

# A Black-Oil Model for Primary and Secondary Oil-Recovery in Stratified Petroleum Reservoirs

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**INTRODUCTION:** Black-oil simulators are commonly used in petroleum reservoir engineering for the prediction of oil production, especially during the earlier stages of oil-field exploitation, while also serving to guide pressure maintenance strategies in the longer term. They account for fluid flow in porous media, where it is assumed that there are three distinct phases: water (w), oil (o), gas (g)<sup>1</sup>.

A numerically stable formulation of the black-oil model is developed and its performance during three common oil-recovery processes in a homogeneous, anisotropic petroleum reservoir is evaluated.

**COMPUTATIONAL METHODS:** The phase formulation based on oil pressure and total velocity with negligible capillary forces is applied<sup>2</sup>:

## 1 Pressure Equation → General form PDE

$$\nabla \cdot \mathbf{u} = \sum_{\beta=o,w,g} B_{\beta} \left( q_{\beta} - \varphi S_{\beta} \frac{\partial}{\partial t} \left( \frac{1}{B_{\beta}} \right) - \mathbf{u}_{\beta} \cdot \nabla \left( \frac{1}{B_{\beta}} \right) \right) - B_g \left( R_{so} q_o + \frac{\varphi S_o}{B_o} \frac{\partial R_{so}}{\partial t} + \frac{1}{B_o} \mathbf{u}_o \cdot \nabla R_{so} \right),$$

$$\mathbf{u} = -\mathbf{K} \lambda (\nabla p - \mathbf{G}_{\lambda}), \quad \mathbf{G}_{\lambda} = \mathbf{g} \sum f_{\beta} \rho_{\beta}, \quad \text{for } \beta = o, w, g$$

## 2 Saturation Equations → Coefficient form PDE

$$\varphi \frac{\partial S_{\alpha}}{\partial t} + \nabla \cdot \mathbf{u}_{\alpha} = B_{\alpha} \left( q_{\alpha} - \varphi S_{\alpha} \frac{\partial}{\partial t} \left( \frac{1}{B_{\alpha}} \right) - \mathbf{u}_{\alpha} \cdot \nabla \left( \frac{1}{B_{\alpha}} \right) \right),$$

$$\mathbf{u}_{\alpha} = f_{\alpha} \mathbf{u} - \mathbf{K} f_{\alpha} \sum \lambda_{\beta} (\rho_{\beta} - \rho_{\alpha}) \mathbf{g}, \quad \text{for } \alpha = o, w,$$

**Initially:** Reservoir in undersaturated state with prevailing hydrostatic pressure, saturated with 90% PV oil and 20% PV water

**Primary recovery:** Pressure reduction BC at outlet

**Water- & Gas-flooding:** Constant rate BD at inlet & Constant pressure BC at outlet

## RESULTS:

### Process: Pressure Depletion, Solution-gas Drive Mechanism

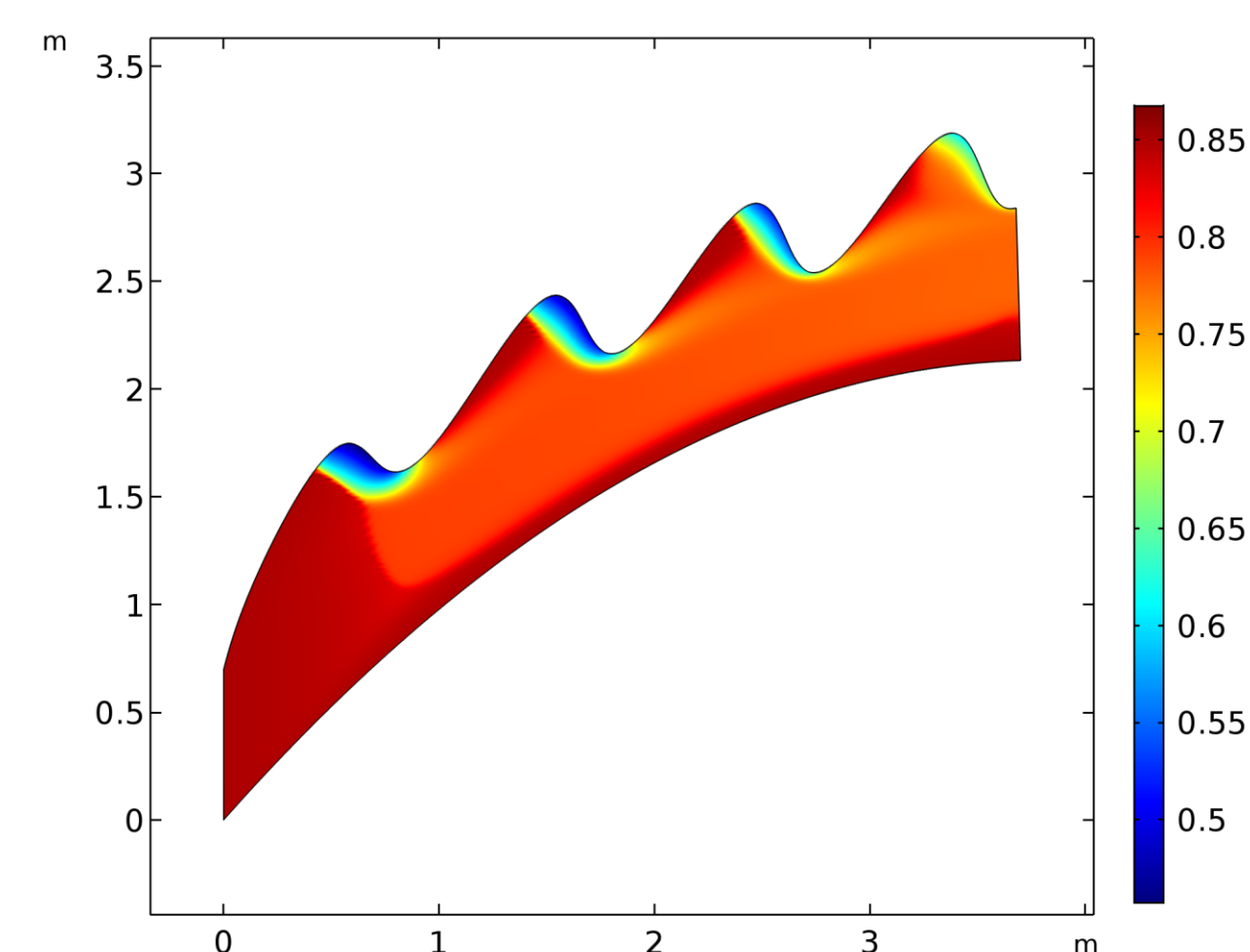


Figure 1. Oil phase saturation at the end of primary recovery

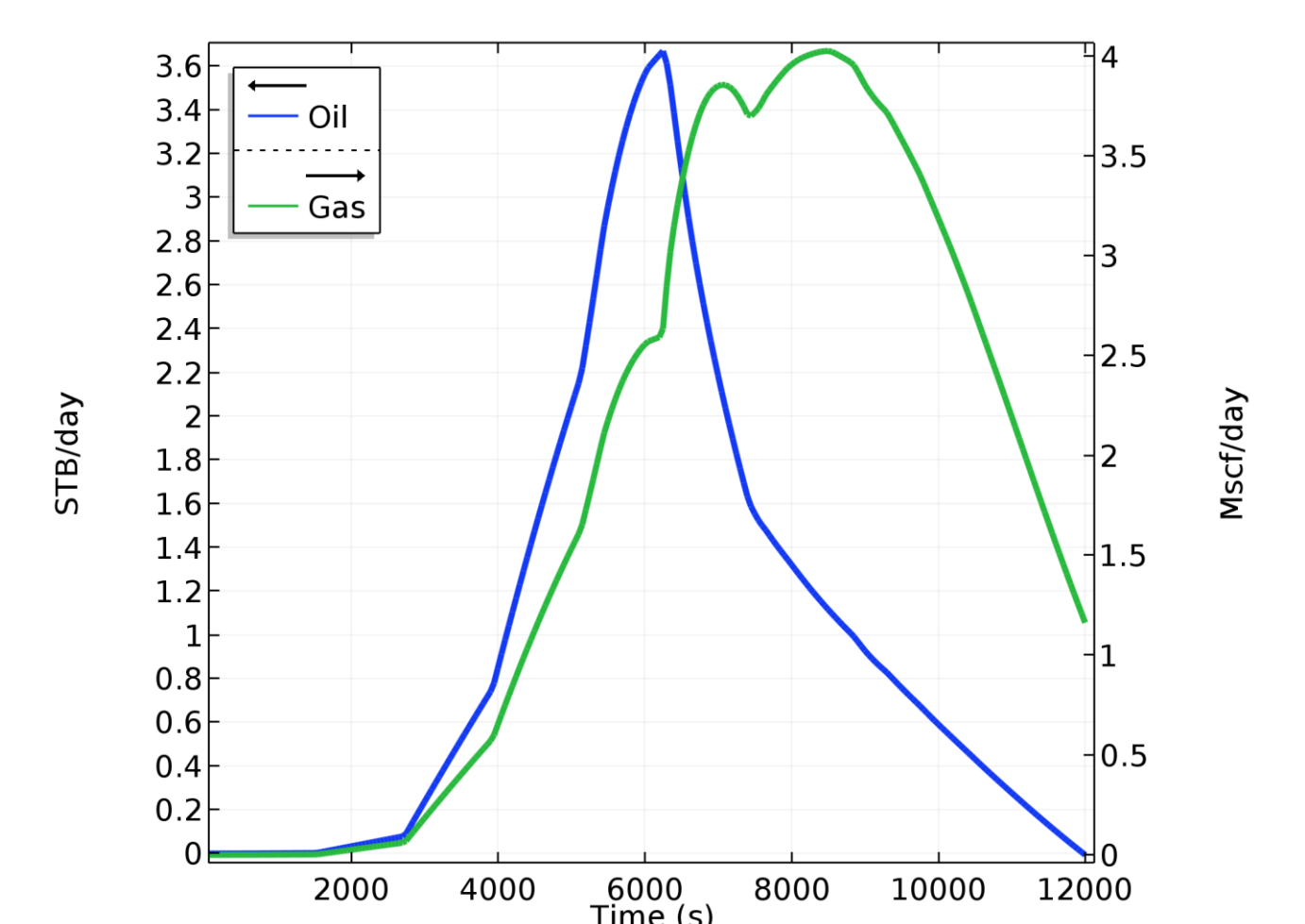


Figure 2. Production rates at standard conditions during primary recovery

### Process: Waterflooding

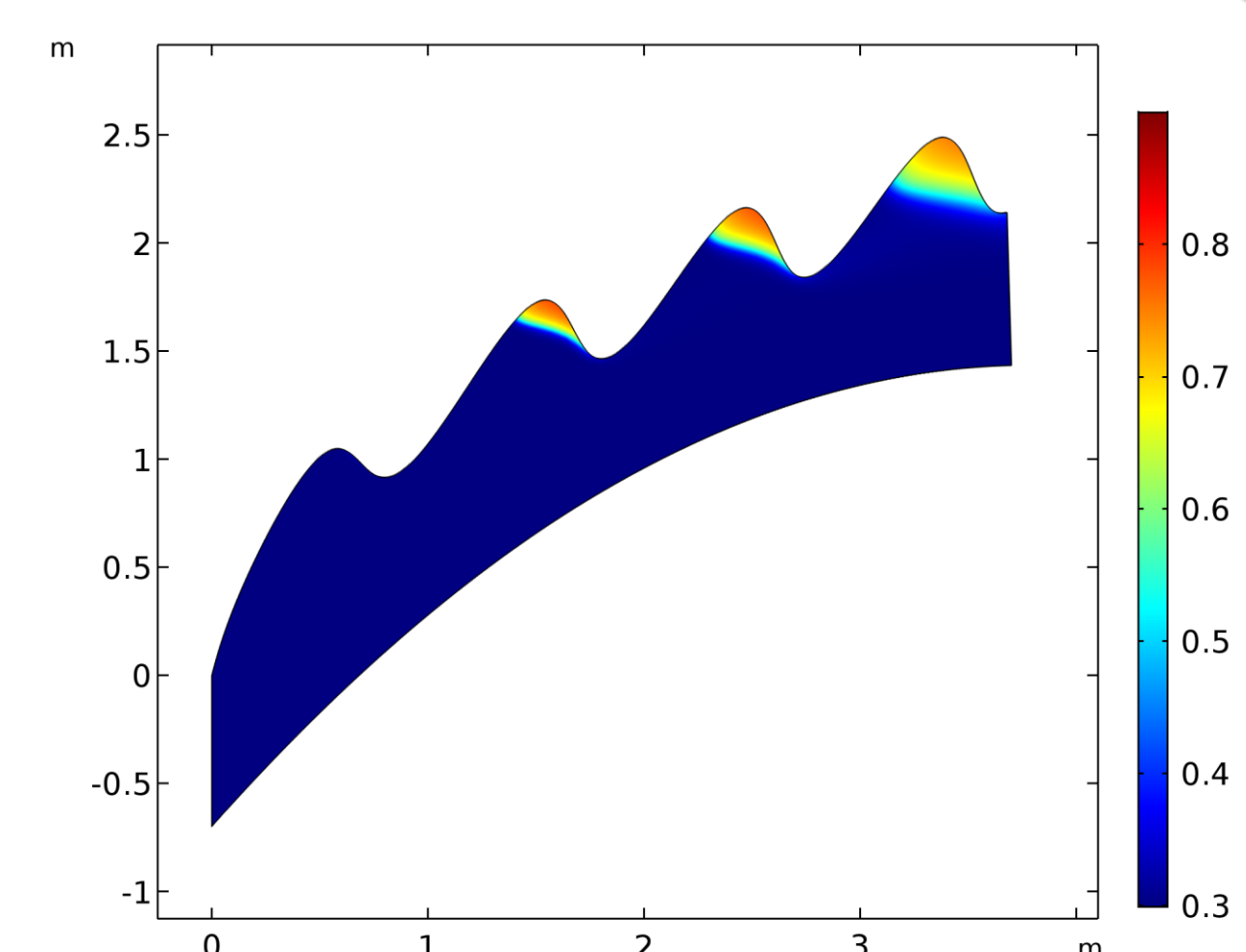


Figure 3. Oil phase saturation at the end of waterflooding

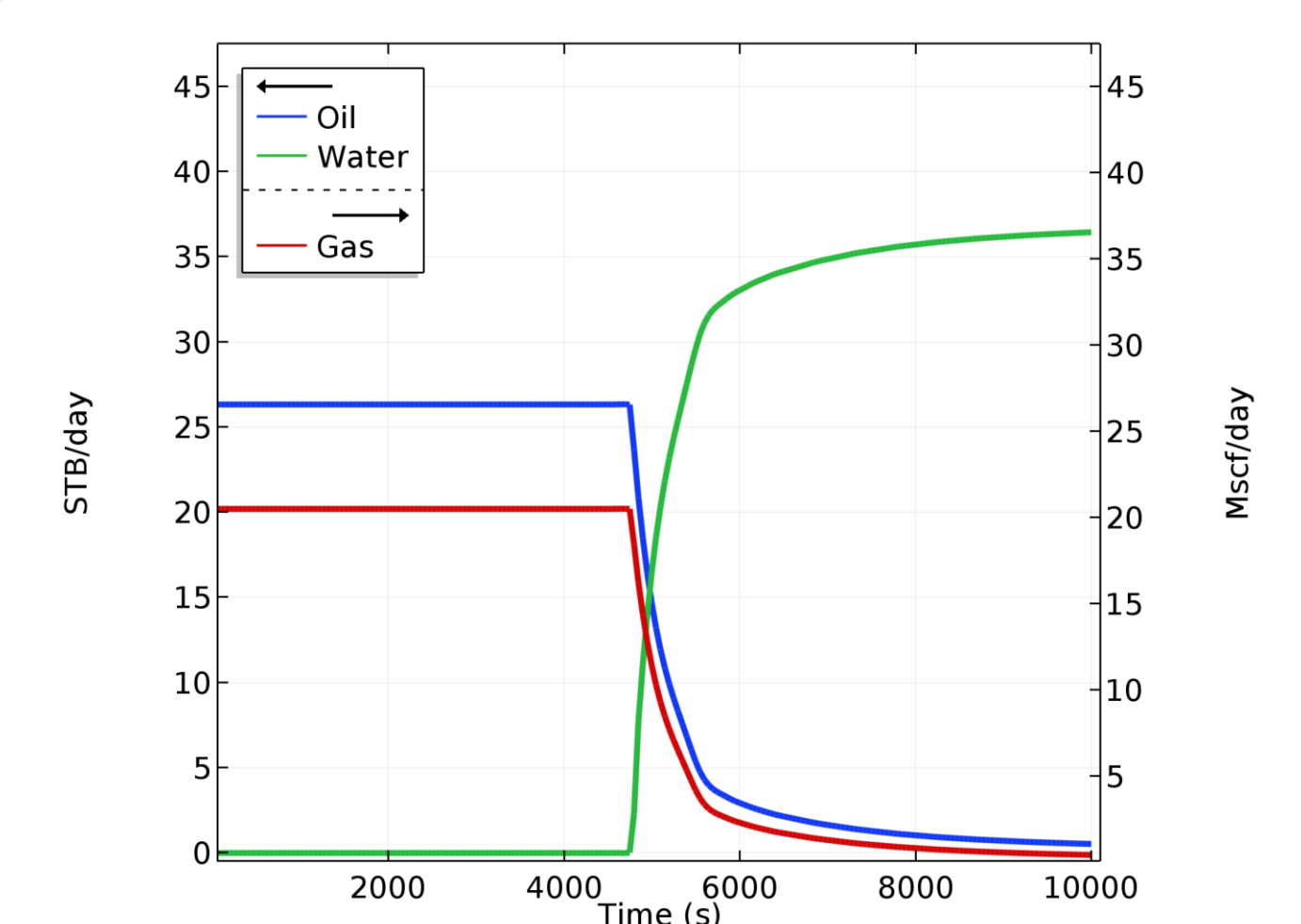


Figure 4. Production rates at standard conditions during waterflooding

### Process: Gas Injection

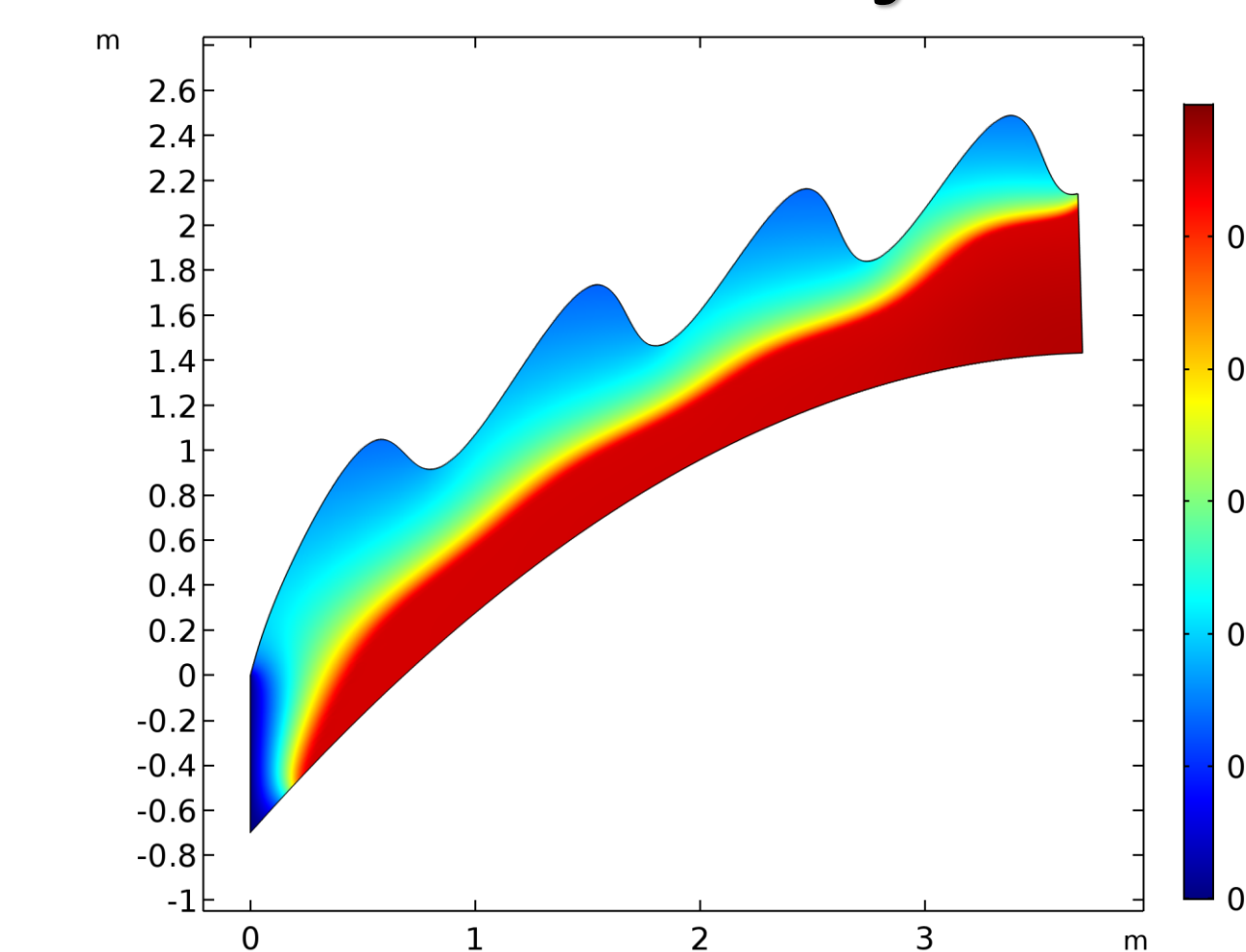


Figure 5. Oil phase saturation at the end of gas injection

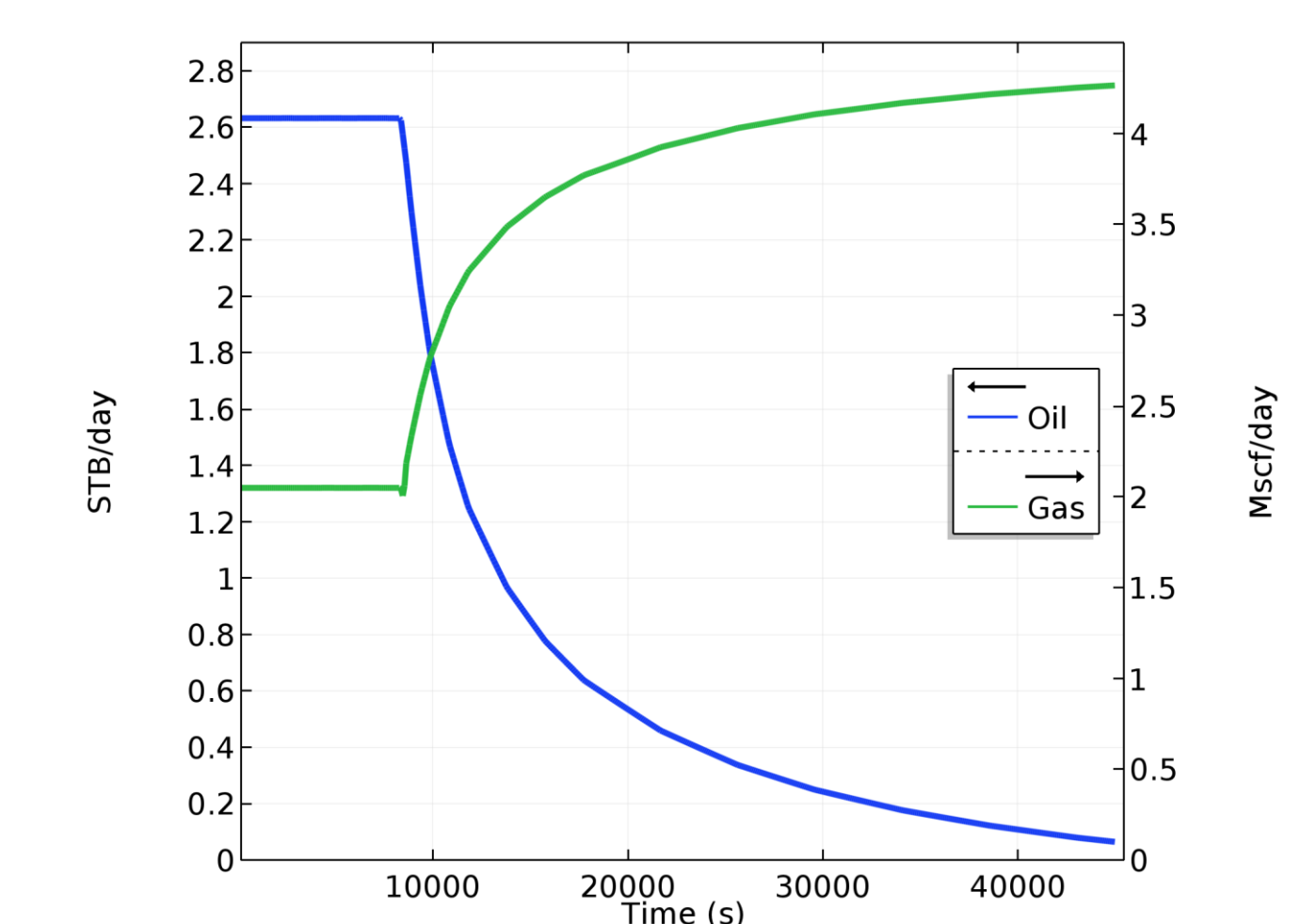


Figure 6. Production rates at standard conditions during gas injection

**CONCLUSIONS:** Successful description of the physical phenomenon and accurate estimation of recoveries. Simulation of more complex problems in EOR by implementing also the appropriate PDEs.

## REFERENCES:

1. D. Peaceman, Fundamentals of Numerical Reservoir Simulation, (1977)
2. Z. Chen, Formulations and Numerical Methods of the Black Oil Model in Porous Media, Society for Industrial and Applied Mathematics, 38, 489-514 (2000)