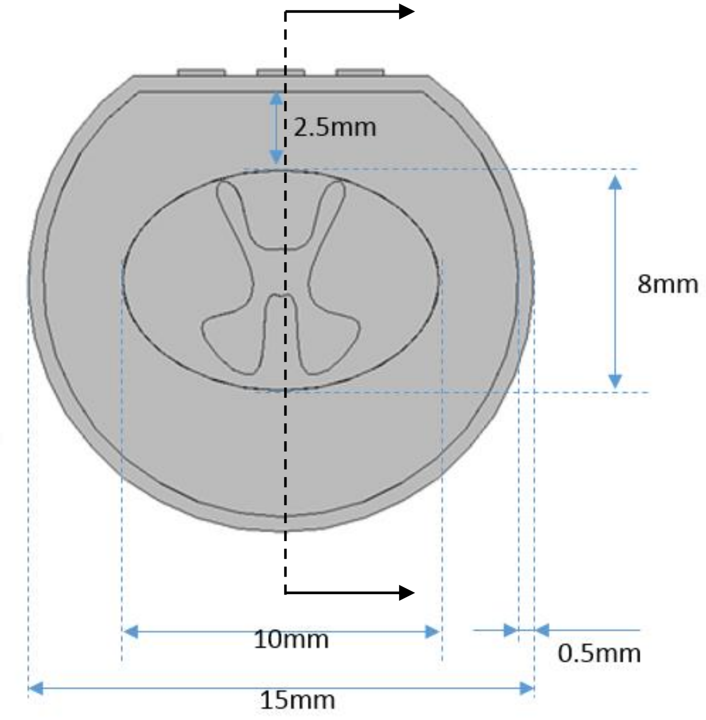
$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I \qquad (4)$$

$$\frac{du}{dt} = a(bv - u) \qquad (5)$$

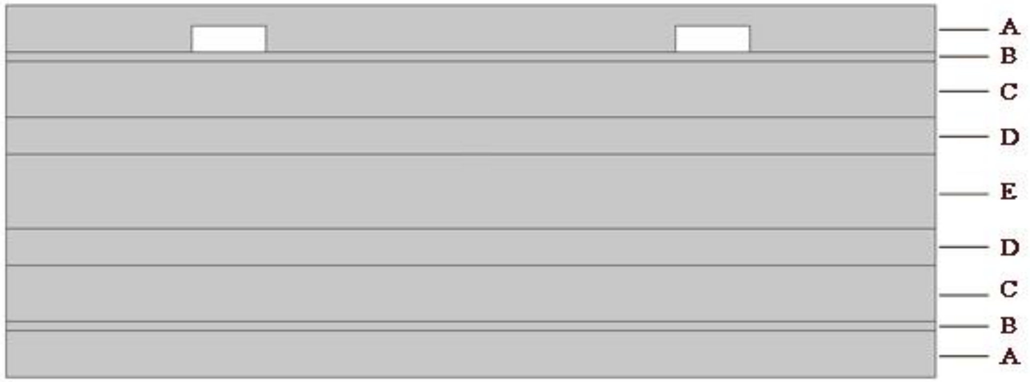
$$if\ v \geq 30mV, then \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases} \qquad (6)$$

<u>Computationally less complex model for simulating neurons</u>

# Modeling in COMSOL Multiphysics




Cross section of the human spinal cord with average dimensions of L1-L5 region[6]



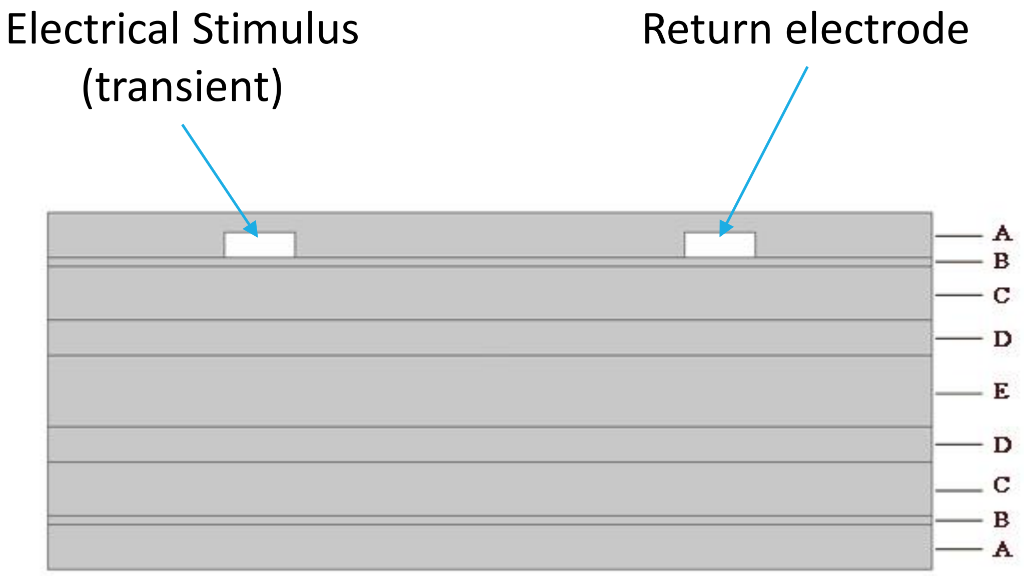

2D model used for simulations representing the longitudinal section of the spinal cord with two embedded electrode; where A, B, C, D and E represent epidural fat, Dura matter, CSF, white matter and gray matter, respectively.

# Materials

| Item | Conductivity (S/m) | Permittivity |
| --- | --- | --- |
| White matter (longitudinal) | 0.6 | 38.79 |
| White matter (transverse) | 0.083 | 1846.05 |
| Gray matter | 0.23 | 458.89 |
| CSF | 2 | 108.89 |
| Dura matter | 0.6 | 141.25 |
| Epidural fat | 0.04 | 38.72 |

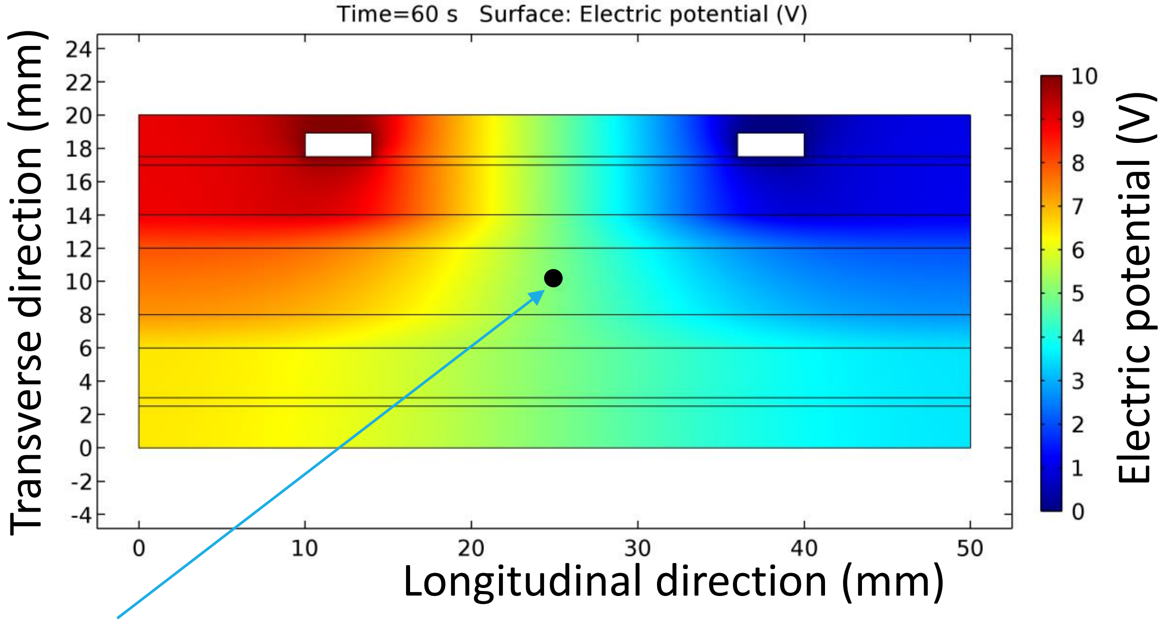
The dielectric properties [7, 8] used in this study.

# Simulation to find the electric potential at point of interest




Maxwell's equations to solve for the potential distribution

$$\nabla \cdot \mathrm{J} = -\nabla(\sigma \nabla V) - \nabla\left(\epsilon_o \epsilon_r \nabla \frac{\partial V}{\partial t}\right) = 0 \qquad (7)$$




Point of interest at which potential data is extracted

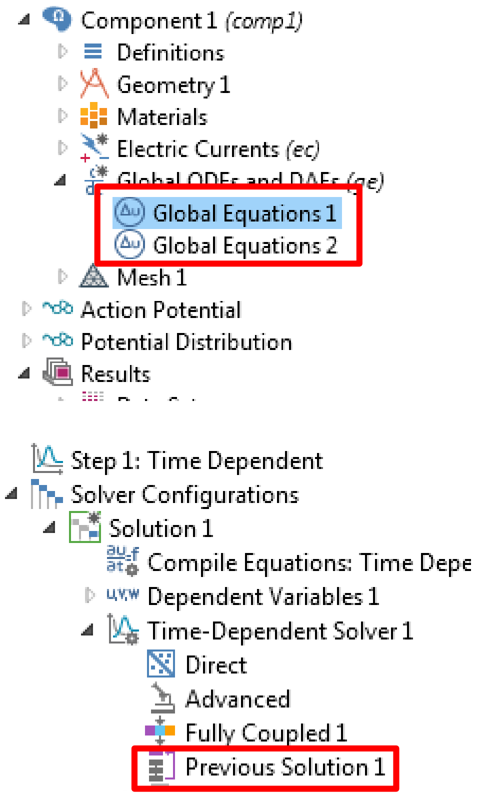
# Simulation to find the action potential at point of interest

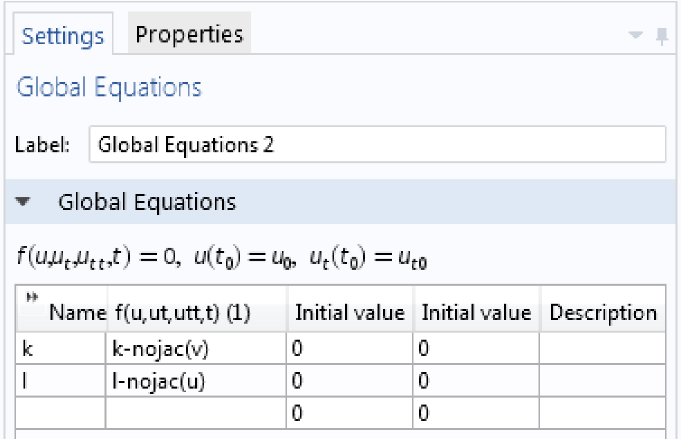
Use of previous solution operator

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I$$

$$\frac{du}{dt} = a(bv - u)$$

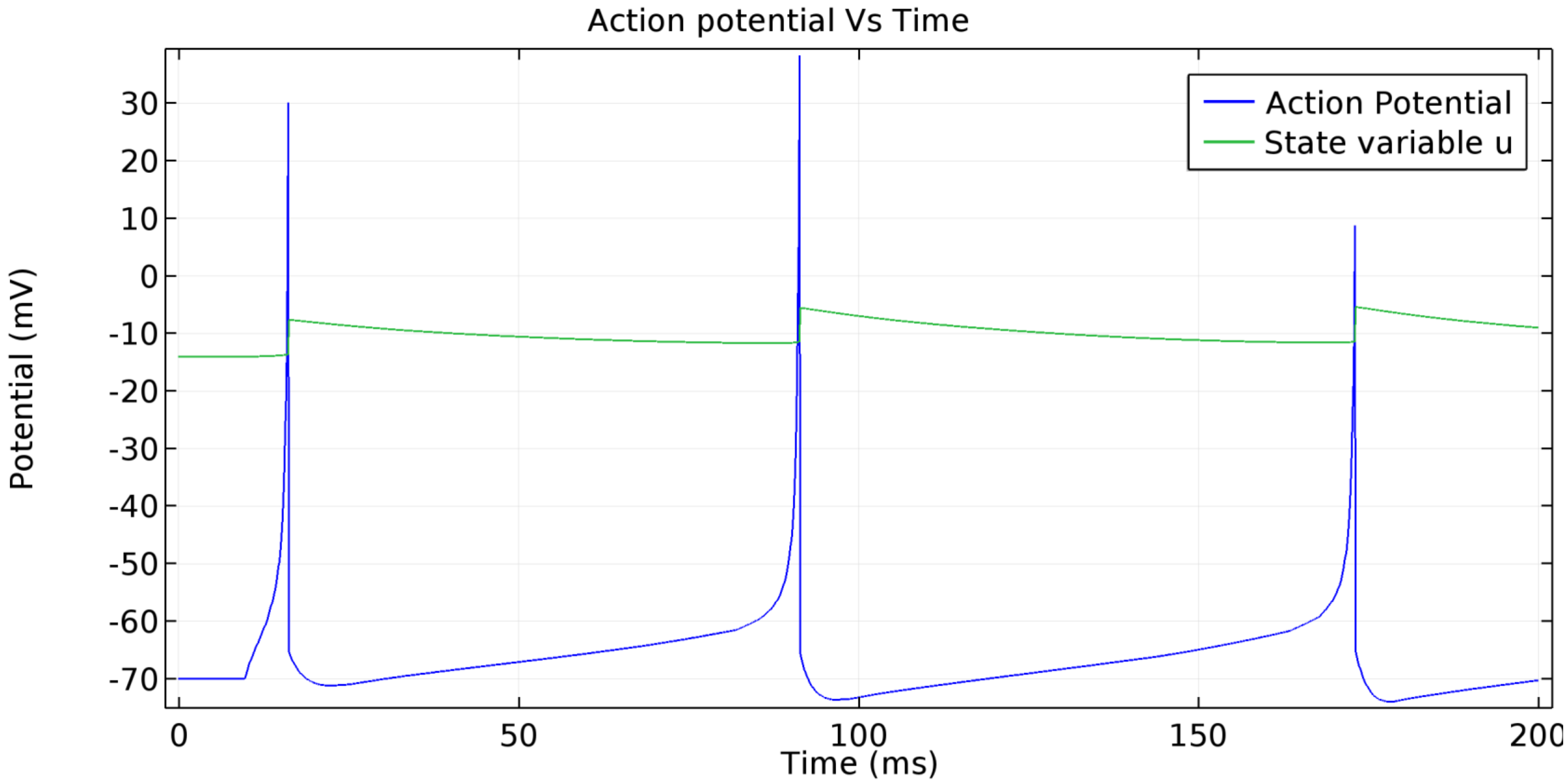
$$if\ v \geq 30mV, then \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases}$$



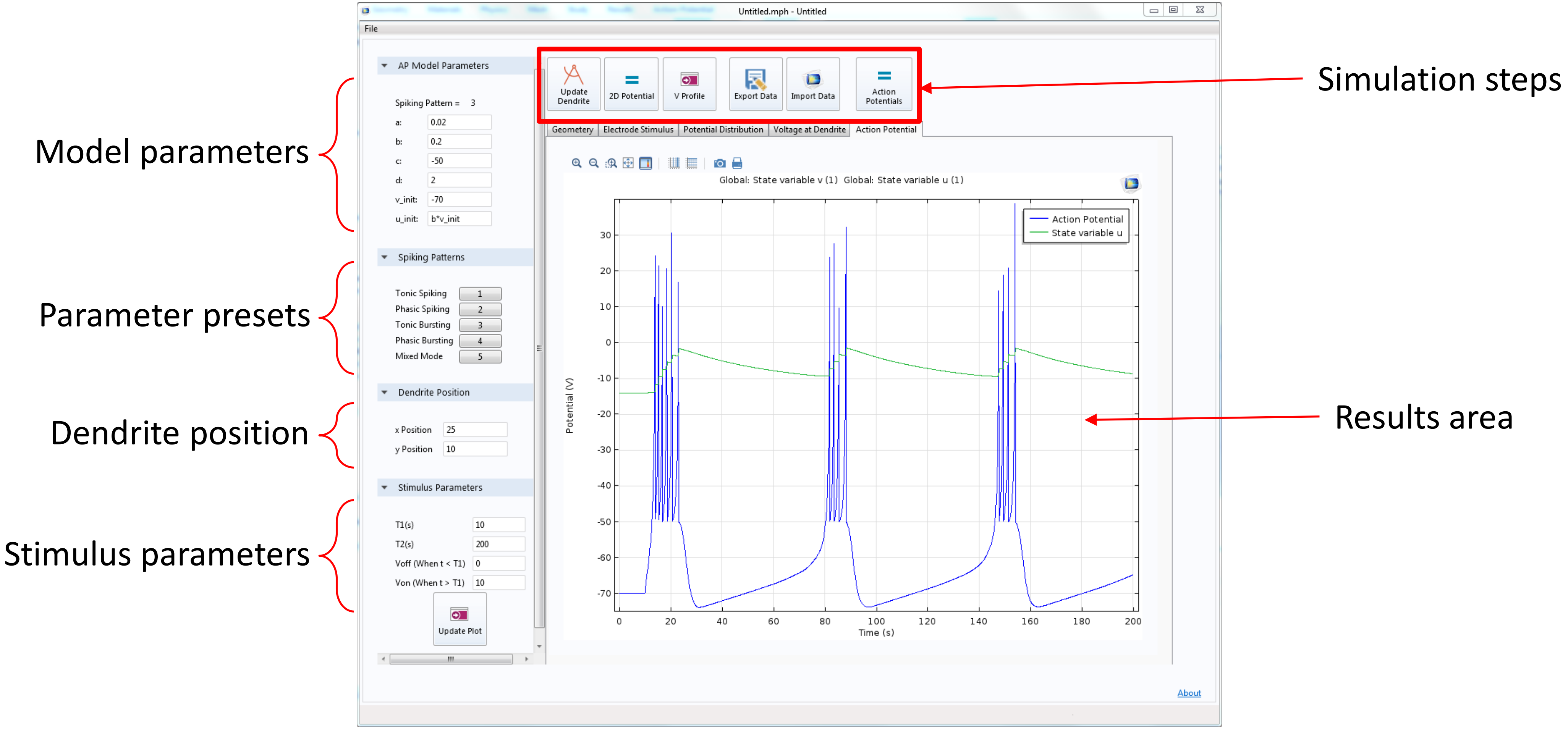



# Simulation results

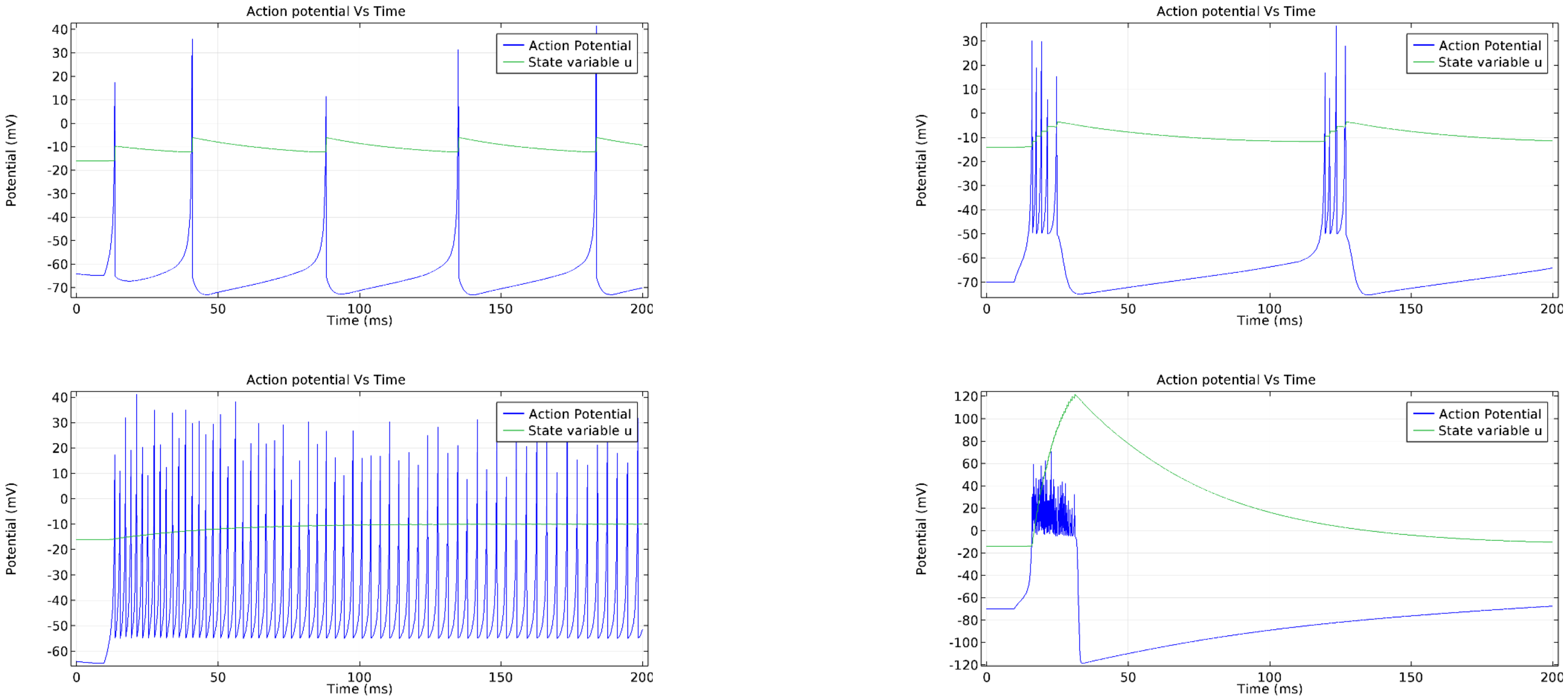
Typical spiking pattern generated for the stimulus extracted from the previous step



# COMSOL app to simplify simulation process

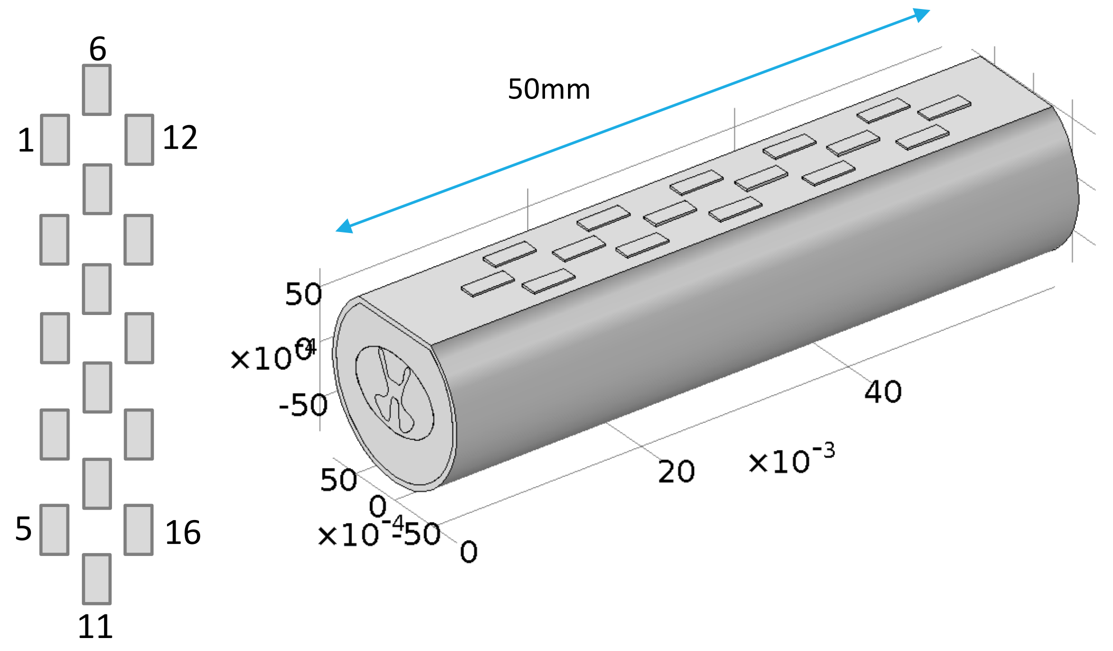


# Different spiking patterns



# Next steps

- Modeling with realistic 3D model
- Use of multiple neurons
- Introduce interaction between neurons




Spinal cord model with 16 electrodes

**Goal: To Improve the effectiveness of simulation**

# Conclusion

- Izhikevich model was utilized to simulate spiking patterns of neurons due to external stimuli provided through a 2D spinal cord model

- COMSOL provides the ability to easily combine external electrical stimuli to a neural model

- COMSOL application builder was very useful in simplifying the simulation process

# References

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[7] Min X, Kent AR, Rosenberg SP, Fayram TA, Modeling dermatome selectivity of single-and multiple-current source spinal cord stimulation system, 36th annual international conference of the IEEE, Chicago, 6246-6249, (2014)

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# Thank You