Presented at the COMSOL Conference 2010 India

Two Dimensional FEM Simulation of Ultrasonic Wave Propagation in Isotropic Solid Media Using COMSOL®

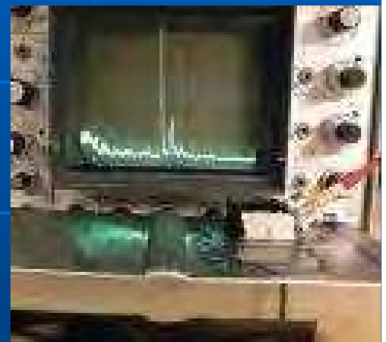
Bikash Ghose[1] *, Krishnan Balasubramaniam[2] #
C V Krishnamurthy[3], A Subhananda Rao[1]

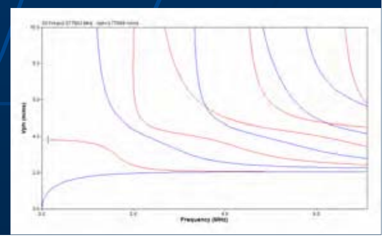
[1] High Energy Materials Research Laboratory, Sutarwadi, Pune - 21
[2] Center for Non Destructive Evaluation, IIT Madras, Chennai - 36
[3] Department of Physics, IIT Madras, Chennai -36

E-mail: * ghose.bikash@hemrl.drdo.in , # balas@iitm.ac.in

Motivation

- Application of Ultrasonic wave propagation
  - Under Water Acoustics
  - Medical Diagnostic
  - Structural Health Monitoring (SHM)
  - Non-Destructive Evaluation (NDE)
  - Material Characterization
- Non-Destructive Evaluation
  - Bulk wave
    - Longitudinal wave (Pulse echo technique, Through transmission etc)
    - Transverse Wave
  - Surface Wave
  - Guided Wave / Lamb Wave etc.
- Guided Wave
  - Many different modes of ultrasonic vibration (Symmetric, Anti symmetric)
- Numerical Simulation of Wave propagation
  - Fluid (Only Longitudinal wave)
  - Solid (Longitudinal, Transverse, Surface, Lamb etc)
- Use of COMSOL for simulation of wave propagation in solids

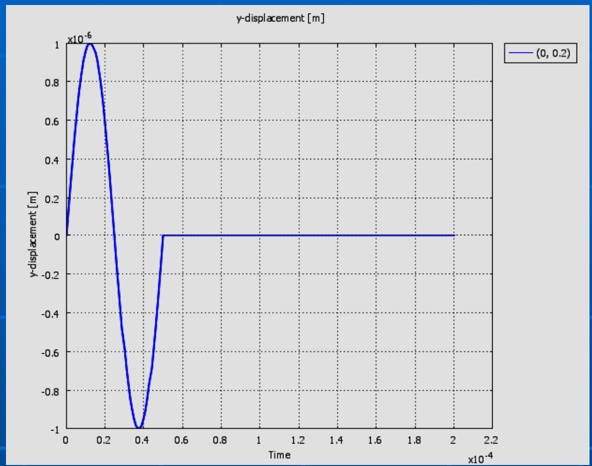




Transient Analysis for Wave Propagation

- Transient Analysis always pose difficult problems
- Length of element (Δx)
- Time steps (Δt)
- Type of Elements (Triangular, Quad)
- Integration scheme for integration over time
- Issues
  - Instability
  - Numerical Dispersion
  - Convergence

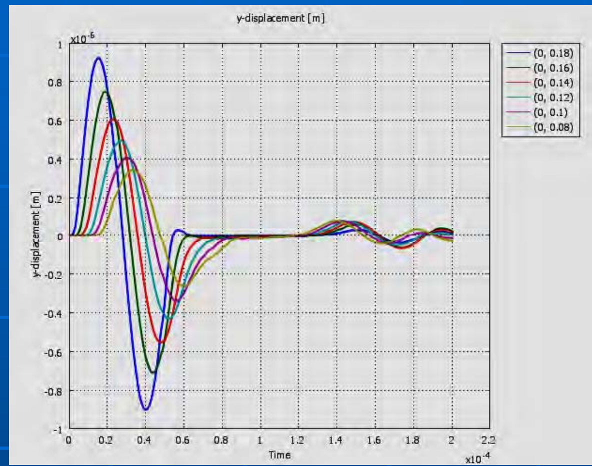
Numerical Simulation of Ultrasonic Wave Propagation using COMSOL®



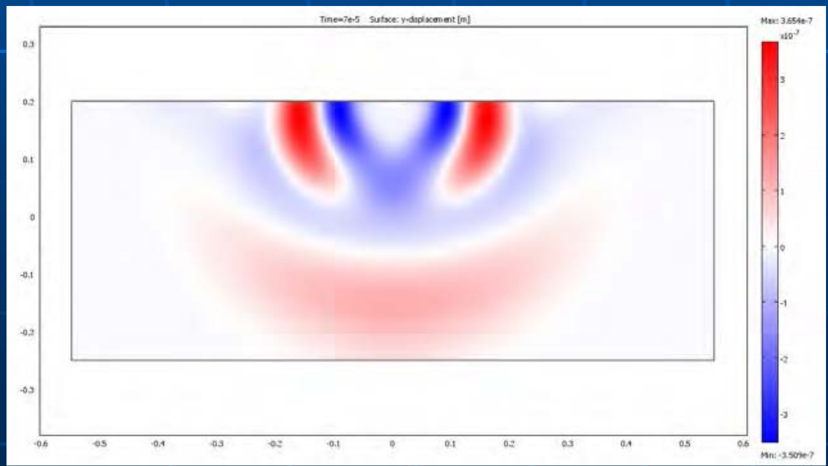
Initial Displacement

Thumb Rule ?

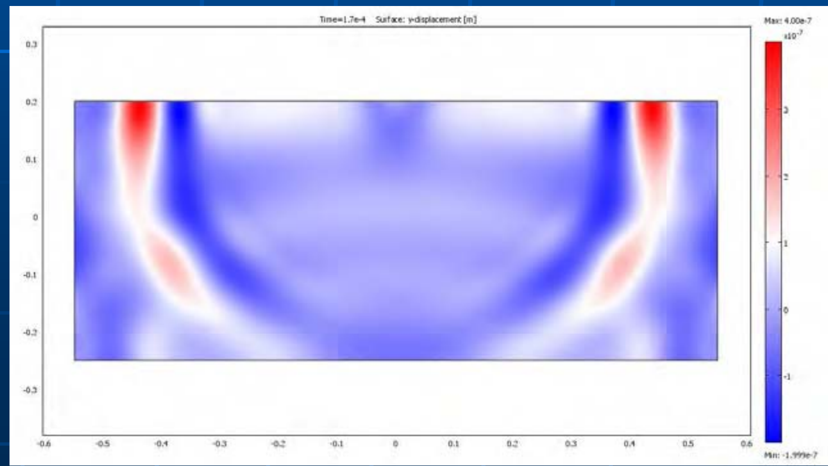
- Length of Element (Δx) = $\lambda/12$
- Time Step = $\Delta x/C_{ph}$



Displacement at diff positions with time



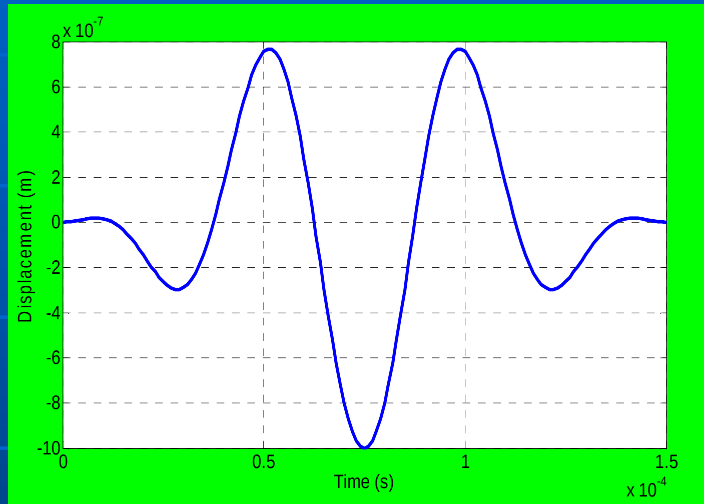
Solution at 7.0×10^{-5} s

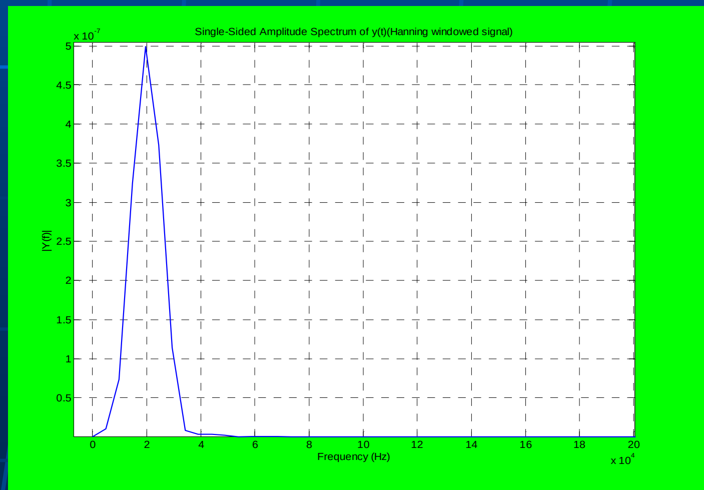


Solution at 1.7×10^{-4} s

Materials & Ultrasonic Wave Properties

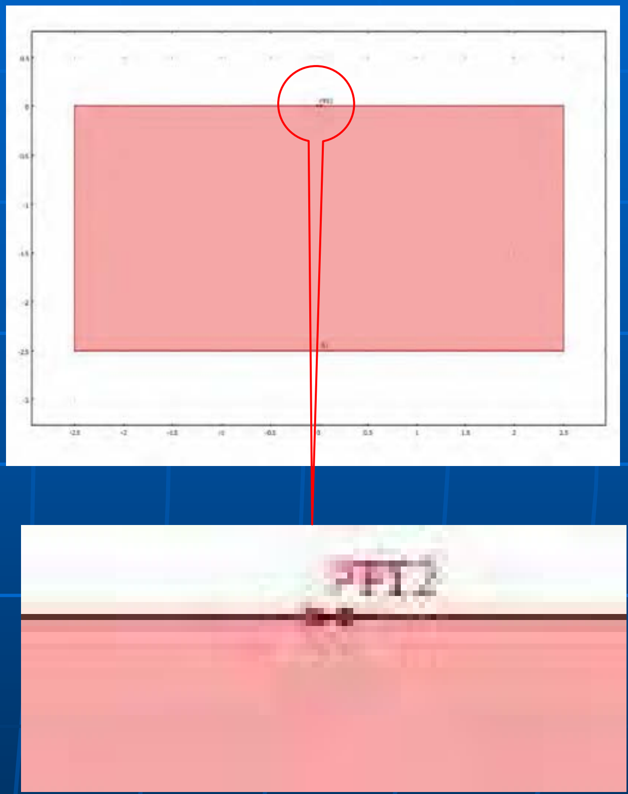
- Young's Modulus (E) = 2×10^{11} Pa
- Poisson's ratio (ν) = 0.33
- Density (ρ) = 7850 kg/m3
- Ultrasonic velocity (C_L) (longitudinal wave) = 5850 m/s
- Frequency of incident ultrasonic wave (f) = 20 kHz = 20×10^3 s^{-1}
- Wavelength of the longitudinal ultrasonic wave (λ_L) = C_L/f = 0.2925 m
- Source Length : 0.04 m (40 mm) (located at the middle)
- Source Excitation : On a line and points on line
- Simulation Domain : 5.0 m (L) x 2.5 m (H)





Meshing and Application Modes

- Triangular Elements
- Elements Automatically Generated
- Approximation : Plane Stain
- Element Type : Lagrange Quadratic
- Analysis : Transient
- Solver : Time Dependent



Effect of Length of Element (Δx)

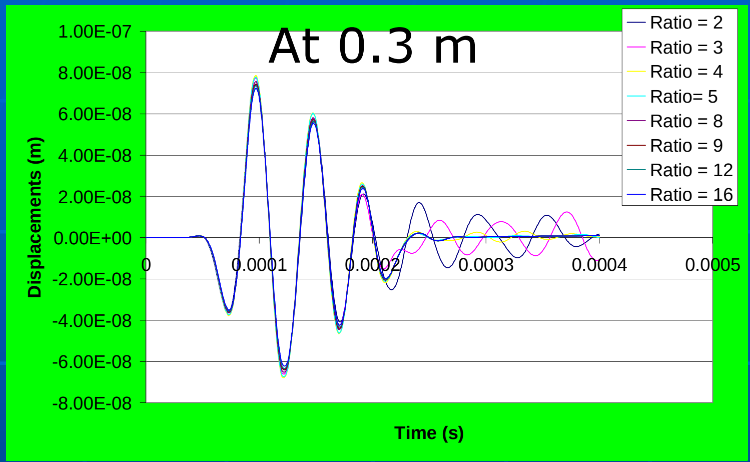
Ratio of Wavelength to Δx_{max} ($\lambda_L / \Delta x_{max}$)	Maximum element size (Δx_{max}) (m)
2	0.1462
3	0.0975
4	0.0731
5	0.0585
8	0.0366
9	0.0325
12	0.0244
16	0.0182

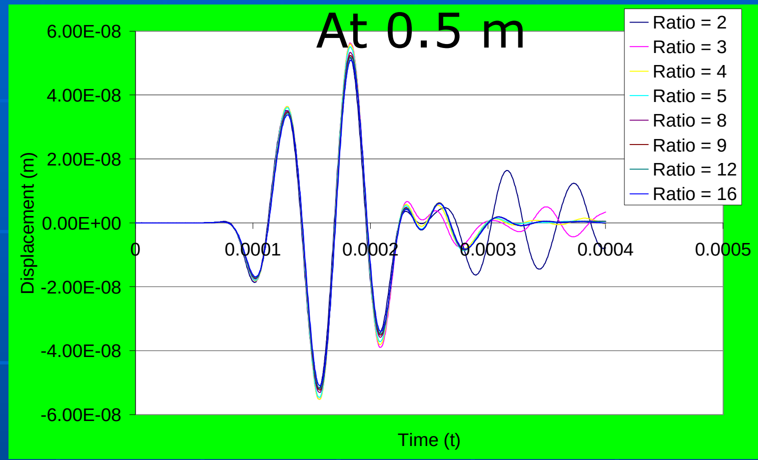
As per CFL Criteria

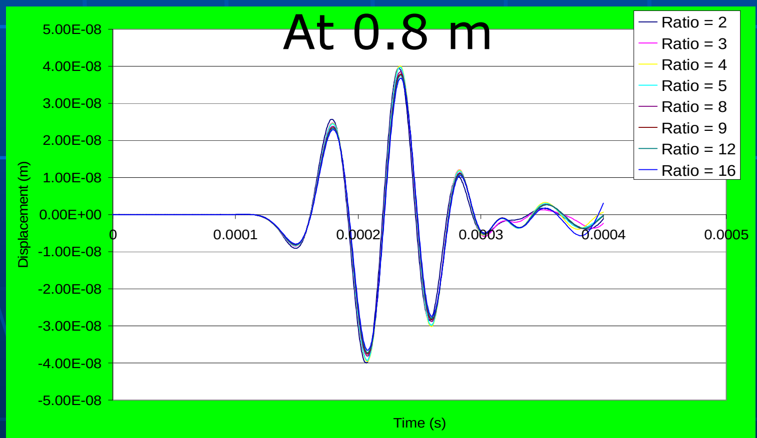
$$\Delta t_{critical} = \frac{\Delta x_{max}}{C_{ph}} = \frac{\Delta x_{max}}{C_L} = \frac{0.0182m}{5850m/s} = 3.1\times10^{-6}s$$

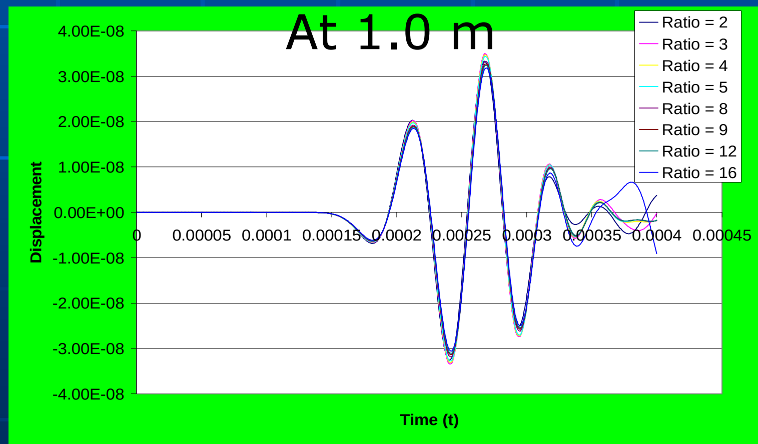
The time steps chosen initially for simulation for different Δx_{max} is 2.5×10^{-6} *s*

Onward Propagating Wave at various Distances for Different $\lambda_L/\Delta x_{max}$ at $t = 4\times10^{-4}$ s



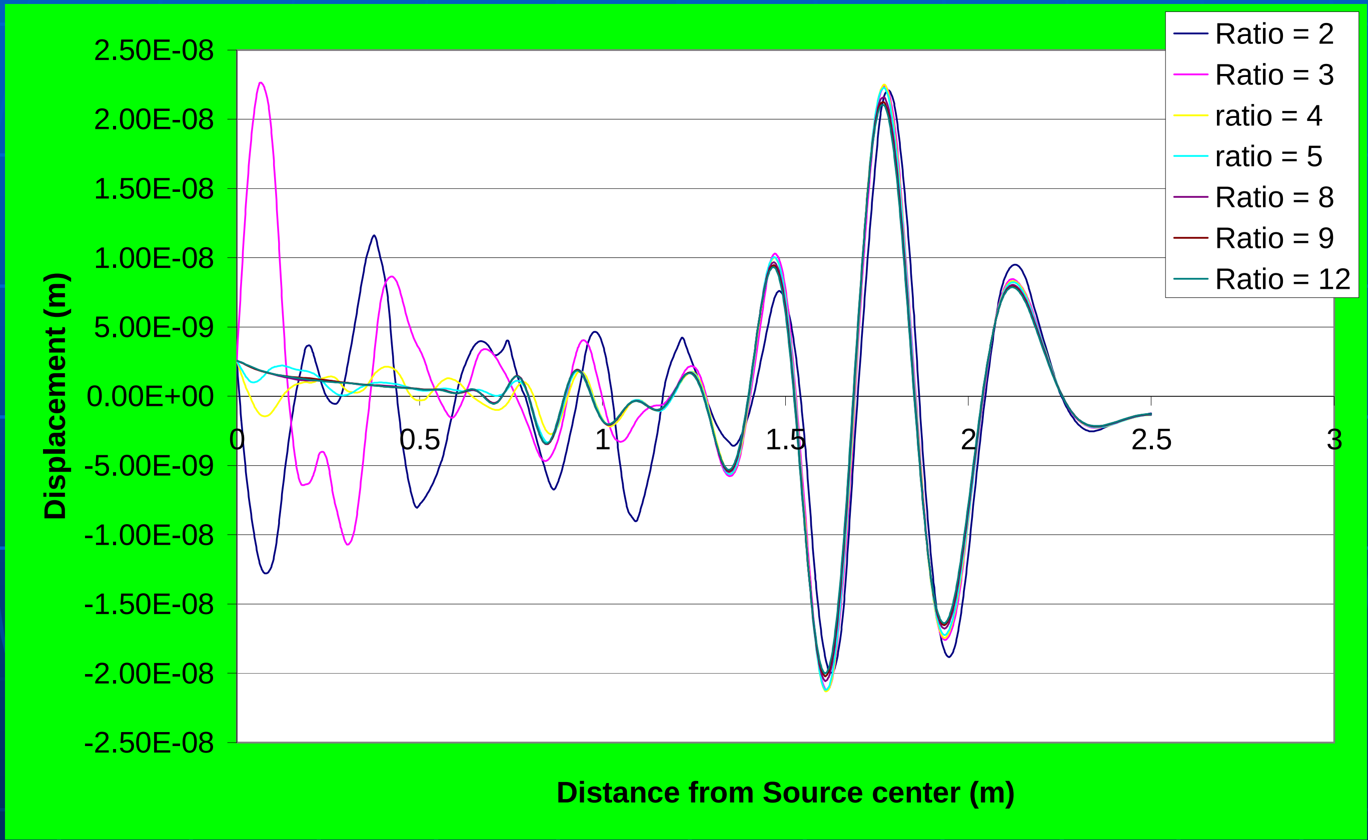






- Oscillations remains after passing by of signal
- Convergence of the solution for the ratio $(\lambda_L/\Delta x_{max}) \geq 8$
- Time steps are taken from solver or exactly what has been given does not make any difference to the propagating signal

Onward Propagating Wave for Different $\lambda_L/\Delta x_{max}$ at $t = 4\times10^{-4}$ s



Line profile (Displacement Vs Distance from source) at 0.0004s for different wavelength to element length ratio

Effect of Time Steps (Δt)

- As per CFL criteria the time steps should be less than $\Delta x/C_L$ for example, for the maximum element length of $\lambda_L /10$ the time steps should be ≤ $(\lambda_L /10 \times C_L)$ that means if

$$\Delta x_{\max} = \frac{\lambda_L}{10}$$

$$\Rightarrow \Delta t \le \frac{\Delta x_{\max}}{C_L} = \frac{\lambda_L}{10 \times C_L}$$

$$\Rightarrow \Delta t \le \frac{1}{10 \times f}$$

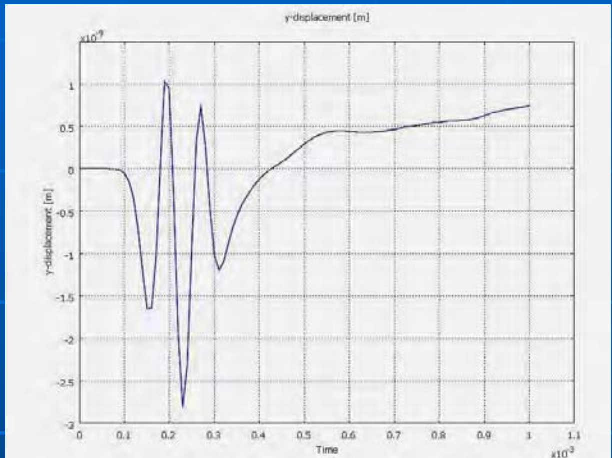
- Signal so far obtained is not as expected
- Major change in the shape of signal
- Time steps further decreased and solution was checked for convergence

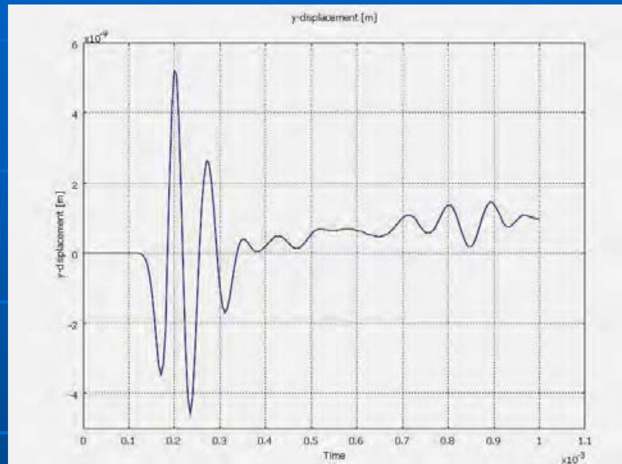
Case: I $\lambda_L/\Delta x_{max} = 5$

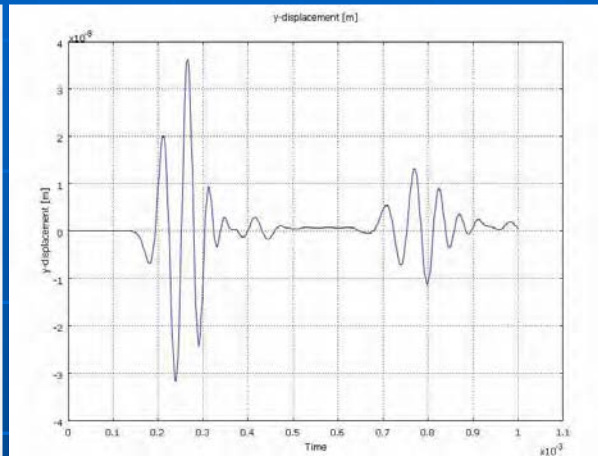
$\Delta t = 10\times10^{-6}s$

$\Delta t = 5\times10^{-6}s$

$\Delta t = 2.0\times10^{-6}s$



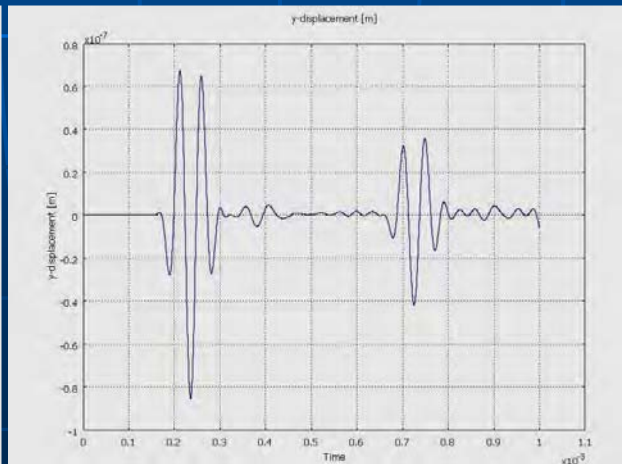


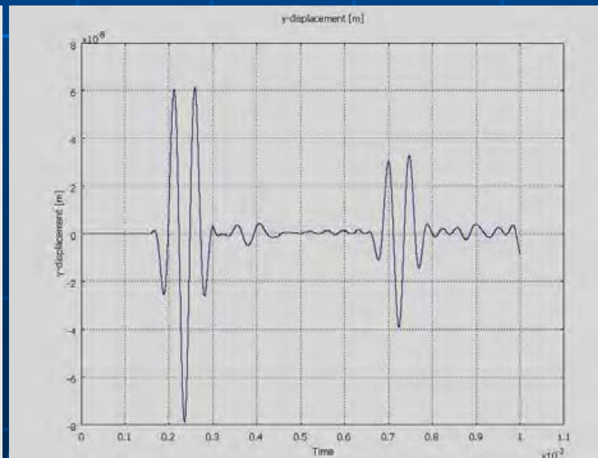


$\Delta t = 1.0\times10^{-6}s$

$\Delta t = 0.5\times10^{-6}s$

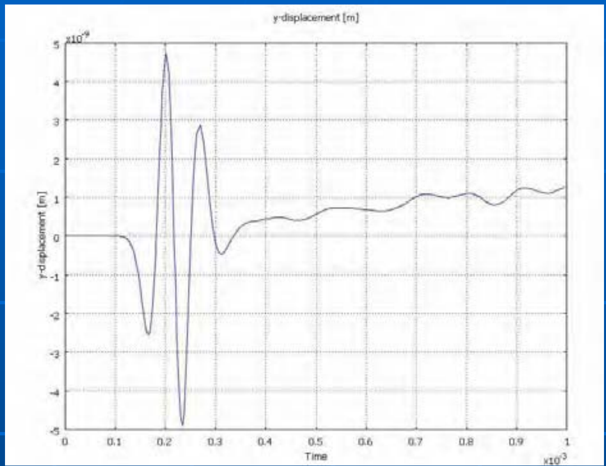
$\Delta t = 0.2\times10^{-6}s$



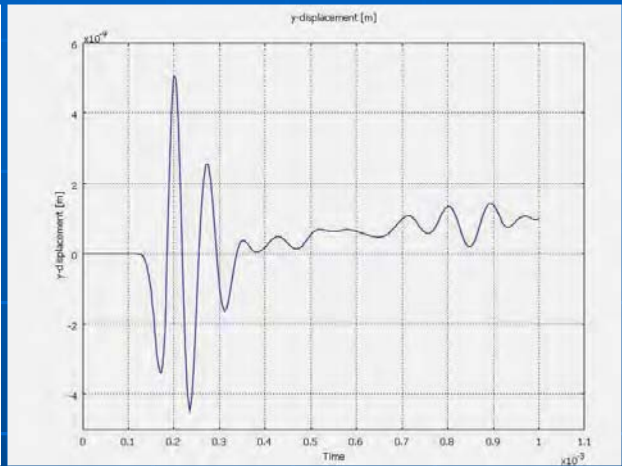


Case: II $\lambda_L/\Delta x_{max} = 8$

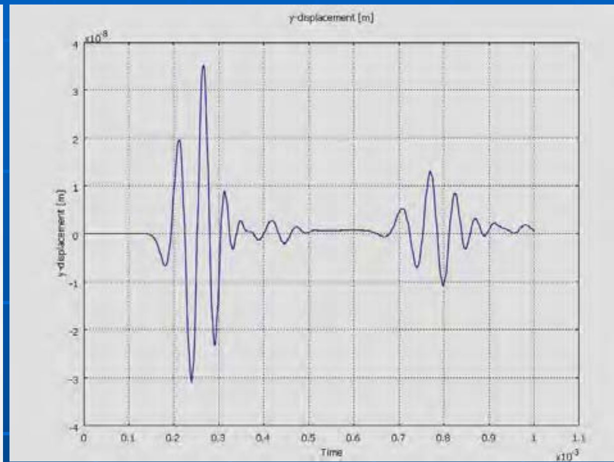
$\Delta t = 6.26\times10^{-6}s$



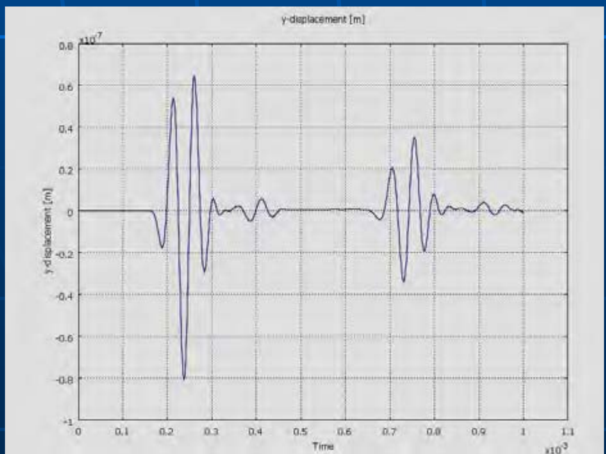
$\Delta t = 5\times10^{-6}s$



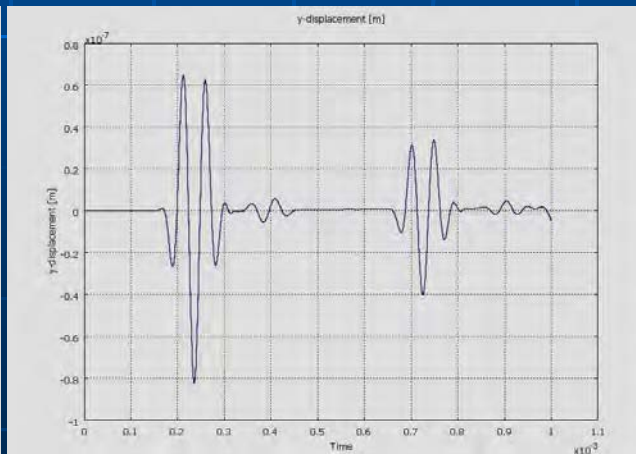
$\Delta t = 2.0\times10^{-6}s$



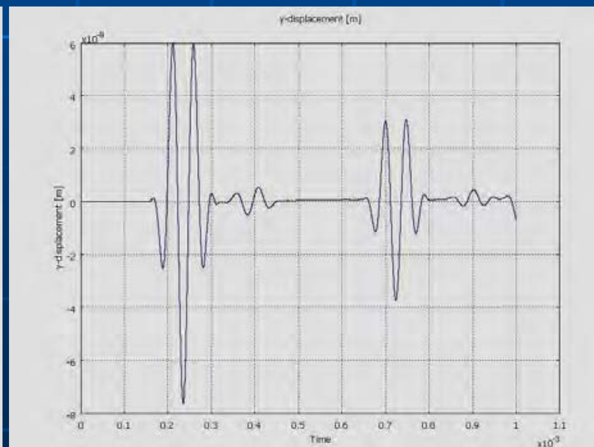
$\Delta t = 1.0\times10^{-6}s$



$\Delta t = 0.5\times10^{-6}s$

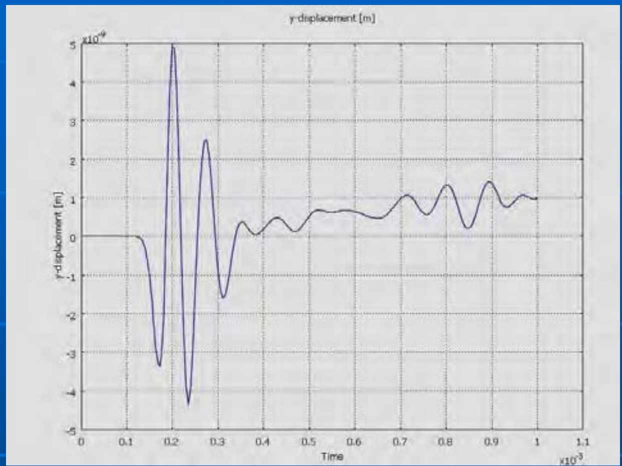


$\Delta t = 0.2\times10^{-6}s$

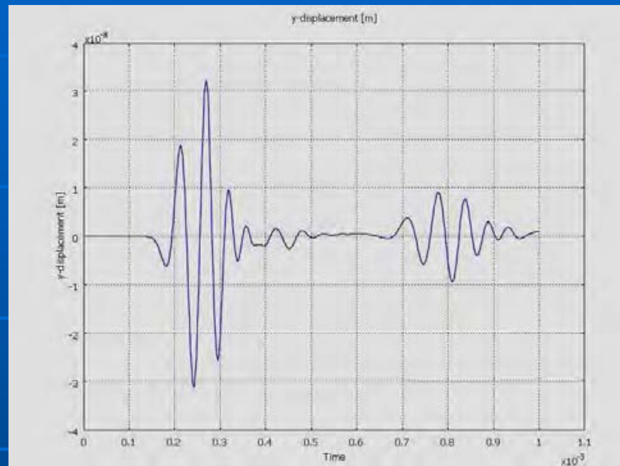


Case: III $\lambda_L/\Delta x_{max} = 12$

$\Delta t = 5\times10^{-6}s$

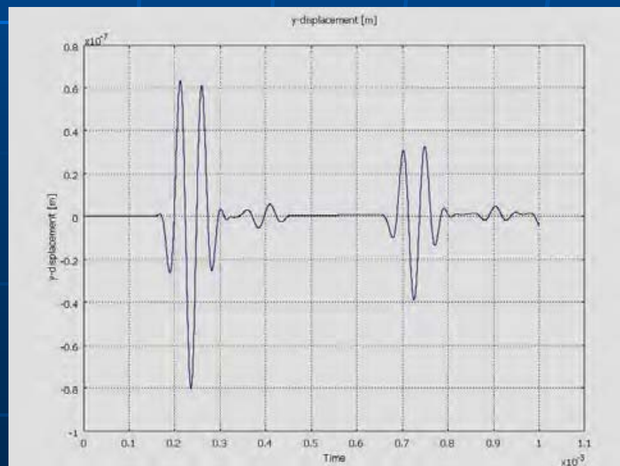


$\Delta t = 2.5\times10^{-6}s$



$\Delta t = 1.0\times10^{-6}s$

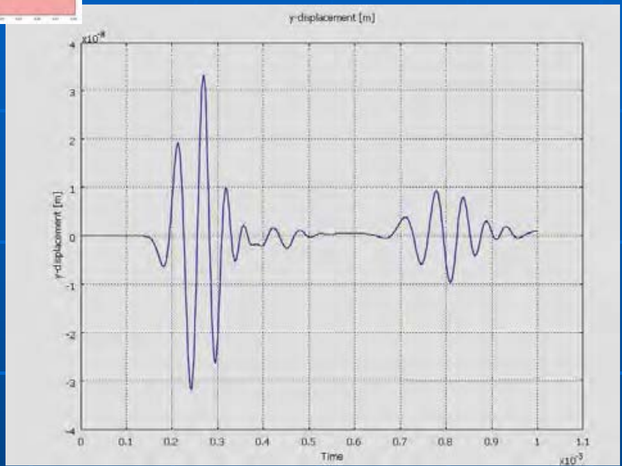
$\Delta t = 0.5\times10^{-6}s$



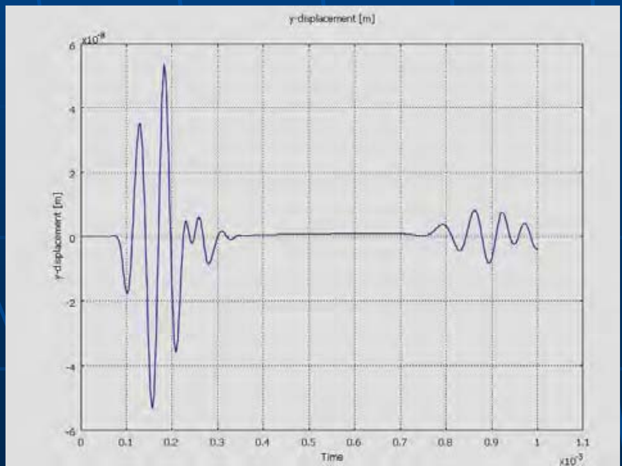
Effect of Excitation on Line and Points on Line for $\lambda_L / \Delta x_{max} = 8$ and $\Delta t = 2.5 \times 10^{-6}$ s

(Line source)
At 1.0 m

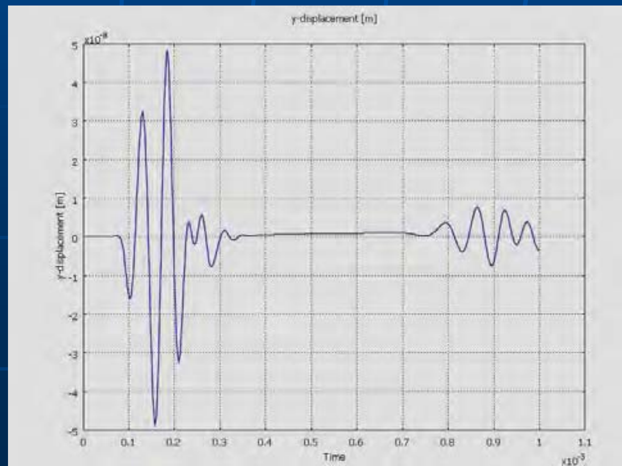
(Points on Line source)
At 1.0 m



At 0.5 m



At 0.5 m

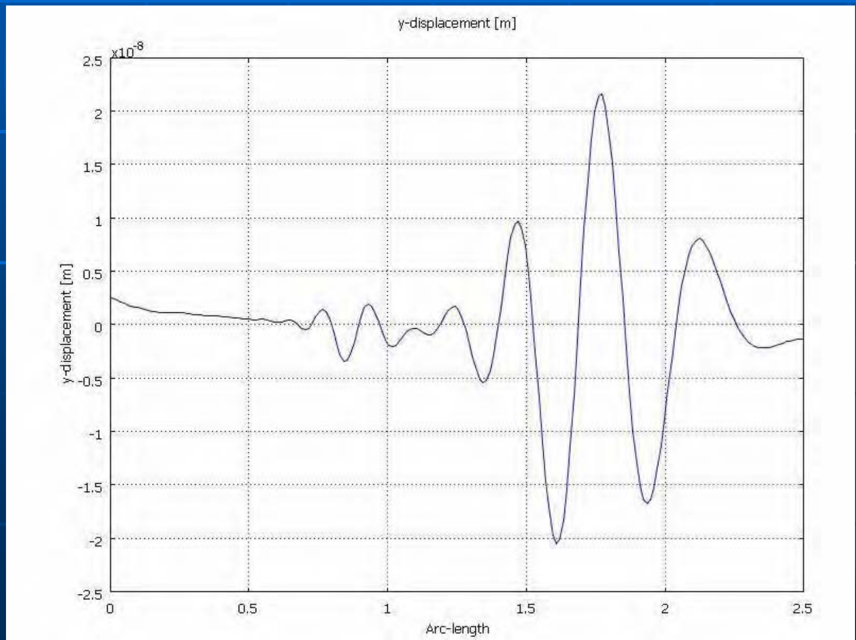


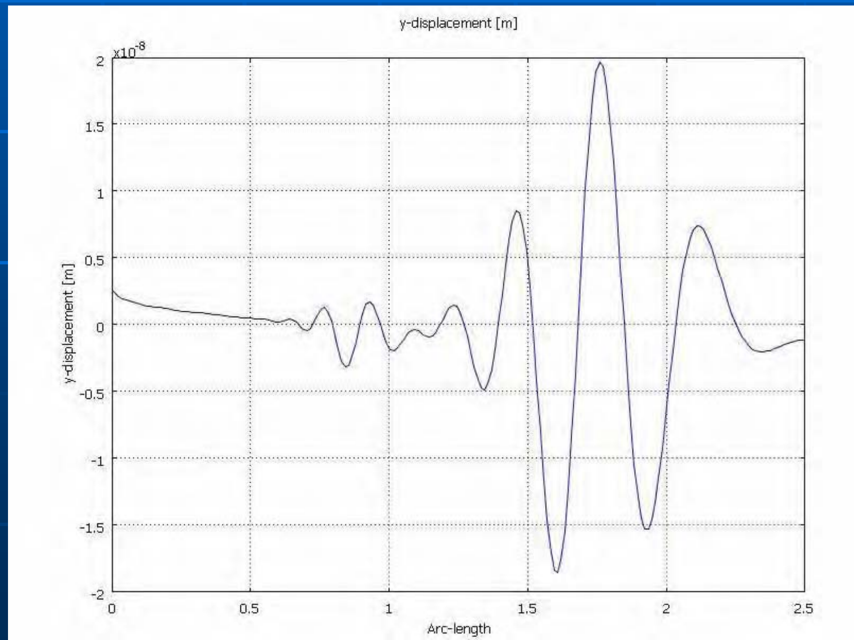
Effect of Excitation on Line and Points on Line for $\lambda_L/\Delta x_{max} = 8$ and $\Delta t = 2.5 \times 10^{-6}$ s

Line profile at $t = 4 \times 10^{-4}$ s

(Line source)

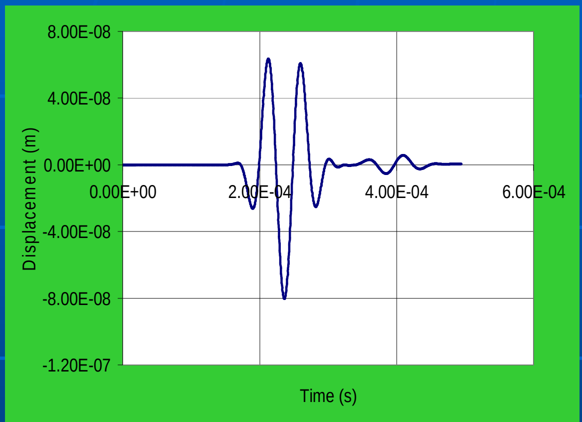
(Points on Line source)

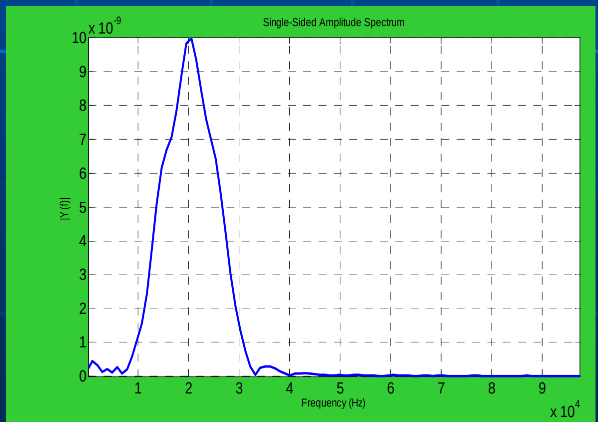




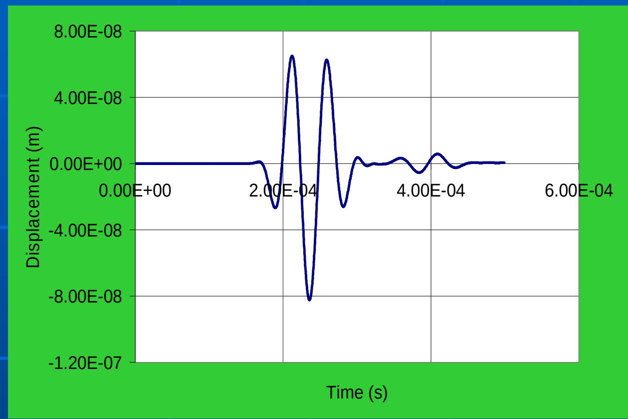
Time and Frequency Domain Signal for Forward Propagating Wave

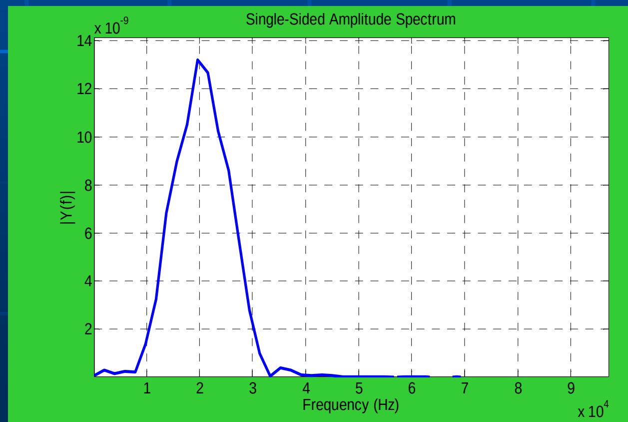
$$\frac{\lambda_L}{\Delta x_{\max}} = 12$$



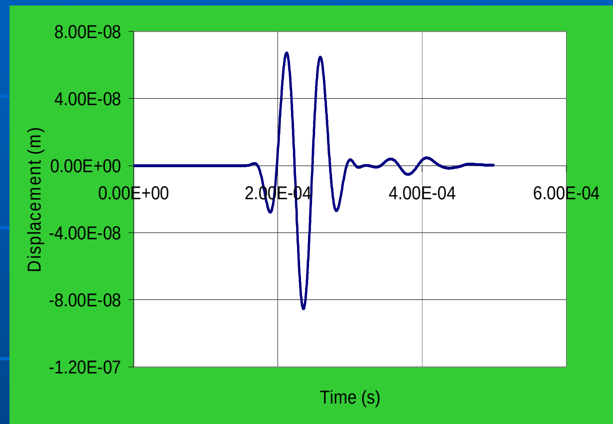


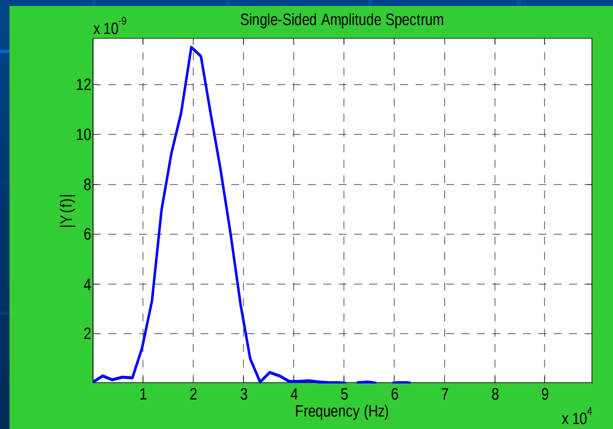
$$\frac{\lambda_L}{\Delta x_{\max}} = 8$$





$$\frac{\lambda_L}{\Delta x_{\max}} = 5$$

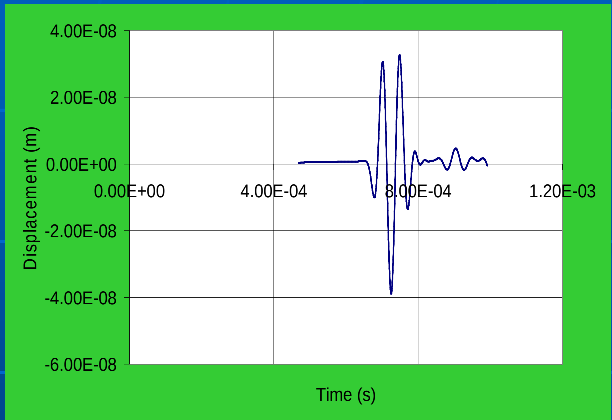


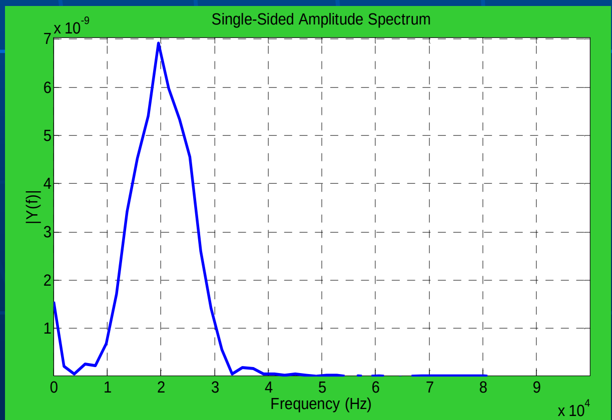


All signals are for $\Delta t = 0.5 \times 10^{-6}$ s

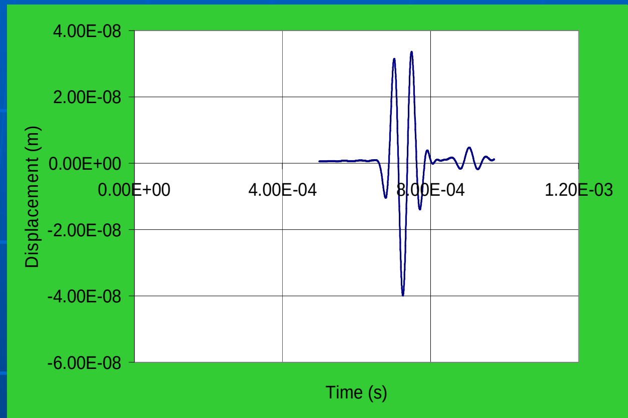
Time and Frequency Domain Signal for Back Wall Reflected Wave

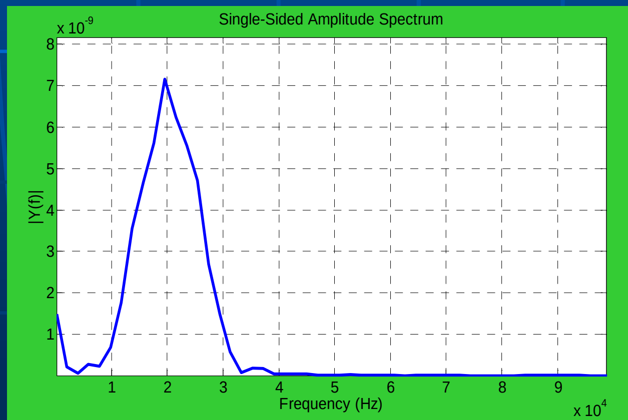
$$\frac{\lambda_L}{\Delta x_{\max}} = 12$$



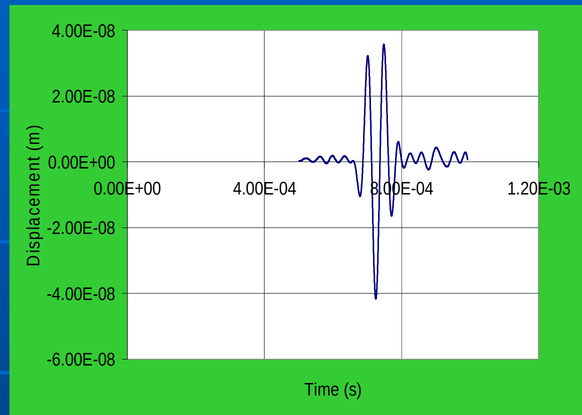


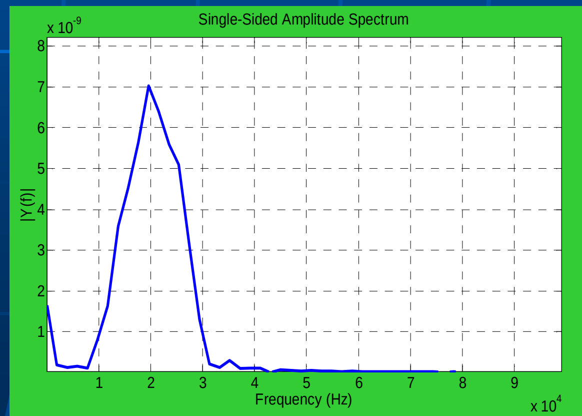
$$\frac{\lambda_L}{\Delta x_{\max}} = 8$$





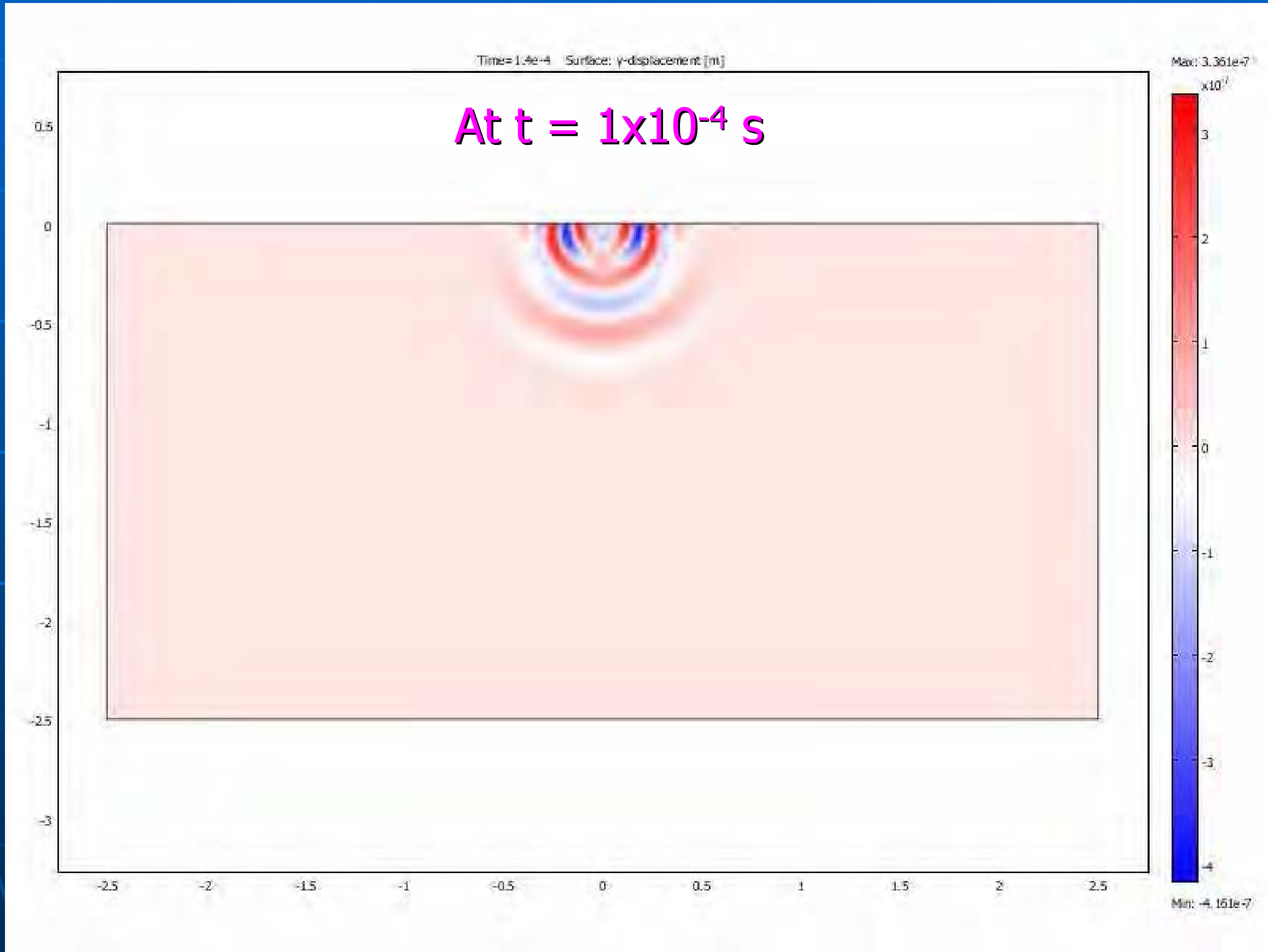
$$\frac{\lambda_L}{\Delta x_{\max}} = 5$$



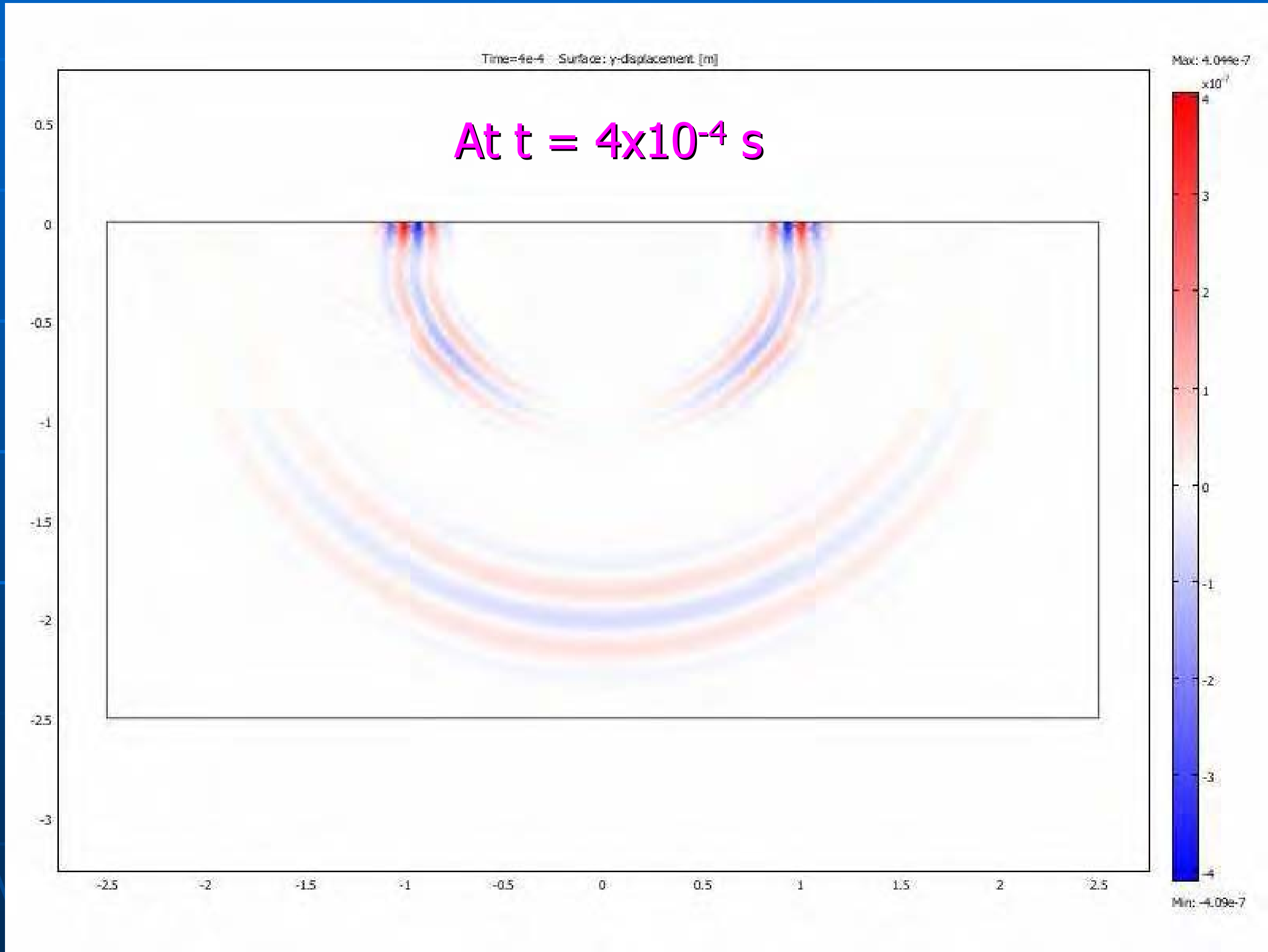


All signals are for $\Delta t = 0.5 \times 10^{-6}$ s

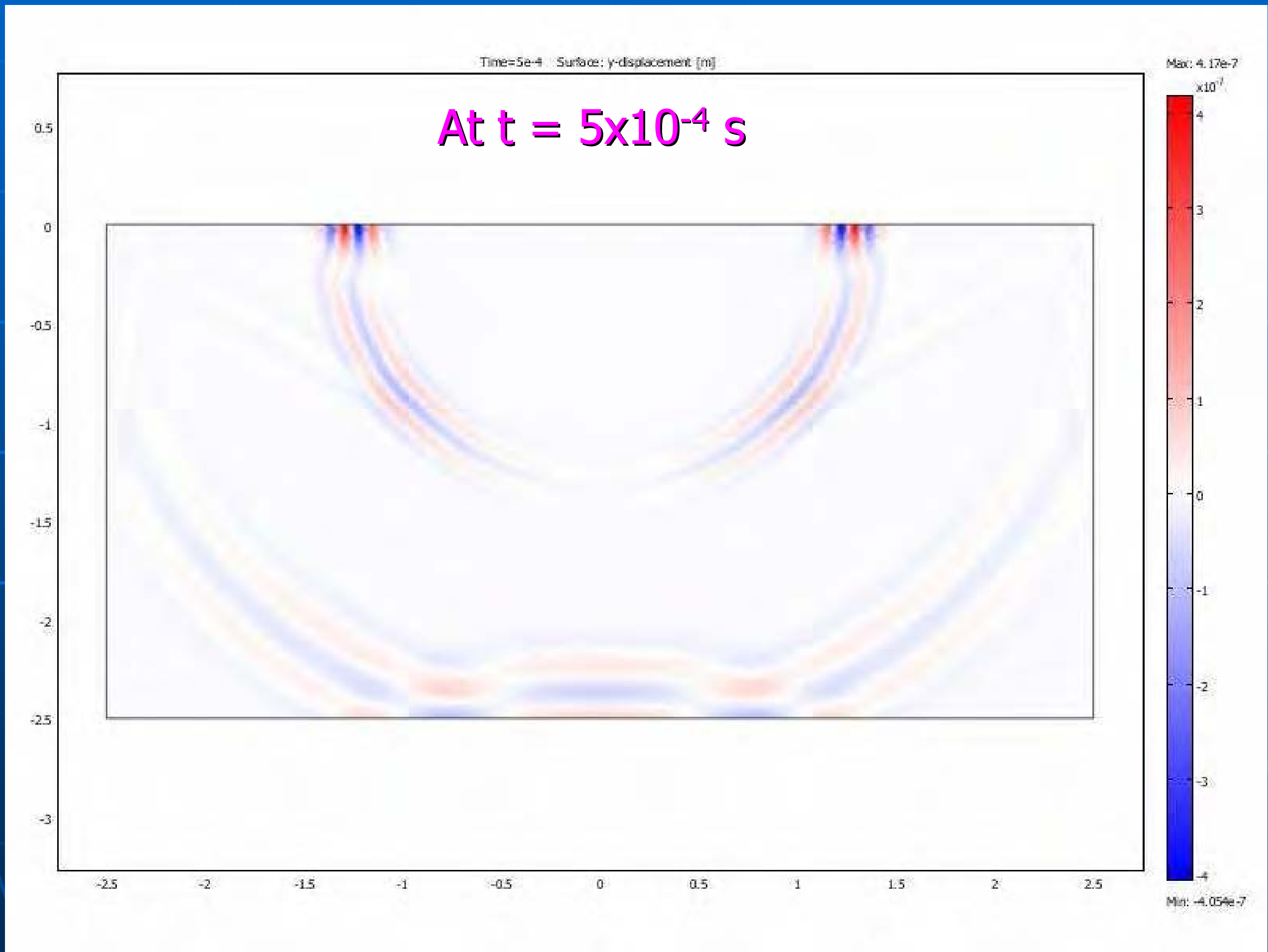
Simulation Results for $\lambda_L/\Delta x_{max} = 8$ $\Delta t = 0.5 \times 10^{-6}$ s at different time



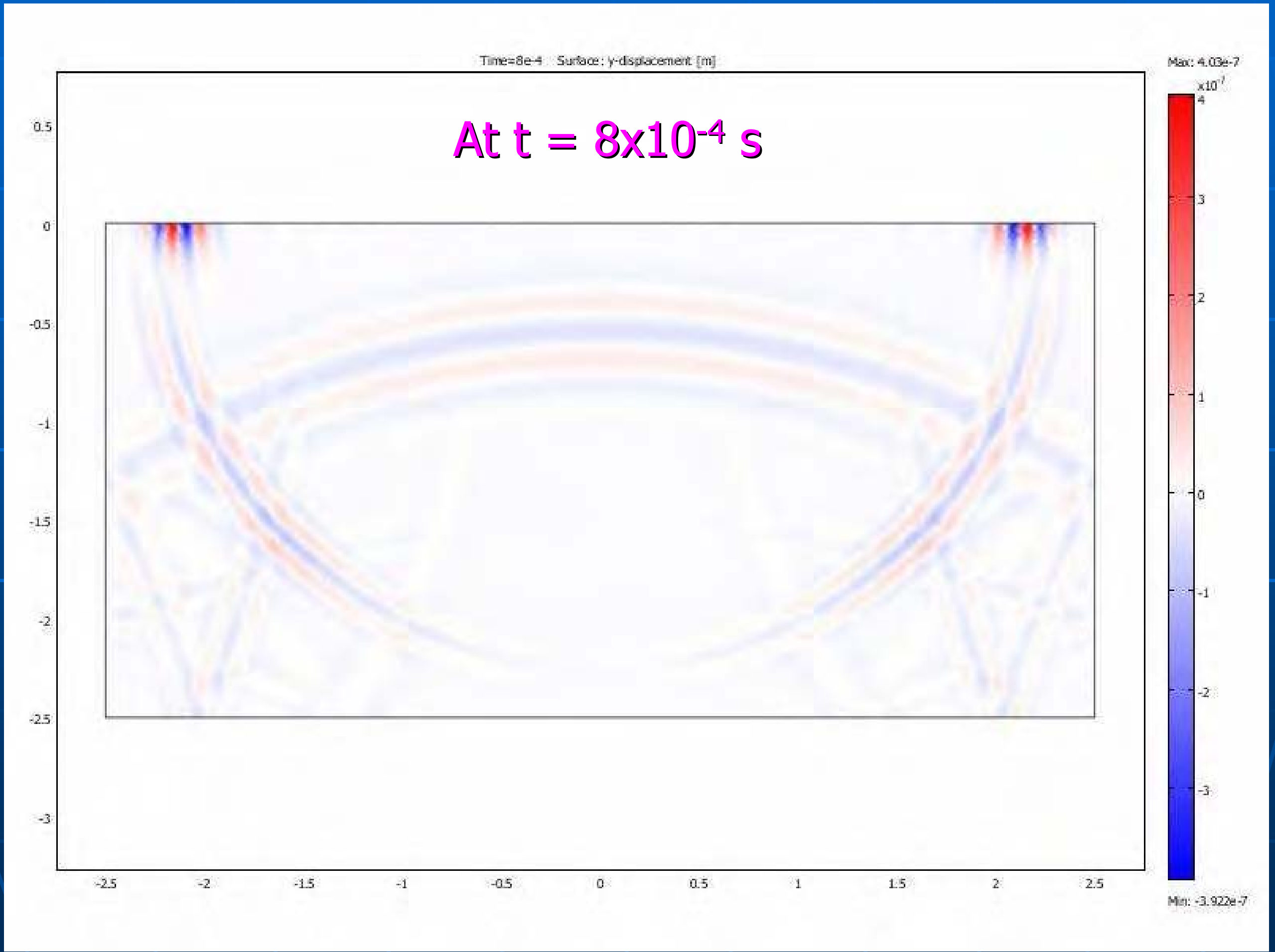
Simulation Results for $\lambda_L/\Delta x_{max} = 8$
$\Delta t = 0.5 \times 10^{-6}$ s at different time



Simulation Results for $\lambda_L/\Delta x_{max} = 8$ $\Delta t = 0.5 \times 10^{-6}$ s at different time

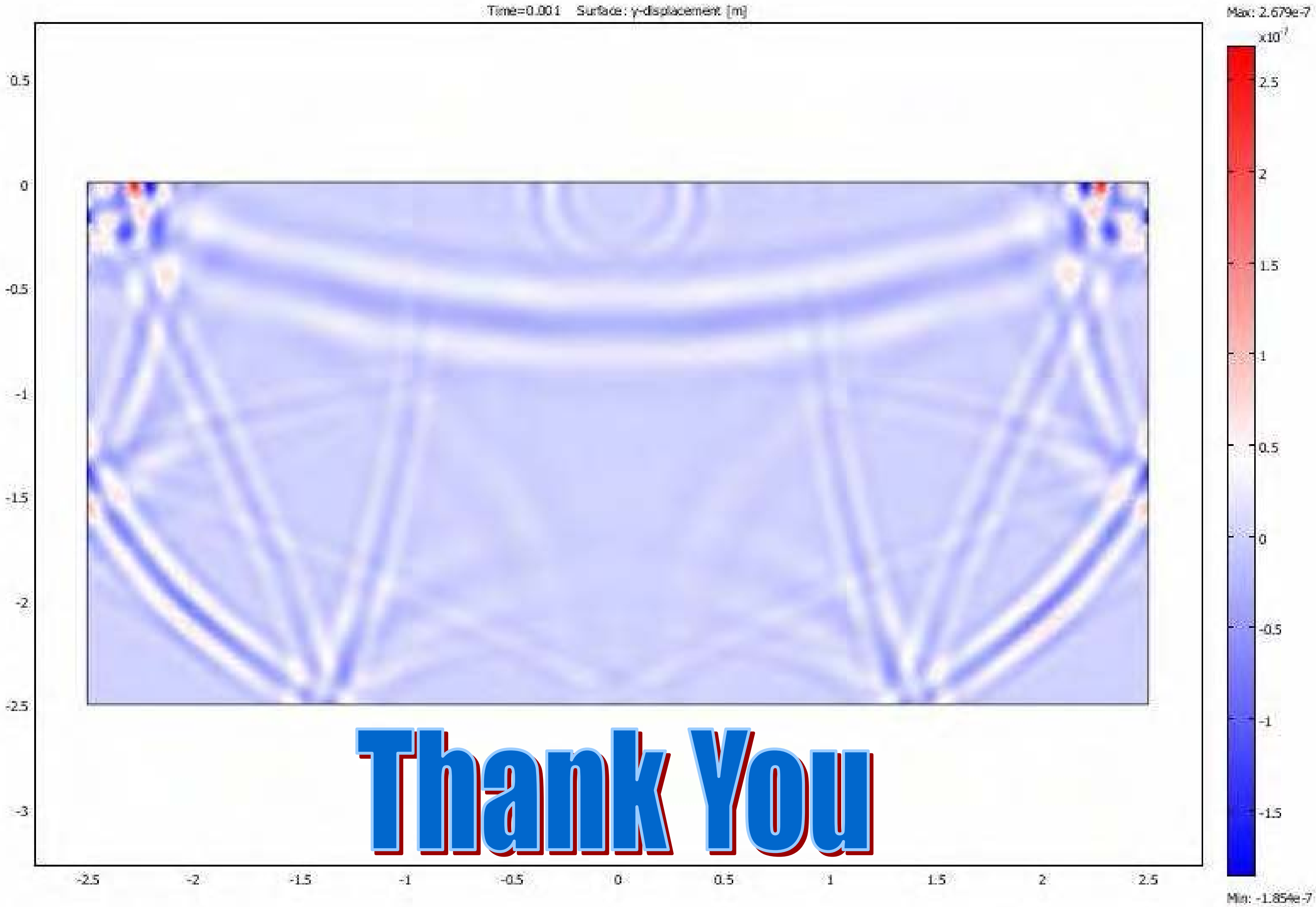


Simulation Results for $\lambda_L/\Delta x_{max} = 8$ $\Delta t = 0.5 \times 10^{-6}$ s at different time



Conclusions

- Wave Propagation can well be modeled using COMSOL with excitation on line segment and / or on points on line (Difference only in the maximum displacements)
- $\lambda_L/\Delta x_{max} \geq 8$ irrespective of any time steps less than calculated by CFL criteria (For triangular free meshing)
- Time step Δt should be about T/100 even if $\lambda_L/\Delta x \approx 16$
- No substantial difference in frequency content of forward as well as back wall reflected ultrasonic signal observed


Time=0.001 Surface: y-displacement [m]
Max: 2.679e-7
x10^-7
Min: -1.854e-7

Thank You

?

E-mail: ghose.bikash@hemrl.drdo.in , balas@iitm.ac.in