Presented at the COMSOL Conference 2010 Boston

Finite element analysis for electronics

October 8, 2010
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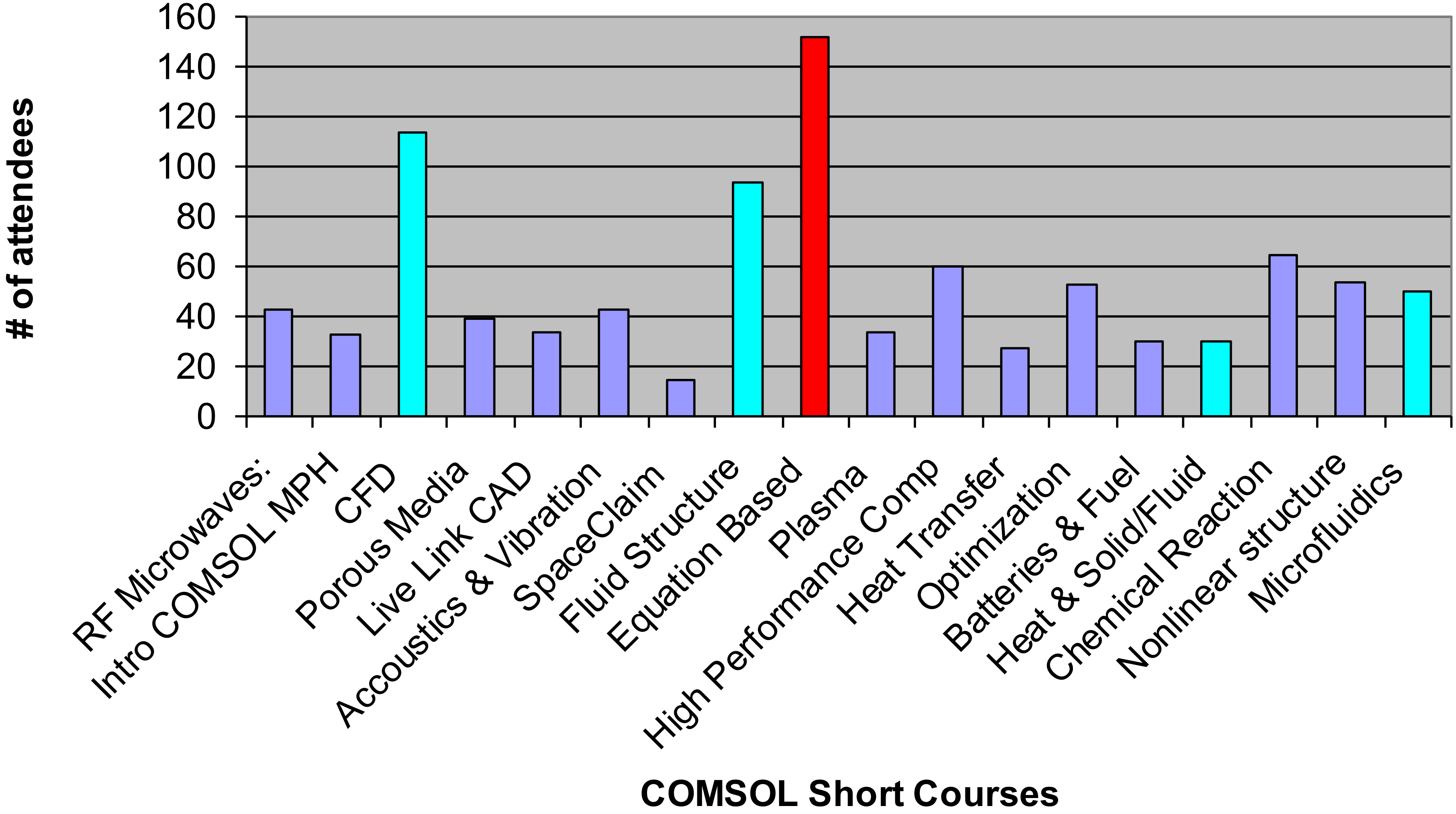
Yosuke Mizuyama, Ph.D.
Panasonic Boston Laboratory

Panasonic ideas for life

of COMSOL short courses attendees



Outline

- *Inkjet printhead*

  Piezoelectric + free surface fluid dynamics coupling

- *Grayscale photolithography for optical pickup*

  Irreversible thermal process computation using COMSOL+MATLAB

- *Holographic data storage*

  Electromagnetic for infinite region

Electronics from Panasonic

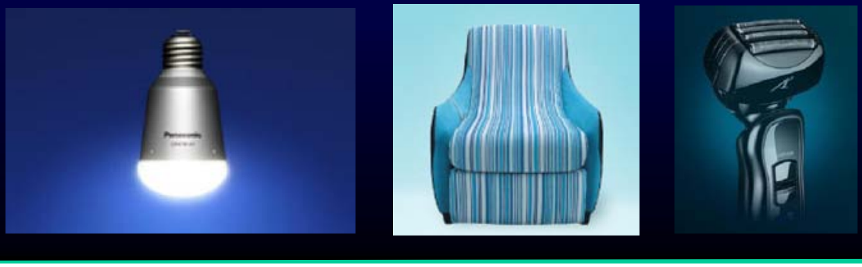
http://panasonic.jp

Home appliances

Laptop & cell phone

Automotive

Lighting & Health

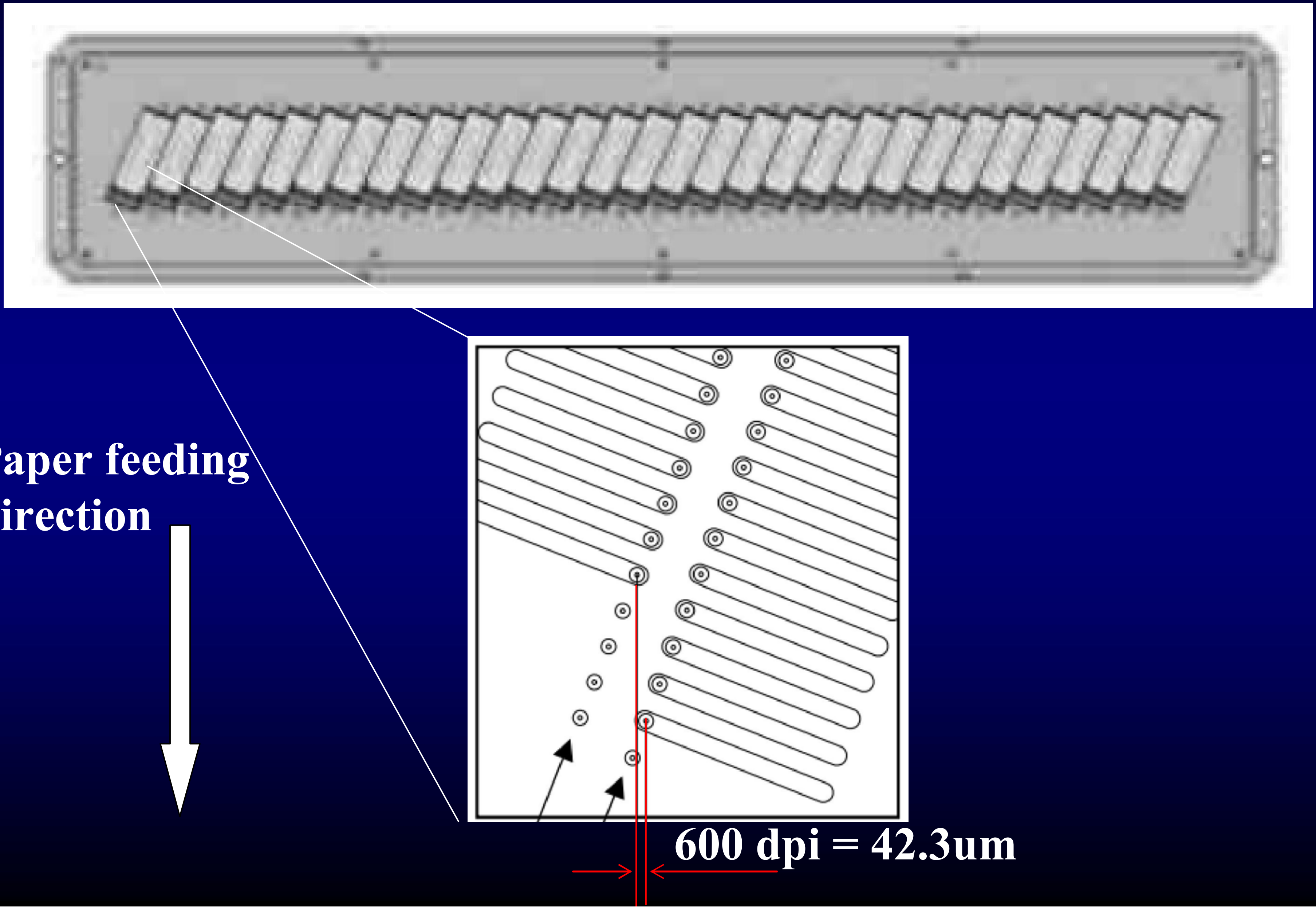


Energy

Industrial inkjet printhead

- 48,000 Jets
- 600 dpi
- 19.5” Width
- CMYK
- 240 ppm
- Aqueous Inks

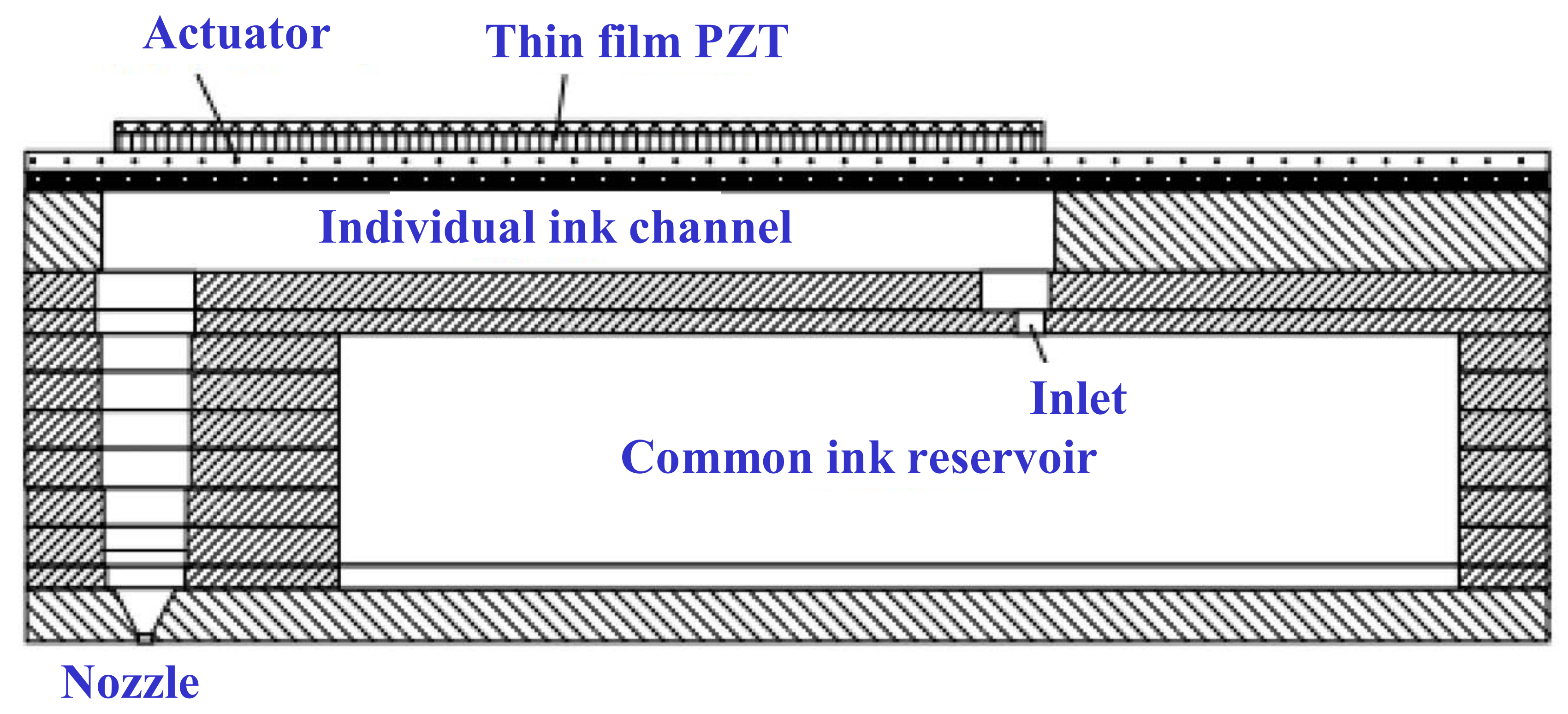
Industrial inkjet printhead



Paper feeding direction

600 dpi = 42.3um

Industrial inkjet printhead



Cross section of the head

Principle of PZT inkjet

Compliance Piezoelectric coupling

Strain

Charge density displacement

6x6 6x3

$$\begin{array}{c} 6\text{x}1 \\ 3\text{x}1 \end{array} \begin{bmatrix} S \\ D \end{bmatrix} = \begin{bmatrix} s_E & d^t \\ d & \varepsilon_T \end{bmatrix} \begin{bmatrix} T \\ E \end{bmatrix} \begin{array}{c} 6\text{x}1 \\ 3\text{x}1 \end{array}$$

3x6 3x3

Stress

Electric field

Permittivity

d-matrix

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ \bigcirc\!\!\!\!\!\!d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$

Strain-charge form constitutive equation

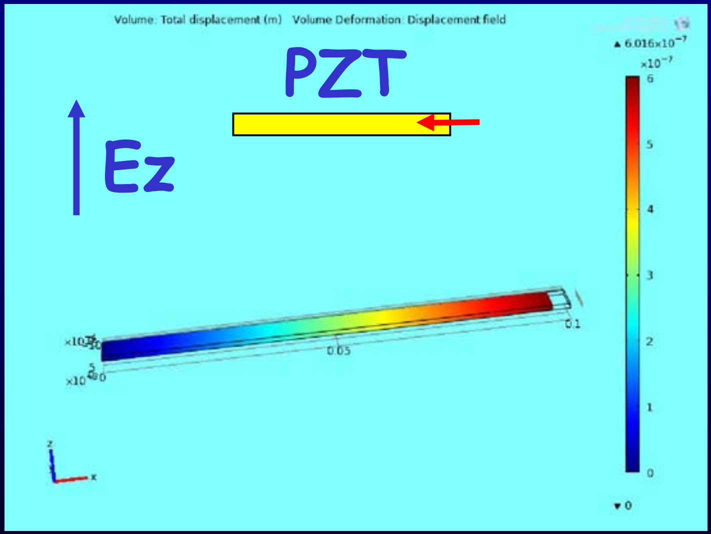
Principle of PZT inkjet

Use this strain

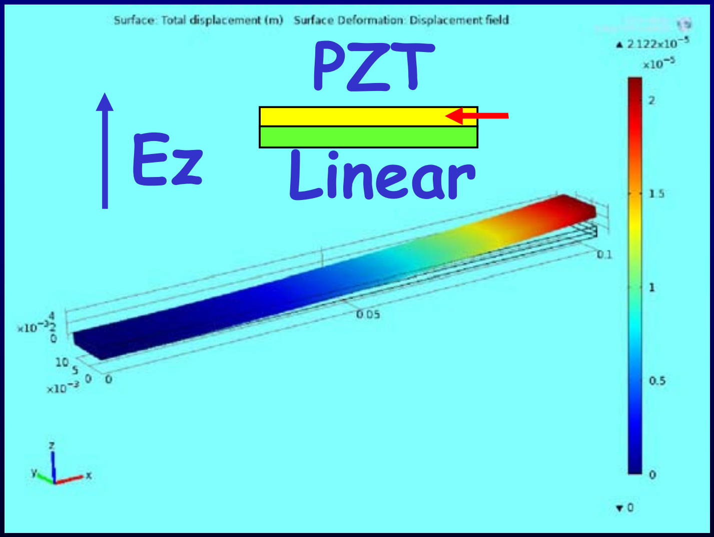
Normal strain

Shear strain

$$\begin{bmatrix} S_{xx} \\ S_{yy} \\ S_{zz} \\ S_{xy} \\ S_{yz} \\ S_{xz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & * & 0 \\ * & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$



PZT only



Unimorph: PZT+ linear material

Statement of the problem

Piezoelectric

$$\frac{\partial^2 U}{\partial t^2} + \nabla \cdot \mathcal{T} \qquad \text{in } \Omega_{\text{PZT}}$$

$$\nabla \cdot \mathcal{E} = 0 \qquad \text{in } \Omega_{\text{PZT}}$$

$$U = 0 \qquad \text{on } \Gamma_{\text{D0PZT}}$$

$$\mathcal{T} \cdot n = 0 \qquad \text{on } \Gamma_{\text{N0PZT}}$$

$$\varphi = \varphi_0 \qquad \text{on } \Gamma_{\text{D0ELE}}$$

$$\frac{\partial \varphi}{\partial n} = 0 \qquad \text{on } \Gamma_{\text{N0ELE}}$$

$$\mathcal{T}_{ij} = c_{ijkl} U_{k,lj} + e_{kij} \varphi_{,k}$$

$$\mathcal{E}_i = -e_{ikl} U_{k,l} + \epsilon_{ik} \varphi_{,k}$$

Free surface Navier-Stokes

$$\rho \frac{D^2 u}{D t^2} - \mu \nabla^2 u + \nabla p = f \qquad \text{in } \Omega_{\text{ink}}$$

$$\nabla \cdot u = 0 \qquad \text{in } \Omega_{\text{ink}}$$

$$u \cdot n = 0 \qquad \text{on } \Gamma_{\text{D0ink}}$$

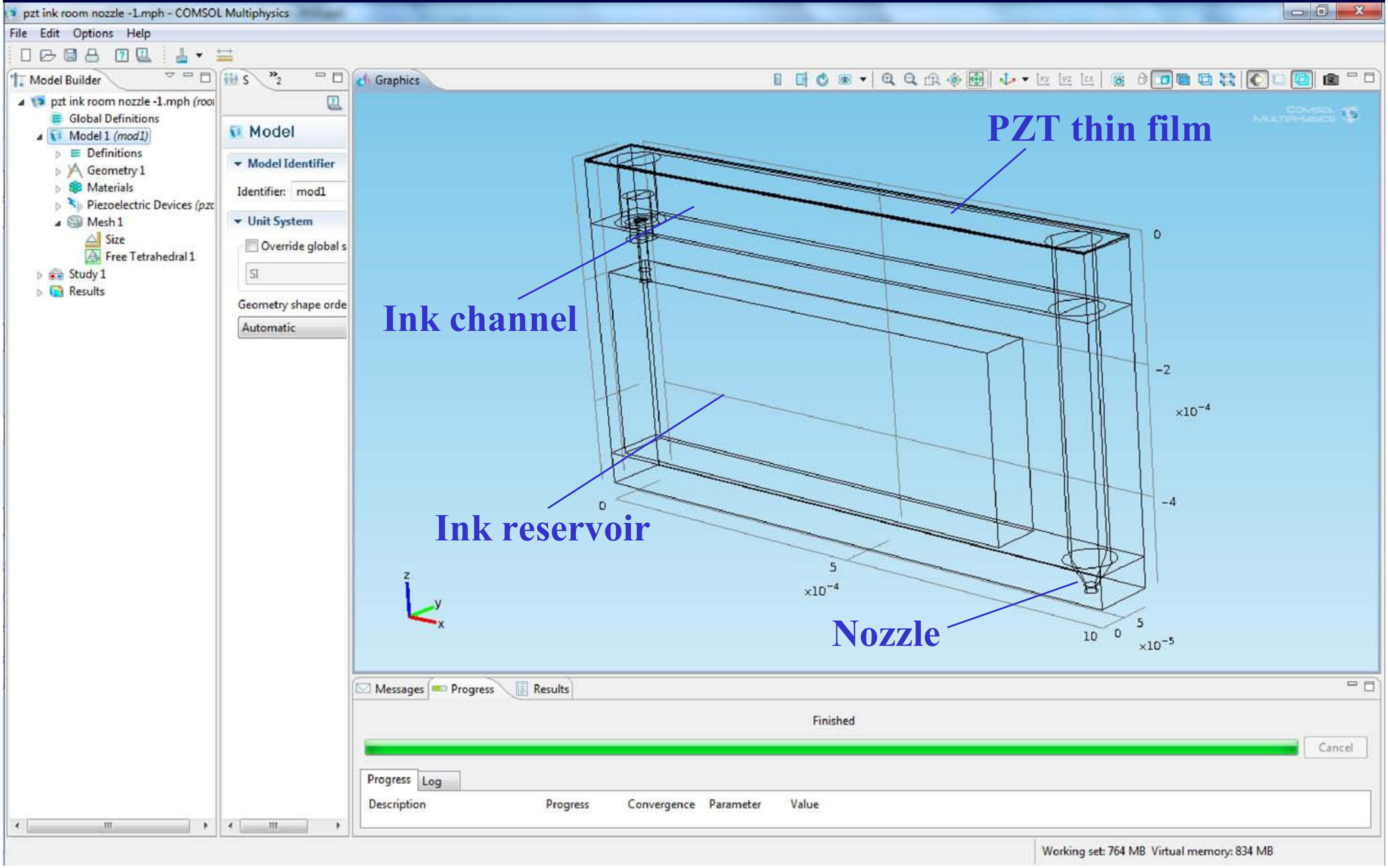
$$p = g \qquad \text{on } \Gamma_{\text{Dpink}}$$

$$-\tau \cdot n + \sigma \kappa n = 0 \qquad \text{on } \Gamma_{\text{N0ink}}$$

$$\tau_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\frac{\partial \eta}{\partial t} + u \cdot \nabla \eta = 0 \qquad \text{in } \Omega_{\text{ink}}$$

COMSOL 4.0a modeling



Identifier: mod1

☐ Override global s

SI

Geometry shape orde

Automatic

Finished

Working set: 764 MB Virtual memory: 834 MB

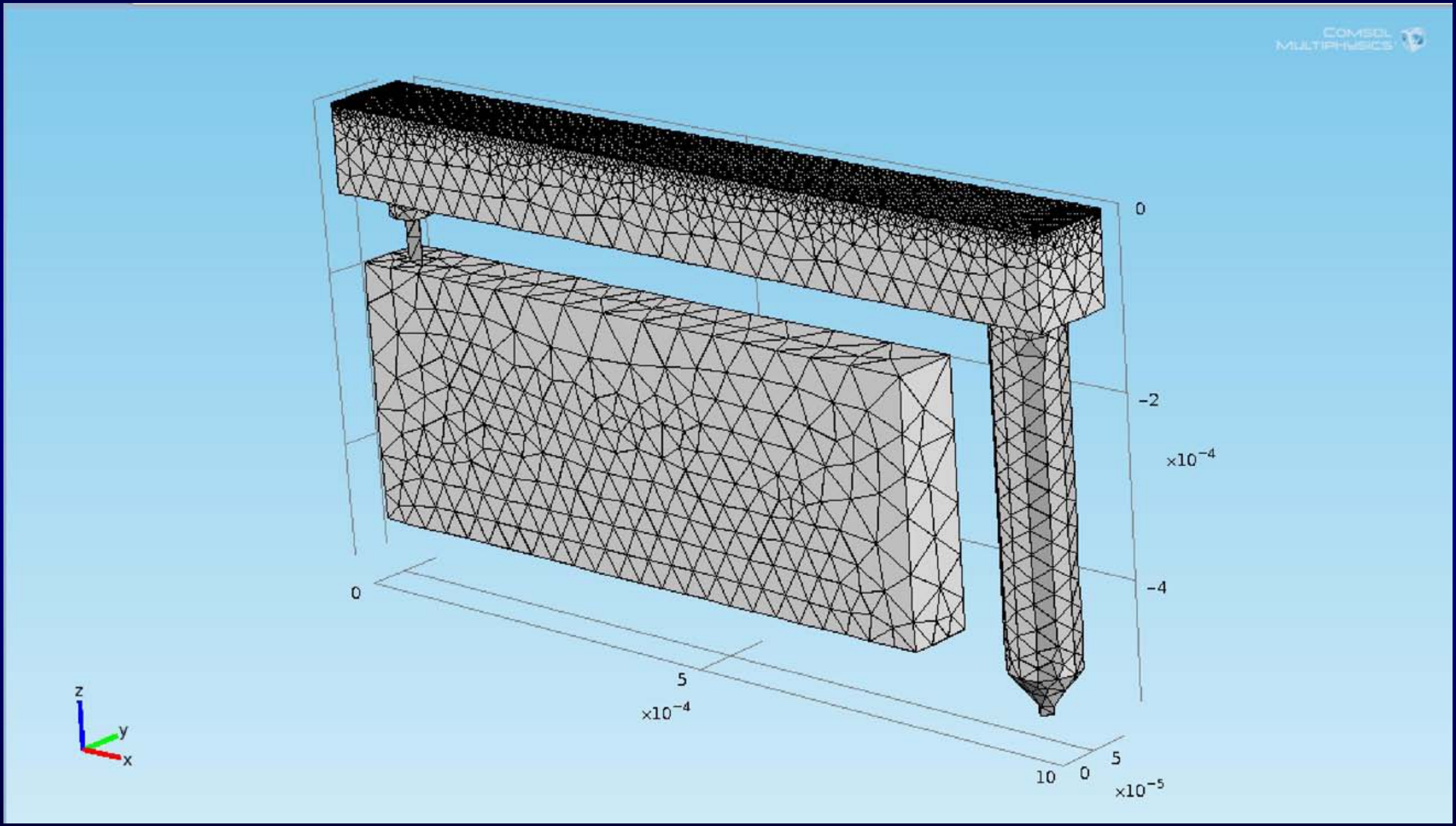
PZT thin film

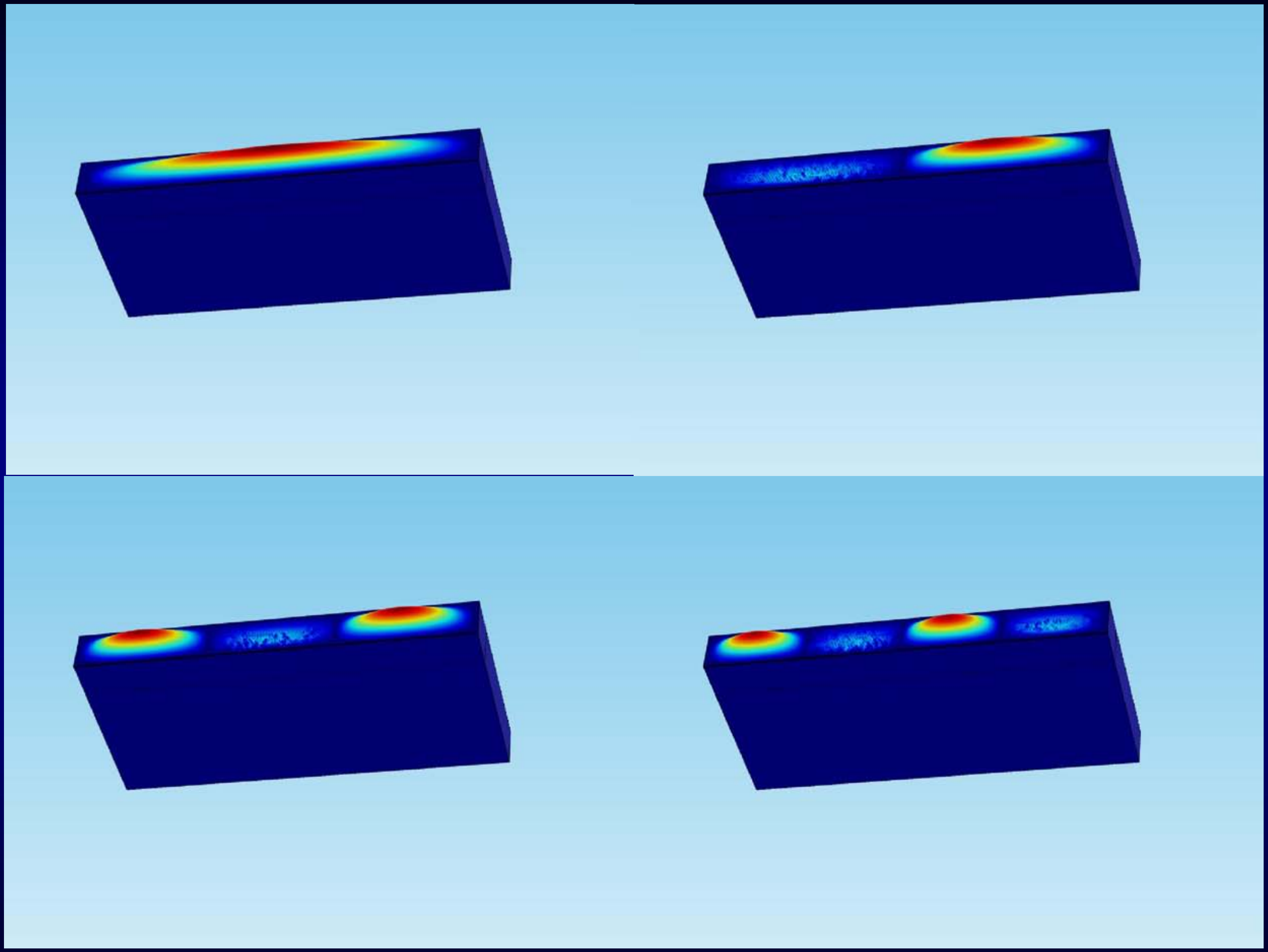
Ink channel

Ink reservoir

Nozzle

Industrial inkjet printhead

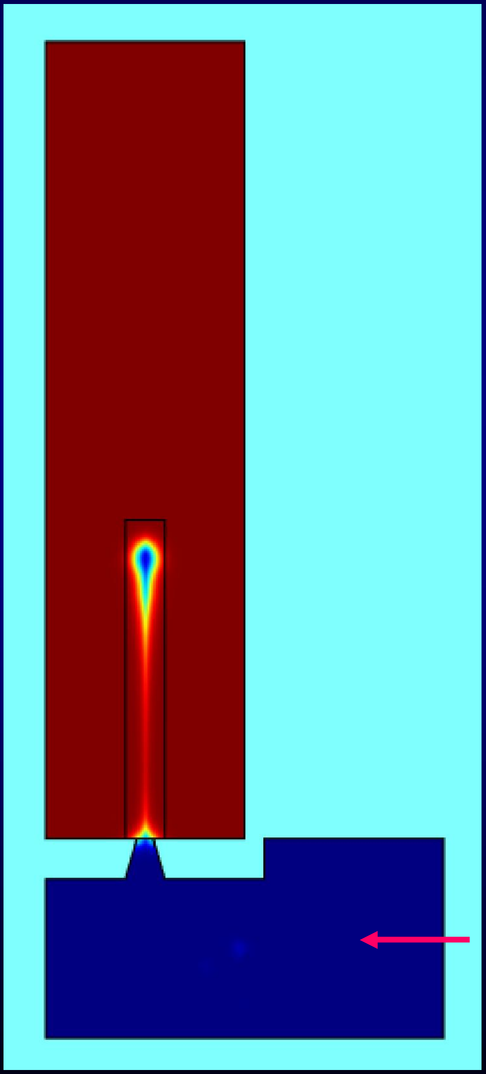


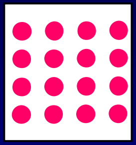


The first 4 modes around 2MHz

Pico-litter droplet formation

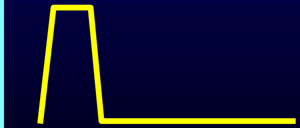
Single pulse



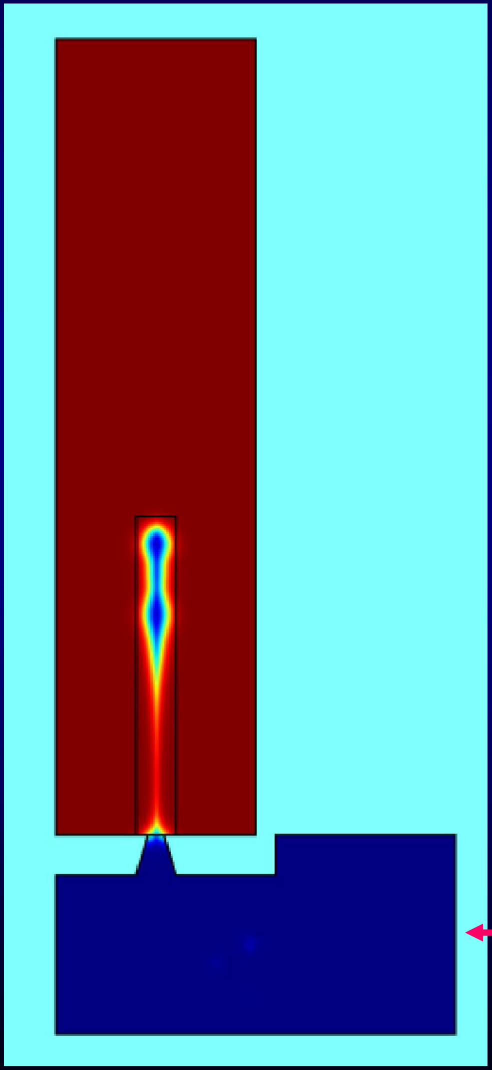


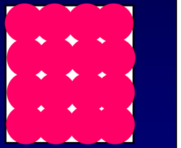
Small dot
Low density

Pressure



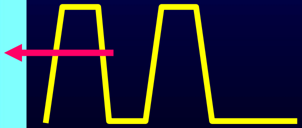
Double pulse





Large dot
High density

Pressure



Surface tension vs pinch off timing

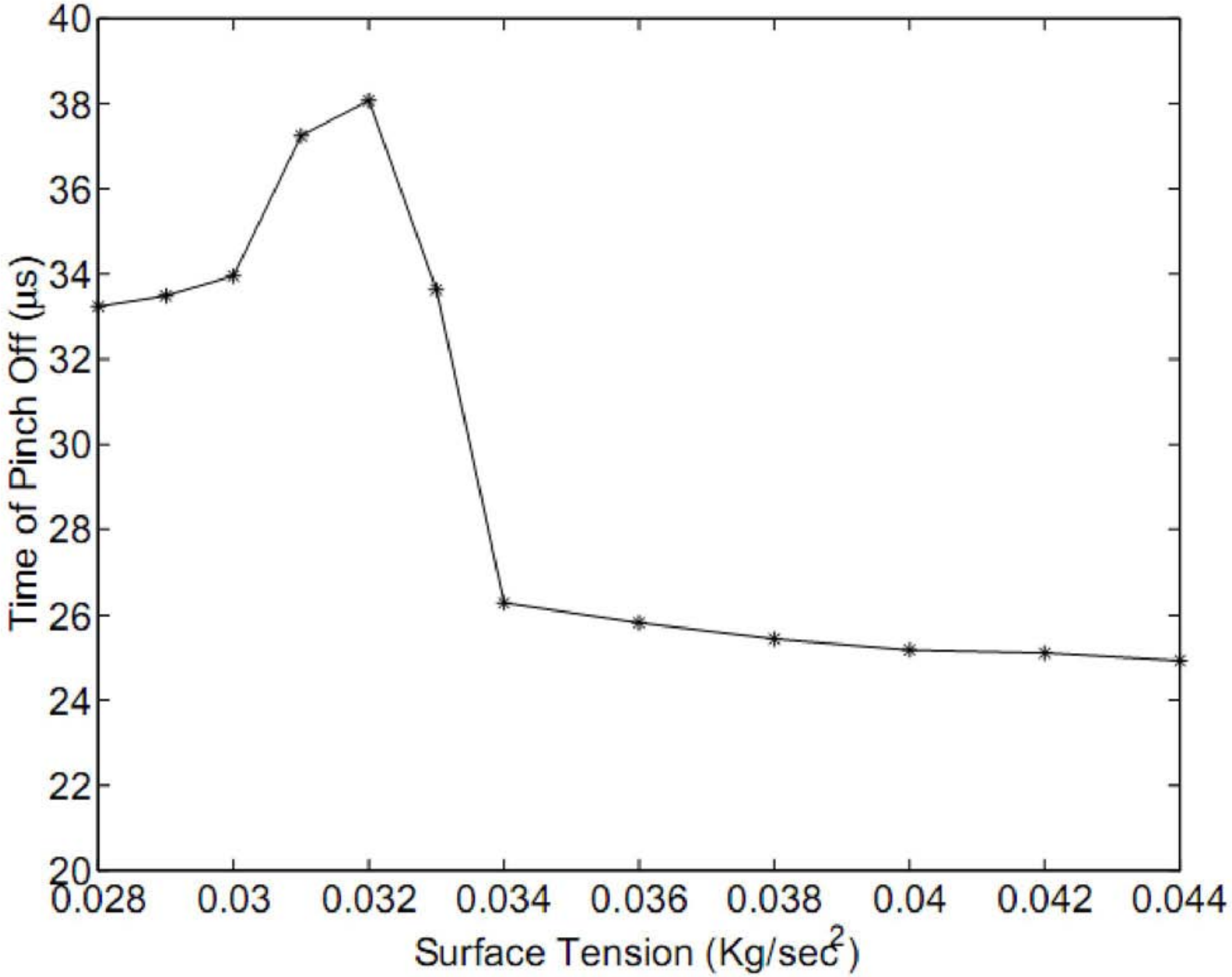


Figure 16: The relation between the time of pinch off and the surface tension.

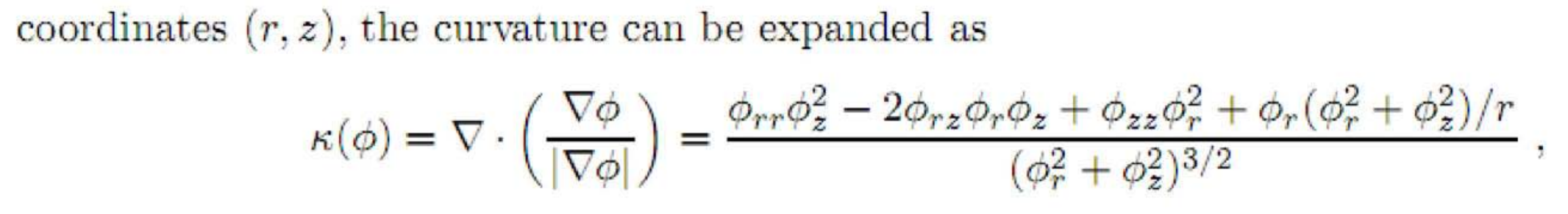
J. Yu, S. Sakai, J. A. Sethian, "A coupled levelset projection method applied to inkjet simulation".

Consideration of surface tension

Definition of curvature using Levelset function

$$n = \frac{\nabla \phi}{|\nabla \phi|} \qquad \kappa = \nabla \cdot n = \nabla \cdot \left(\frac{\nabla \phi}{|\nabla \phi|} \right)$$

⇨ 2nd derivative



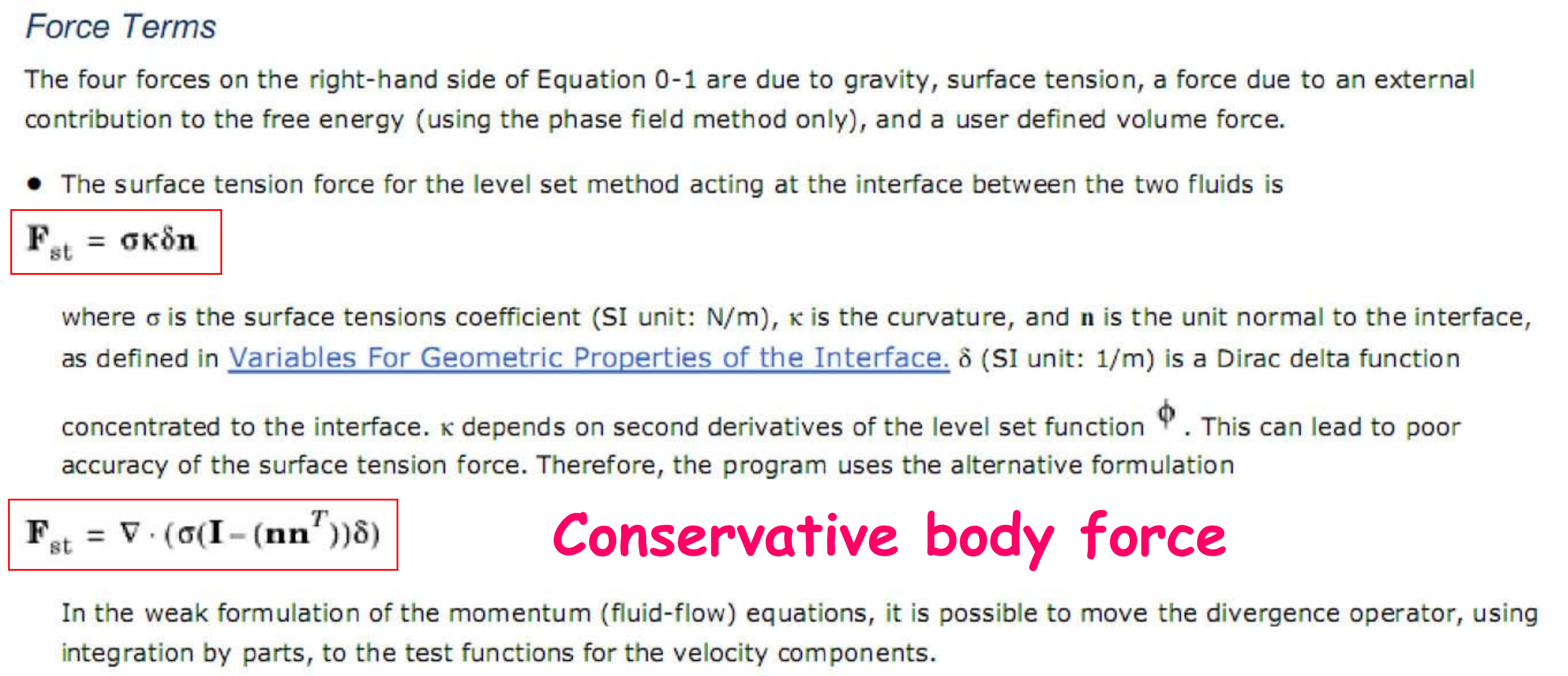
coordinates (r, z), the curvature can be expanded as

$$\kappa(\phi) = \nabla \cdot \left(\frac{\nabla \phi}{|\nabla \phi|} \right) = \frac{\phi_{rr}\phi_z^2 - 2\phi_{rz}\phi_r\phi_z + \phi_{zz}\phi_r^2 + \phi_r(\phi_r^2 + \phi_z^2)/r}{(\phi_r^2 + \phi_z^2)^{3/2}} ,$$

J. Yu, S. Sakai, J. A. Sethian, "A coupled levelset projection method applied to inkjet simulation".

Consideration of surface tension

Surface tension term in COMSOL (from manual)



Force Terms

The four forces on the right-hand side of Equation 0-1 are due to gravity, surface tension, a force due to an external contribution to the free energy (using the phase field method only), and a user defined volume force.

- The surface tension force for the level set method acting at the interface between the two fluids is

$$\boxed{\mathbf{F}_{st} = \sigma\kappa\delta\mathbf{n}}$$

where σ is the surface tensions coefficient (SI unit: N/m), κ is the curvature, and $\mathbf{n}$ is the unit normal to the interface, as defined in Variables For Geometric Properties of the Interface. δ (SI unit: 1/m) is a Dirac delta function concentrated to the interface. κ depends on second derivatives of the level set function ϕ. This can lead to poor accuracy of the surface tension force. Therefore, the program uses the alternative formulation

$$\boxed{\mathbf{F}_{st} = \nabla\cdot(\sigma(\mathbf{I}-(\mathbf{n}\mathbf{n}^T))\delta)}$$

Conservative body force

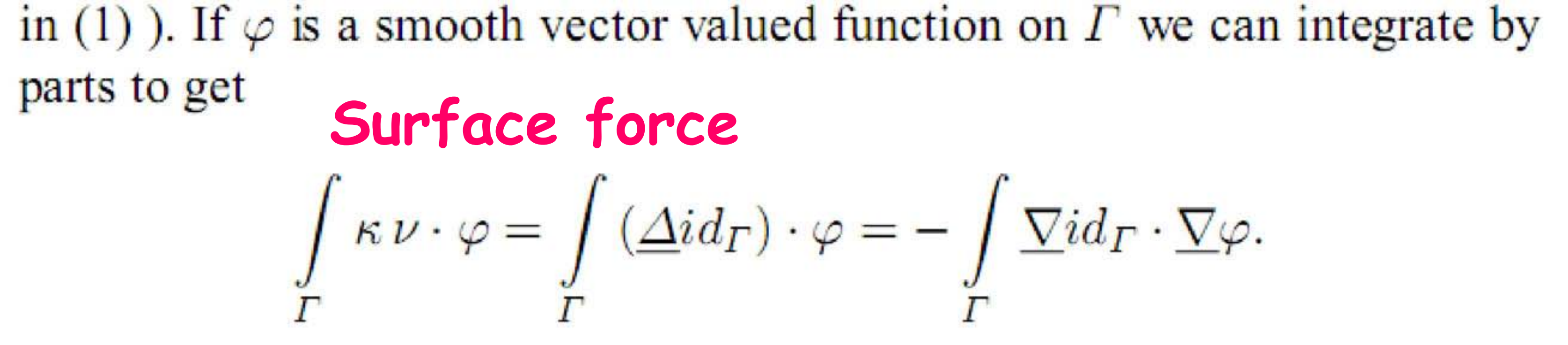
In the weak formulation of the momentum (fluid-flow) equations, it is possible to move the divergence operator, using integration by parts, to the test functions for the velocity components.

Weak form

$$-\int_\Omega (\sigma\kappa n\delta)v\,dx = \int_\Omega \sigma(I-(nn^T))\delta)\nabla v\,dx$$
$$-\int_\Gamma \sigma(I-(nn^T))\delta)v dx$$

Consideration of surface tension

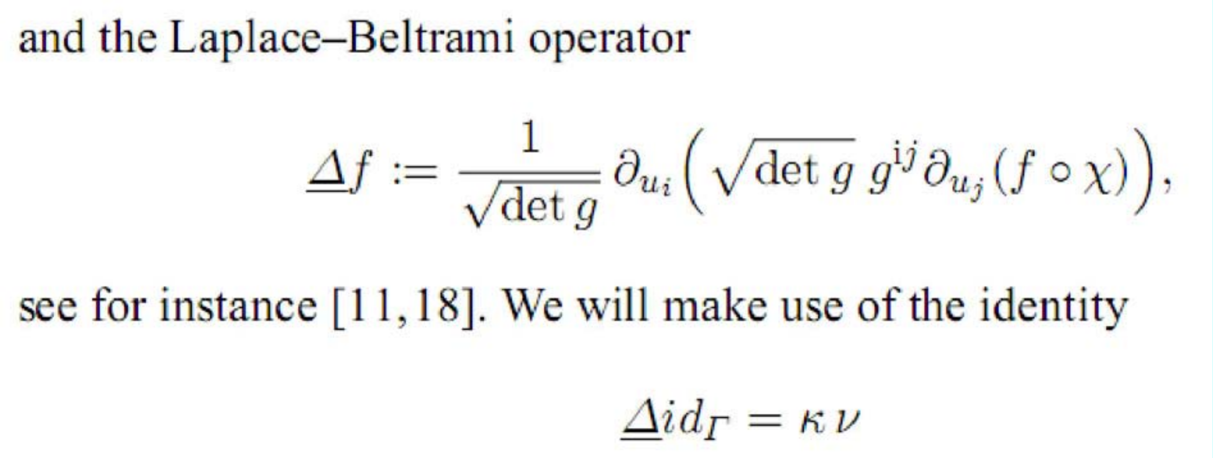
Surface tension as surface force by Melcher, Bansch



in (1)). If φ is a smooth vector valued function on Γ we can integrate by parts to get

Surface force

$$\int_{\Gamma} \kappa\,\nu\cdot\varphi = \int_{\Gamma} (\underline{\Delta} id_{\Gamma})\cdot\varphi = -\int_{\Gamma} \underline{\nabla} id_{\Gamma}\cdot\underline{\nabla}\varphi.$$



and the Laplace–Beltrami operator

$$\underline{\Delta} f := \frac{1}{\sqrt{\det g}}\,\partial_{u_i}\Big(\sqrt{\det g}\;g^{ij}\partial_{u_j}(f\circ\chi)\Big),$$

see for instance [11,18]. We will make use of the identity

$$\underline{\Delta} id_{\Gamma} = \kappa\,\nu$$

Eberhard Bansch, "Finite element discretization of the Navier-Stokes equations with a free capillary surface", ***Numer. Math.*** **(2001)**

Consideration of surface tension

Surface tension as surface force by Mizuyama

$$\kappa = \frac{\ddot{x}\dot{y} - \dot{x}\ddot{y}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$$

Definition from differential geometry

$$\kappa n_i = -\frac{d^2 x_i}{ds^2}$$

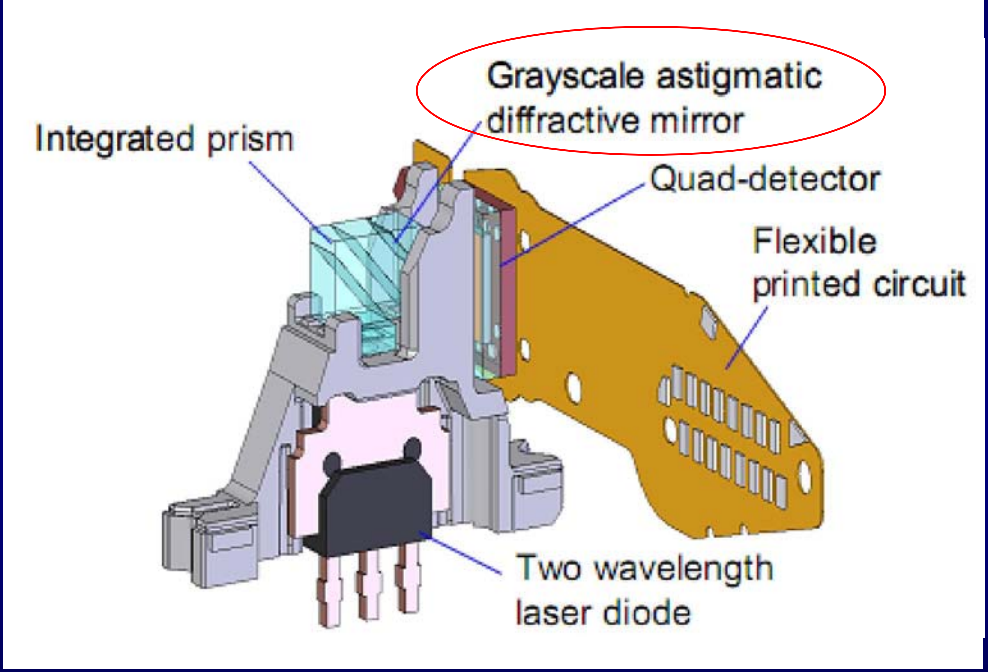
s:Arc length

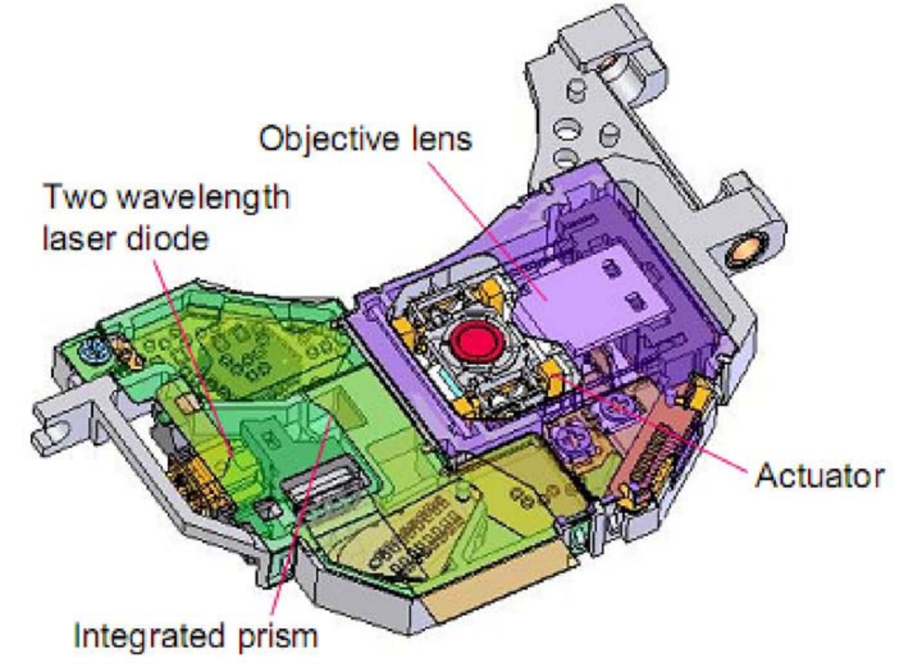
$$\int_\Gamma v\sigma\kappa n ds = -\sigma \int_\Gamma v\frac{d^2x}{ds^2}ds$$

Surface force in weak form

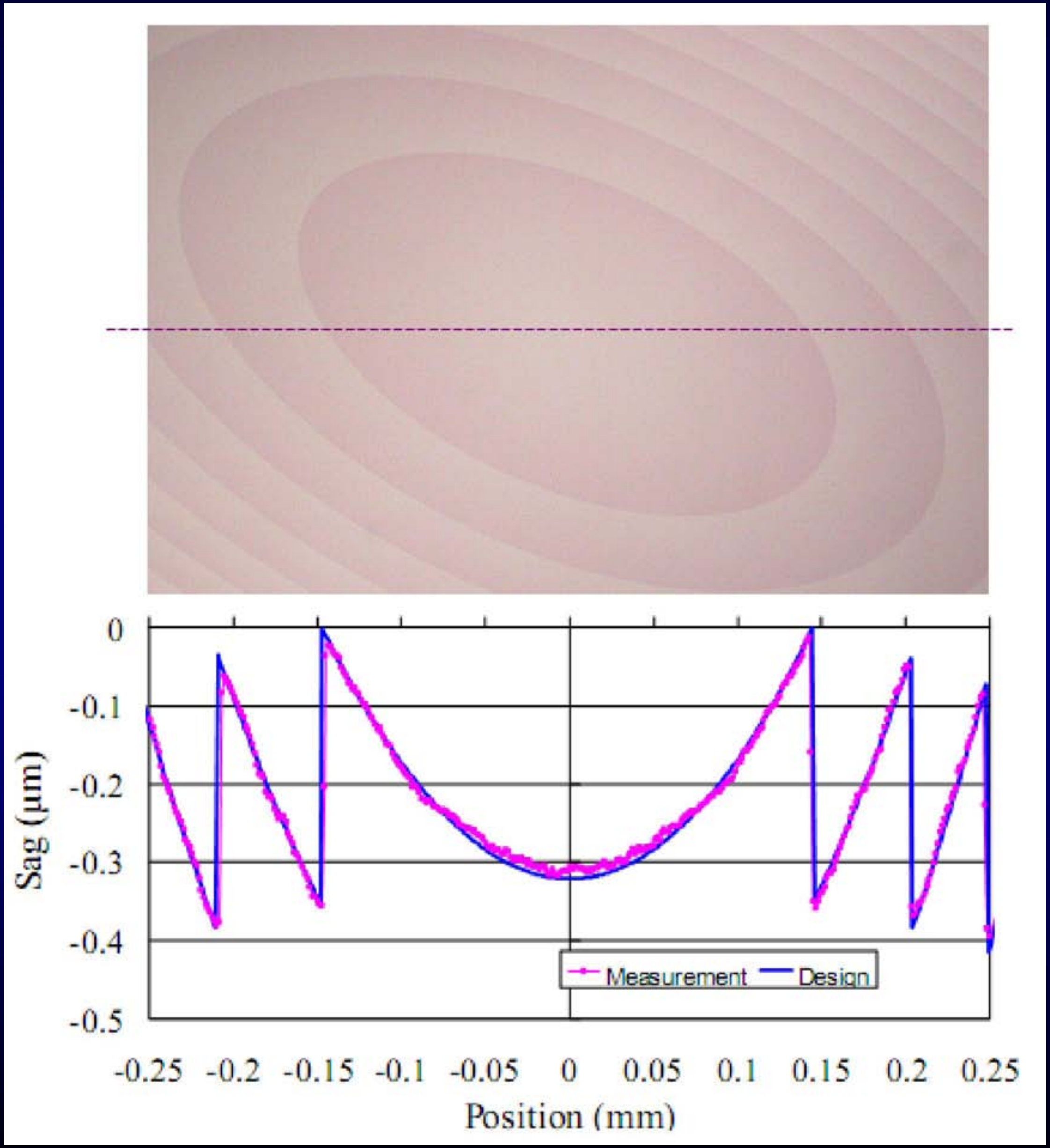
$$= \sigma \int_\Gamma \frac{dv}{ds}\frac{dx}{ds}ds - \sigma \cancel{\left[v\frac{dx}{ds}\right]_{\partial\Gamma}}$$

Grayscale photolithography technology for optical pickup

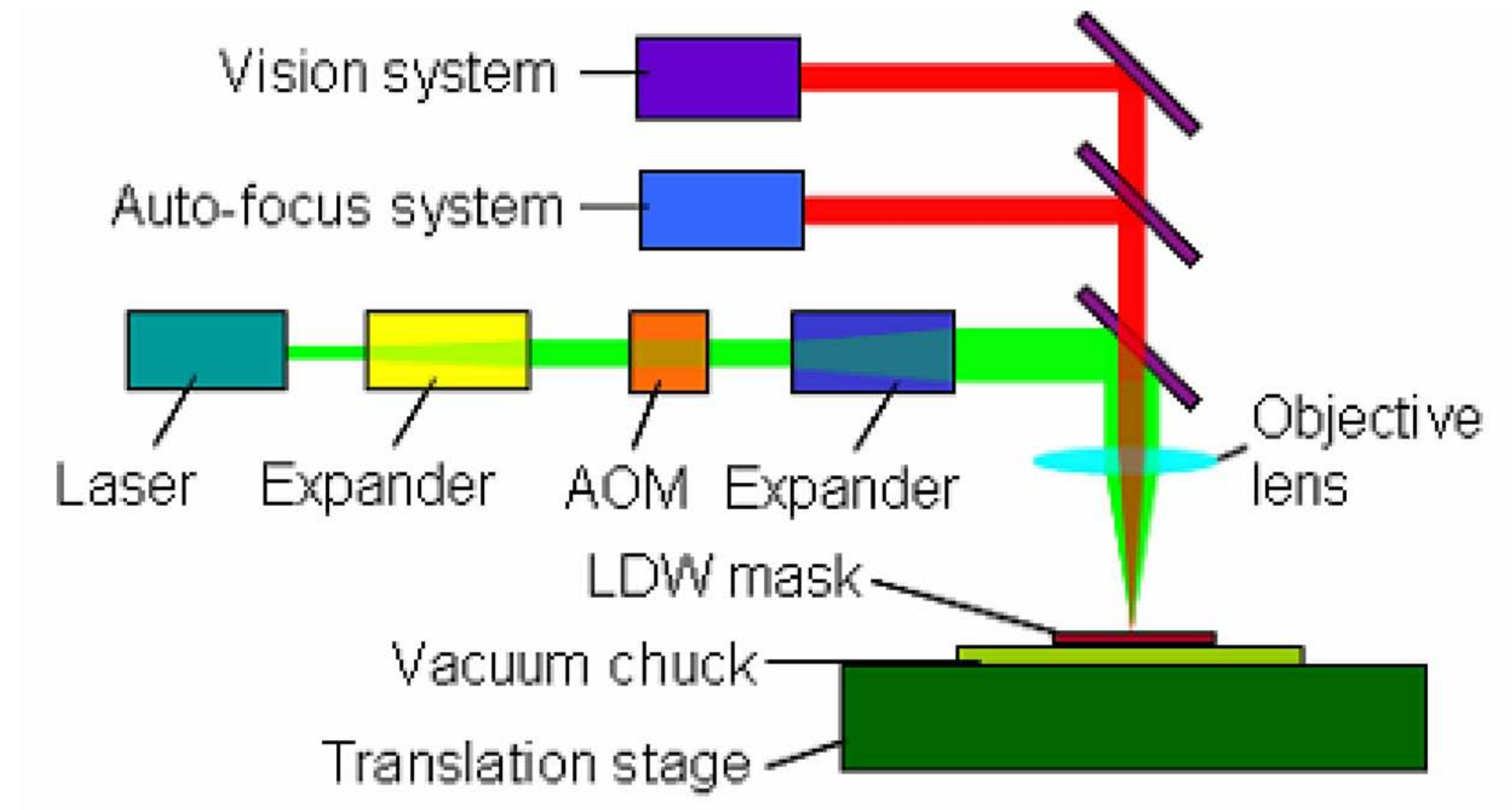




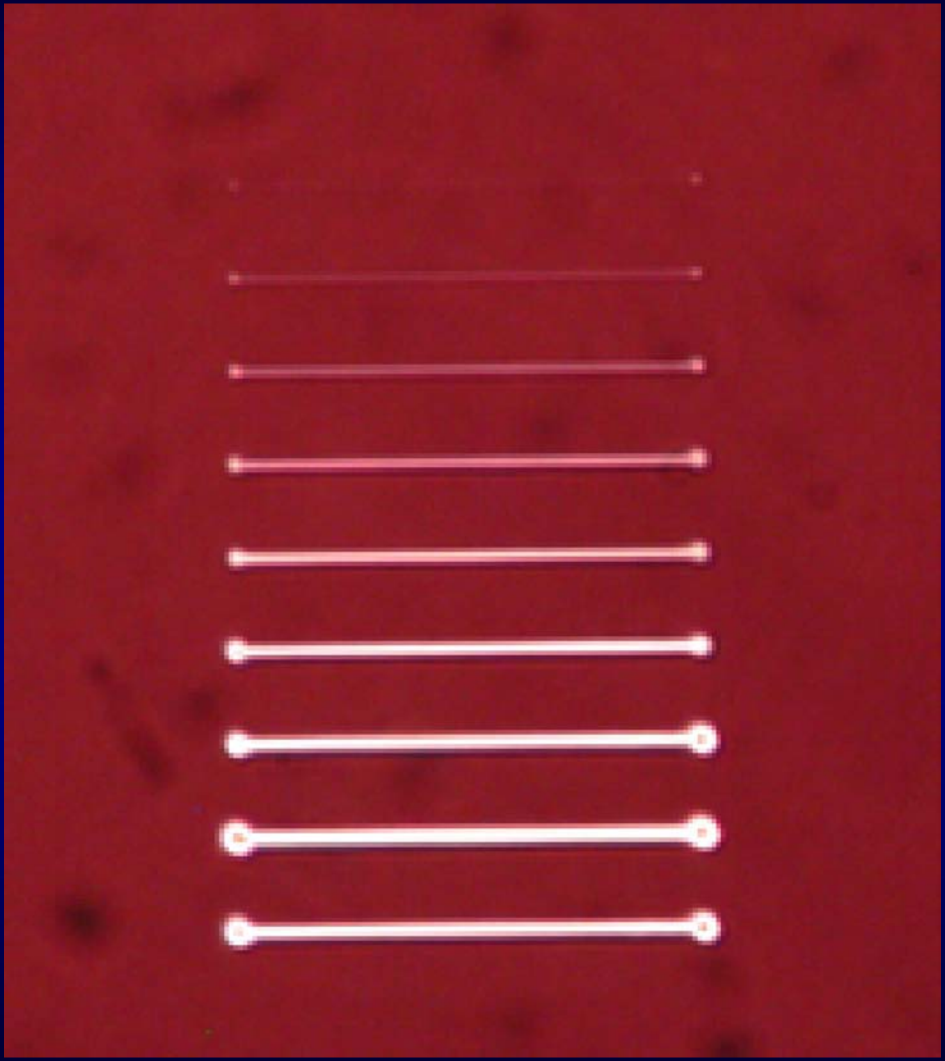
Grayscale mask



Laser direct write



Laser direct write



Laser power

20%
30%
40%
50%
60%
70%
80%
90%
100%

Transmission changes as laser power

Statement of the problem

$$\begin{cases} \rho c_p \dfrac{\partial T}{\partial t} - \nabla \cdot (\kappa \nabla T) = 0 & \text{in } \Omega \\ \kappa \nabla T \cdot n = \boxed{I(x,t)} & \text{on } \Gamma_N \\ \kappa \nabla T \cdot n = 0 & \text{on } \Gamma_{N0} \\ T = T_0 & \text{on } \Gamma_D \end{cases}$$

Surface heat source

Transmission

$$I(x,t) = I_0(x,t)(1 - \boxed{\tau(x,t)} - r_0)$$

Gaussian beam

$$I_0(x,t) = \exp\left\{-(x - x_c - vt)^2/w_x^2 - (y - y_c)^2/w_y^2\right\}$$

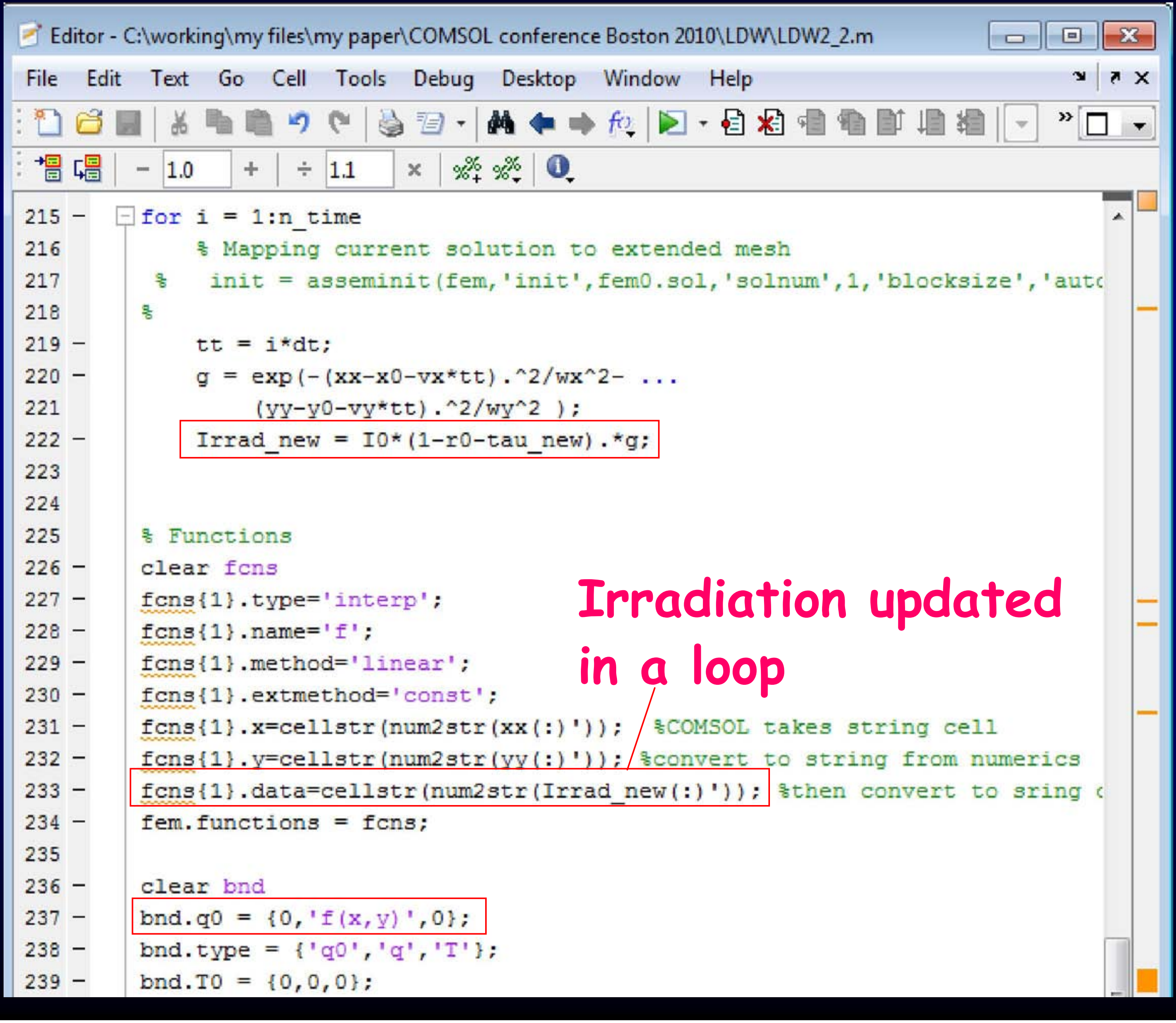
Transmission vs temperature

$$\tau(\bar{T}) = \tau_{min} + (\tau_{max} - \tau_{min})\sin^2(\pi\bar{T}/2T_0) \quad T < T_0$$

$$\bar{T}(x,t) = \max\{T(x,t'), t' < t\}$$

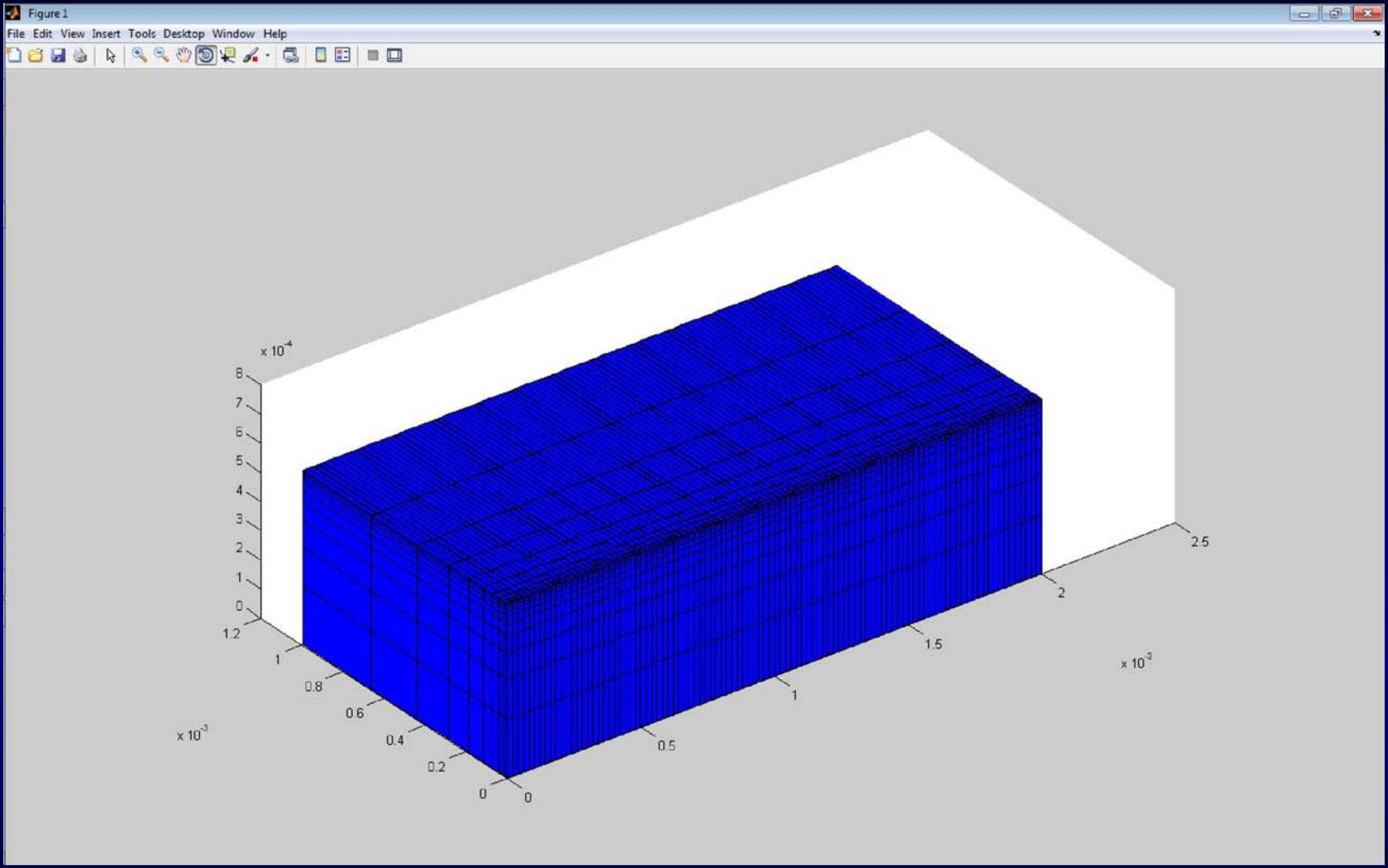
Irreversible process

COMSOL with MATLAB



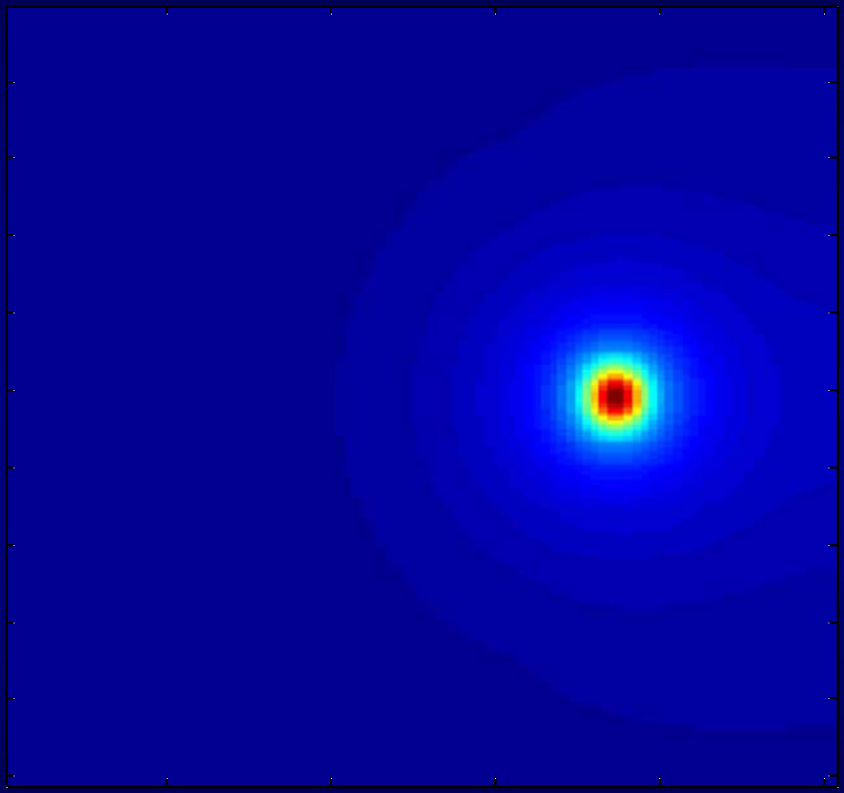
```
for i = 1:n_time
    % Mapping current solution to extended mesh
 %   init = asseminit(fem,'init',fem0.sol,'solnum',1,'blocksize','aut
%

    tt = i*dt;
    g = exp(-(xx-x0-vx*tt).^2/wx^2- ...
        (yy-y0-vy*tt).^2/wy^2 );
    Irrad_new = I0*(1-r0-tau_new).*g;


% Functions
clear fcns
fcns{1}.type='interp';
fcns{1}.name='f';
fcns{1}.method='linear';
fcns{1}.extmethod='const';
fcns{1}.x=cellstr(num2str(xx(:)'));  %COMSOL takes string cell
fcns{1}.y=cellstr(num2str(yy(:)')); %convert to string from numerics
fcns{1}.data=cellstr(num2str(Irrad_new(:)')); %then convert to sring
fem.functions = fcns;

clear bnd
bnd.q0 = {0,'f(x,y)',0};
bnd.type = {'q0','q','T'};
bnd.T0 = {0,0,0};
```

Irradiation updated in a loop

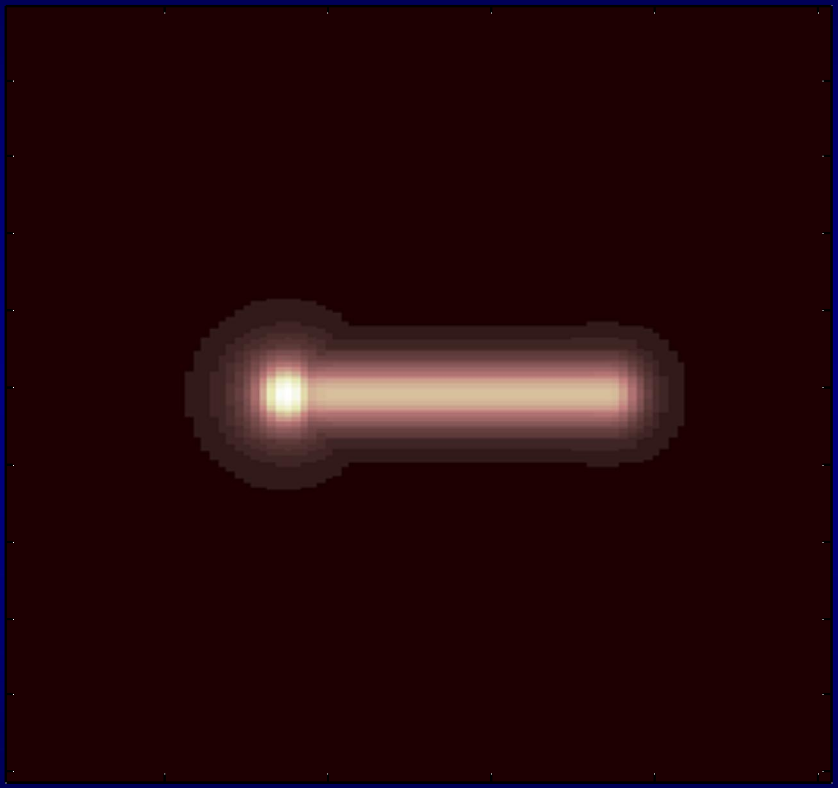


Mesh in 1/4 model

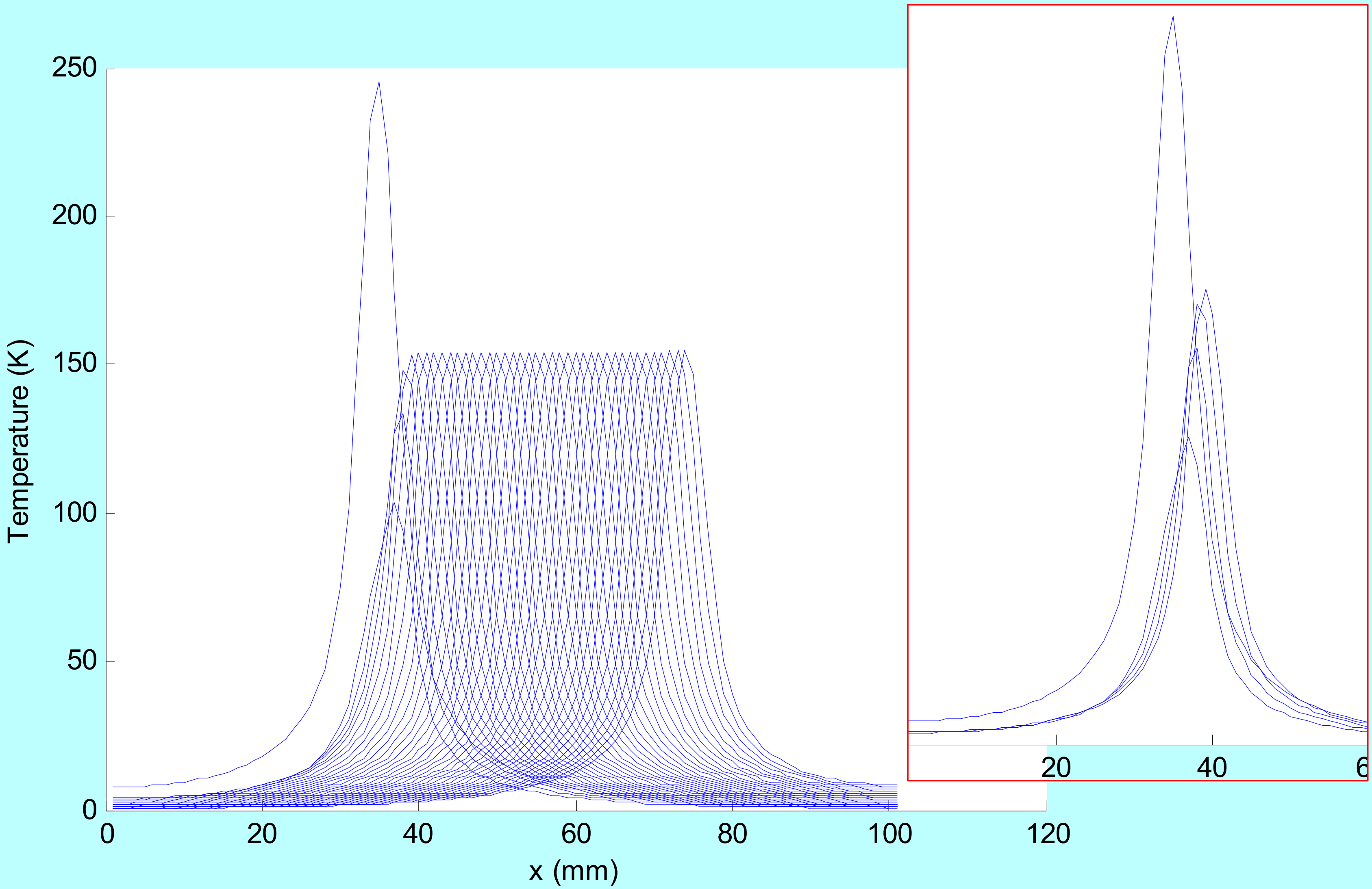
COMSOL+MATLAB simulation



Temperature



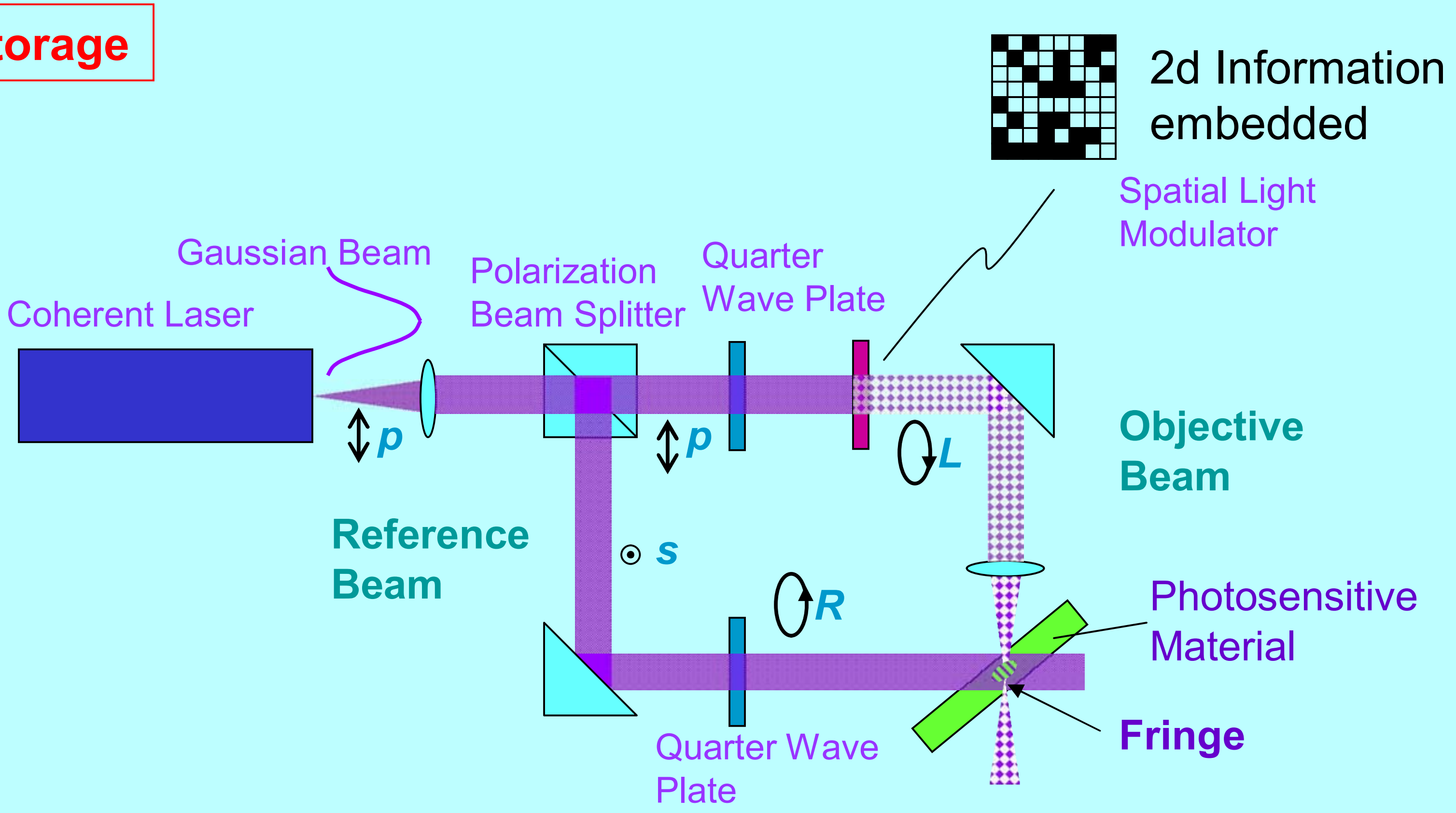
Transmittance



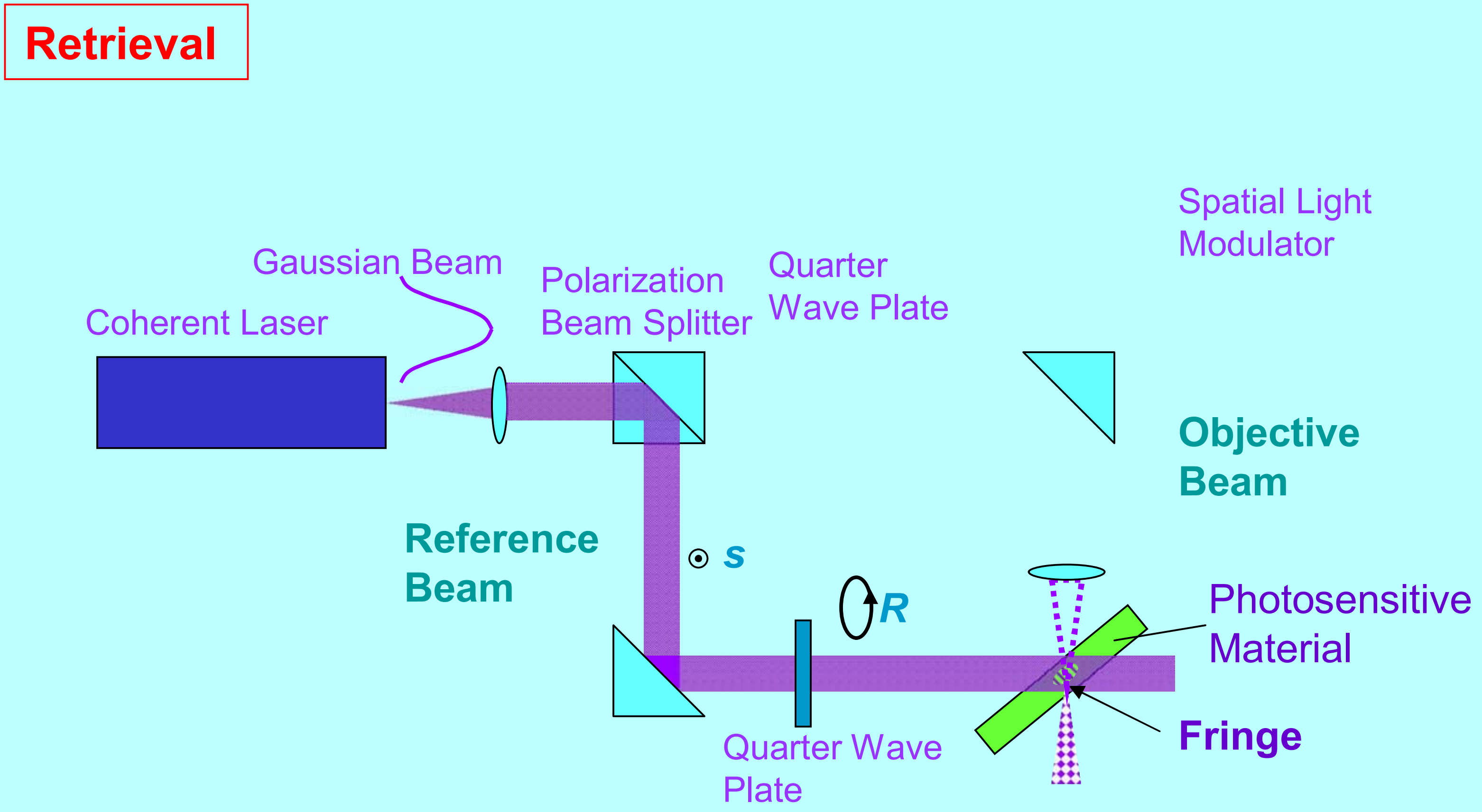
Temperature transition

Holographic data storage

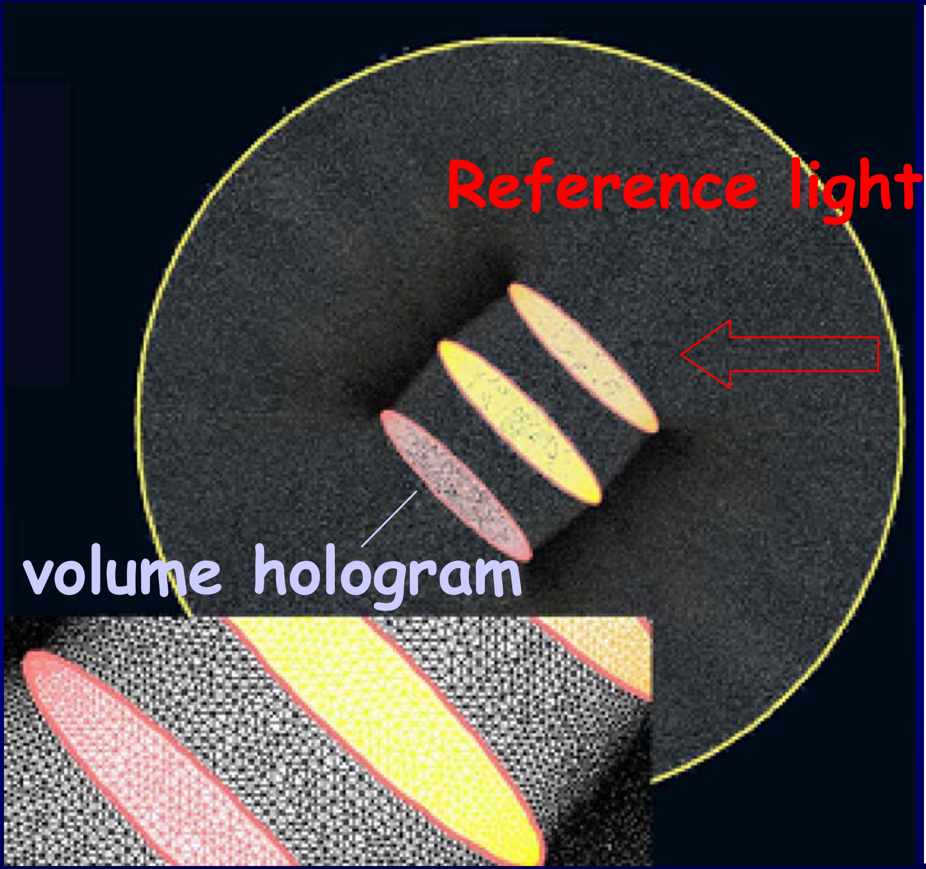
Storage

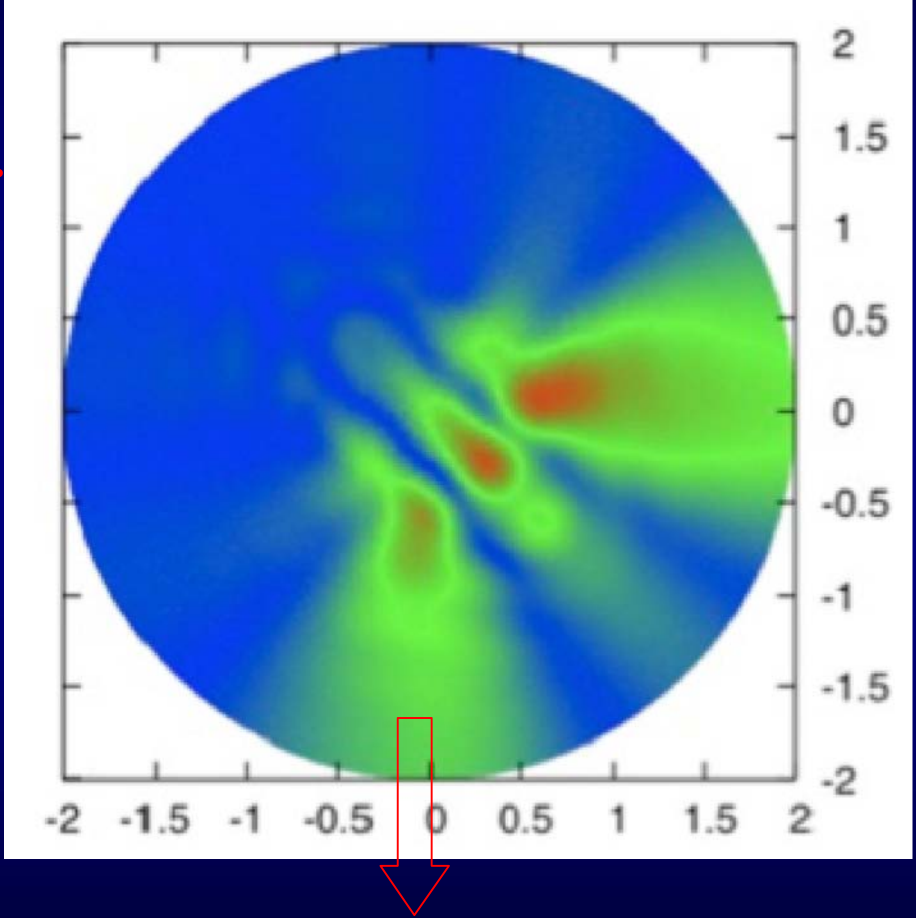


Holographic data storage



Holographic data storage





Fringes in a volume hologram

Information light retrieved

Statement of the problem
Infinite region problem

$$-\nabla^2 u - k_2^2 u = (\nabla^2 + k_2^2) u^{inc} \qquad \text{in } \Omega_1$$

$$-\nabla^2 u - k_1^2 u = 0 \qquad \text{in } \Omega_\infty$$

$$[u]_2^1 = 0 \qquad \text{on } \Gamma_{12}$$

$$\left[\frac{\partial u}{\partial n}\right]_2^1 = 0 \qquad \text{on } \Gamma_{12}$$

Sommerfeld radiation condition

$$\lim_{r \to +\infty} \sqrt{r} \left(\frac{\partial u}{\partial n} - i k_1 u \right) = 0$$

Equivalent problem with DtN map
Finite region problem

$$-\nabla^2 u - k_2^2 u = (\nabla^2 + k_2^2) u^{inc} \qquad \text{in } \Omega_1$$

$$-\nabla^2 u - k_1^2 u = 0 \qquad \text{in } \Omega_2$$

$$[u]_2^1 = 0 \qquad \text{on } \Gamma_{12}$$

$$\left[\frac{\partial u}{\partial n}\right]_2^1 = 0 \qquad \text{on } \Gamma_{12}$$

$$\frac{\partial u}{\partial n} = -\mathcal{S}u \qquad \text{on } \Gamma_2$$

Steklov-Poincare operator

$$\mathcal{S}u = -k_1 \sum_{n=-\infty}^{+\infty} \frac{H_n^{(1)\prime}(k_1 a)}{H_n^{(1)}(k_1 a)} u_n(a) \phi_n(\theta)$$

Weak form

$$a(u,v) + s(u,v) = \langle f, v \rangle \quad \forall u \in V$$

$$a(u,v) = \int_\Omega (\nabla u \cdot \nabla \bar{v} - k^2 u \bar{v}) dx$$

$$s(u,v) = \int_{\Gamma_2} (\boldsymbol{\mathcal{S}} u) \bar{v}) ds$$

$$\langle f, v \rangle := \int_\Omega f\ v\ dx$$

$$f(x) := \begin{cases} 0 & \text{in } \Omega_2 \\ (\Delta + k_2^2) u^{\text{inc}} & \text{in } \Omega_1 \end{cases}$$

$$s(u,v) = -k_1 a \sum_{n=-\infty}^{+\infty} \frac{H_n^{(1)\prime}(k_1 a)}{H_n^{(1)}(k_1 a)} u_n(a) \bar{v}_n$$

Thank you!

Panasonic ideas for life