Are you solving a PDE on an a priori unknown domain?

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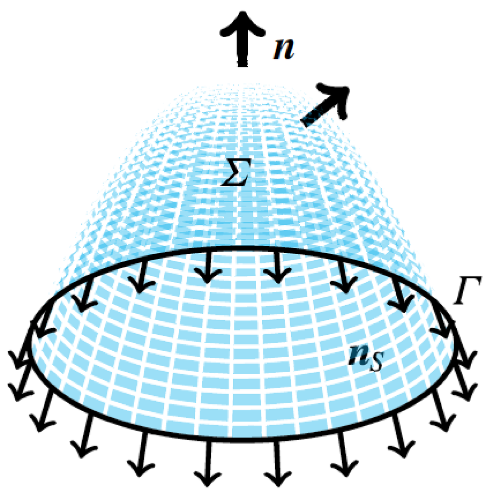
We propose a systematic method to deal with it!

The method relies on the **intrinsic surface differential operator** which is implemented in **COMSOL®**



Definition:

$$\int_{\Sigma} \boldsymbol{D}_S \varphi \, \mathrm{d}\Sigma = \int_{\Gamma} \boldsymbol{n}_S \varphi \, \mathrm{d}\Gamma$$

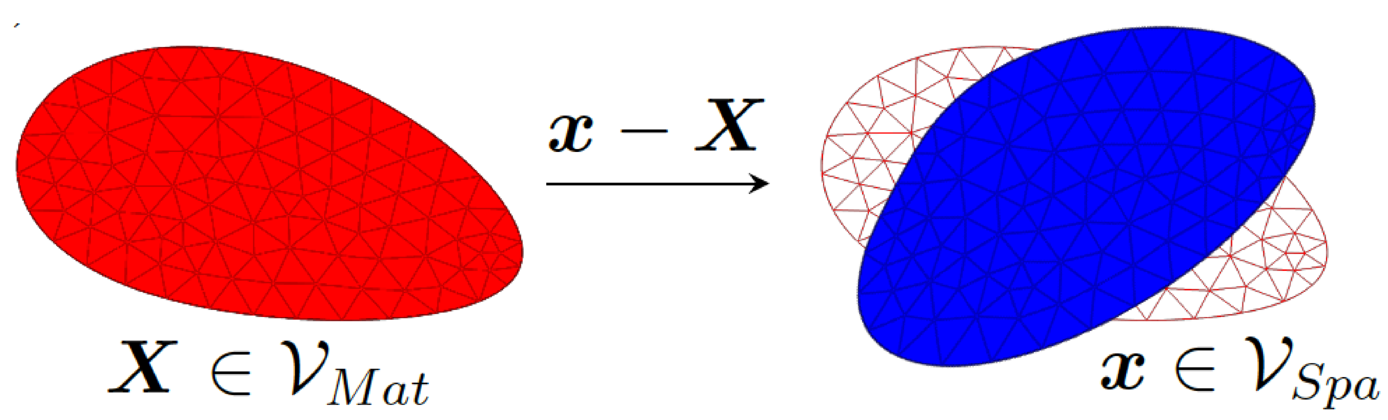
Implementation: surface reciprocal theorem

$$\int_{\Sigma} \psi \boldsymbol{D}_S \varphi \, \mathrm{d}\Sigma = \int_{\Gamma} \boldsymbol{n}_S (\varphi \psi) \, \mathrm{d}\Gamma - \int_{\Sigma} (\nabla_S \psi) \varphi \, \mathrm{d}\Sigma$$

Are the deformations large or nonlinear?

The Arbitrary Lagrangian Eulerian method (BALE) is a very accurate method to deal with deformable domains …

Arbitrary Lagrangian Eulerian



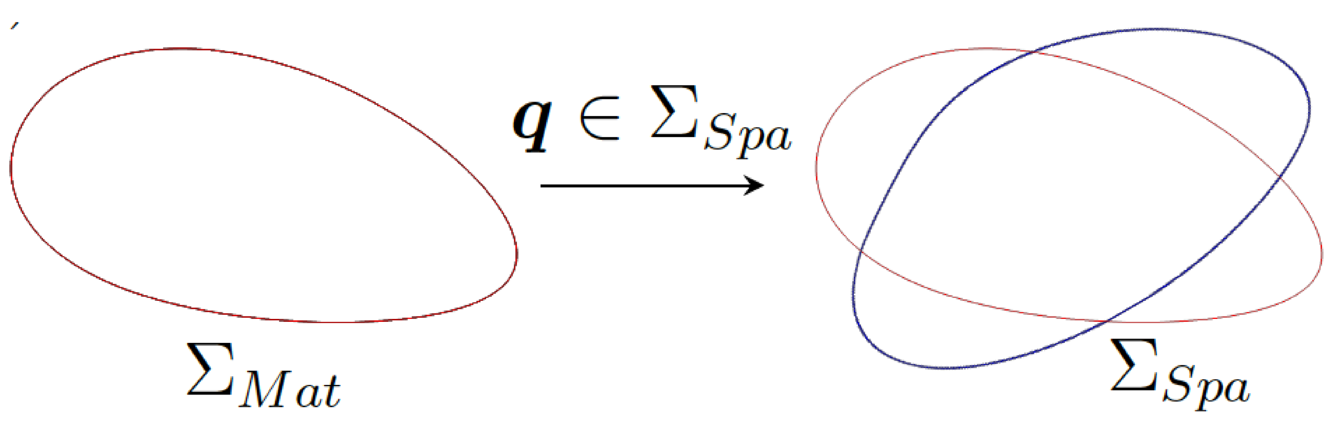
ALE:

$$\nabla \cdot \nabla (\boldsymbol{x} - \boldsymbol{X}) = \boldsymbol{0} \qquad \text{at } \mathcal{V}_{Spa} \,,$$

$$\boldsymbol{x} - \boldsymbol{X} = \boldsymbol{q} \qquad \text{at } \Sigma_{Spa} \,,$$

… but it requires boundary conditions! The Boundary Arbitrary Lagrangian Eulerian method is a systematic and accurate method to impose them.

Boundary Arbitrary Lagrangian Eulerian



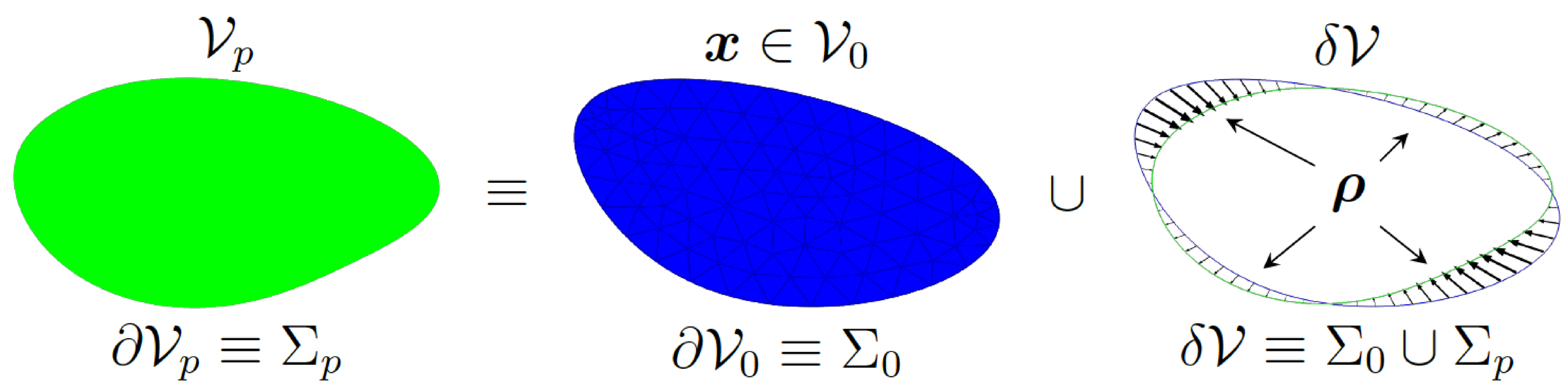
BALE:

$$\boldsymbol{D}_S \cdot \nabla_S \boldsymbol{q} = g \, \boldsymbol{n} \qquad \text{at } \Sigma_{Spa} \,,$$

- It allows you to solve steady problems with boundary conditions in Eulerian coordinates.
- It can be applied to any physics such as fluid mechanics, surface tension, fluid-structure interactions, buckling/flutter phenomena and any physics in general.
- You get rid of cumbersome algebra.
- Minimize the distortion of the mesh boundary.

Are the deformations small or infinitesimal?

If you know the unperturbed geometry, with the Deformable Boundary Perturbation method (DBP) you can write you equations on it.



Perturbed geometry Unknown | Unperturbed geometry Known! | Perturbation domain

Perturbation of integrals

$$\int_{\mathcal{V}_p} \varphi \, \mathrm{d}\mathcal{V} = \int_{\mathcal{V}_0} \varphi \, \mathrm{d}\mathcal{V} + \int_{\Sigma_0} \boldsymbol{n} \cdot \boldsymbol{\rho} \, \varphi \, \mathrm{d}\Sigma$$

Perturbation of boundary conditions

at perturbed geometry

$$\nabla \cdot \nabla \phi = \sigma \qquad \text{at } \mathcal{V}_p$$

$$\boldsymbol{n} \cdot \nabla \phi = c\phi + \gamma \qquad \text{at } \Sigma_p$$

at unperturbed boundary

$$\nabla \cdot \nabla \phi = \sigma \qquad \text{at } \mathcal{V}_0$$

$$\boldsymbol{n} \cdot \nabla \phi - \boldsymbol{D}_S \cdot (\rho \nabla \phi) + \rho \sigma = [1 + \rho (\nabla_S \cdot \boldsymbol{n}) + \rho \boldsymbol{n} \cdot \nabla] (c\phi + \gamma) \qquad \text{at } \Sigma_0$$

Perturbation of surface tension term

$$\int_{S_p} \boldsymbol{D}_S \varphi \, \mathrm{d}\Sigma = \int_{S_0} \boldsymbol{D}_S \varphi \, \mathrm{d}\Sigma + \int_{S_0} \boldsymbol{D}_S \cdot \{ [\mathcal{I}(\rho \nabla_S \cdot \boldsymbol{n}) - \rho \nabla_S \boldsymbol{n} + (\nabla_S \rho) \boldsymbol{n} + \rho \boldsymbol{n} \cdot \nabla] \varphi \} \, \mathrm{d}\Sigma$$

It allows you to carry out:
- stability analysis
- asymptotic analysis
- faster shape optimization