Excerpt from the Proceedings of the COMSOL Conference 2010 Paris

Using the Time Parameter as the Third Geometrical Dimension

J. Krah
Aker Solutions
Snar¢yveien 36, 1364 Fornebu, Norway, julius.krah@akersolutions.com

Abstract: 2D models in Cartesian coordinates are used for long geometries without changes in the third dimension; similarly works a 2D model in axial symmetric coordinates. This paper proofs that for some applications a 2D model can be simulated such that a change of a calculated parameter along the third geometrical dimension is obtained. A practical application in Cartesian coordinates is presented. It is verified in axial symmetric coordinates. In an example heat exchanger the fluid temperature changes non-linearly along the length. All results of the modified 2D model are confirmed by models using full 3D geometry.

Keywords: Flow, Heat exchange, Extrusion, Simplification, Memory.

1. Introduction

For some applications modeled in 2D cartesian coordinates and simulated in steady-state the time parameter can be introduced to extend the 2D model to a 3D model. The physical approach is straightforward yet elegant by avoiding implementation of a 3D geometry. A 3D solution is obtained with a 2D geometry without the disadvantages involved with 3D models such as higher memory consumption, longer computation times and lower convergence. This paper presents the representative and useful application of a cooling tube or more generally a heat exchanger. Warm water is cooled by pumping it into a 10m long tube buried 1m deep in cold ground. The task is to determine the required flow rate to meet a required cooling power.

The problem is that the warm inlet water cools down on the way such that the heat exchange effect towards the environment reduces along the tube. For practical applications this effect can not be neglected as the cooled water has to be considerably colder than the warm inlet water in order to obtain reasonable cooling power. To take that effect into account a 3D model is needed. An example is shown where a U-shaped tube loop cools the water well whereas the tube-in-tube configuration has reduced thermal performance. For both examples the paper shows that a time-dependant 2D model gives the same 3D solution as a steady-state model using 3D geometry. Typical applications becoming more and more popular the last decades are ground-source heat pumps (GSHP) and bore hole heat exchangers (BHE). A 3D model of a BHE can be found in [1]. Often a 2D model is used neglecting the temperature distribution along the tube length [2]. Also some known analytical approaches have similar limitations [3].

2. Principle

A steady-state thermal model does not use the specific heat capacity of the modeled materials, whereas a time-dependent thermal model does. By setting the specific heat capacities of all used materials to zero a transient model will immediately jump to the final or asymptotic solution, which is a steady-state solution. In a 2D model a cooling tube has the geometry of a cross section, typically a circle. The material of a water tube has the specific heat capacity of water and high effective heat conductivity due to the so-called heat tube effect of liquids. A time-dependent calculation gives a solution as a function of the time parameter t. In our example the parameter t will translate to distance along the z-axis or along the cooling tube in the following way:

The thermal energy Q of a water unit with heat capacity c, mass m, density ρ, volume V, cross-sectional area A, depth Δz in z-direction heated by temperature ΔT is:

$$Q = c \cdot m \cdot \Delta T = c \cdot \rho \cdot V \cdot \Delta T = c \cdot \rho \cdot A \cdot \Delta z \cdot \Delta T \quad (1)$$

Due to contact with the surrounding materials a time-dependent model will calculate thermal energy absorbed by the water tube over time giving the heat flow power P:

$$P = \frac{dQ}{dt} = \frac{c \cdot r \cdot A \cdot \Delta z \cdot \Delta T}{dt} \quad (2)$$

Applying the time parameter t to the depth in z-direction the velocity v of water flow in z-direction can be introduced:

$$P = c \cdot \rho \cdot A \cdot \Delta T \cdot v \qquad (3)$$

If the water material has been set to the specific heat capacity of water, which is around 4200J/K, the resulting water speed will be 1m/s, which means that the solution of 1s will translate to 1m in z-direction. Entering a different specific heat capacity for the cooling tube, i.e. the value for oil, which is about half of the value for water, will, of course model a cooling tube with oil content - or it can be interpreted as water flowing with twice the speed, meaning that 1s will translate to 1m for an oil tube or alternatively to 2m for a water tube.

3. An application example in Comsol: A water tube in ground.

3.1 The Task

Consider a water-cooled device dissipating around 1kW. The cooling water in the device should not exceed 30ºC. The aim is to design a heat exchanger that cools the warm water with at least 1kW such that it can be reused in the device. Due to cost, simplicity, robustness and reliability a passive heat exchanger is preferred.

3.1 Designing a solution

A straight long water tube buried in ground is the most robust application. The warm water with constant temperature is pumped into the tube, cooled by the low ambient ground temperature and returns to the pump. Fig. 1 demonstrates the cross section.

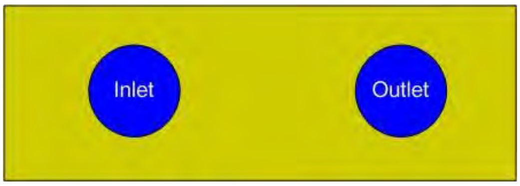

Figure 1. Sketch of tube loop cross section.

As the two parallel tubes in Fig. 1 have a distance to each other and thus need space a tube-in-tube arrangement was considered as an alternative, see Fig. 2. Then only one tube needs to be buried in ground.

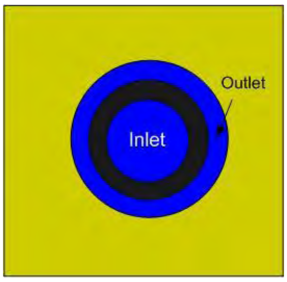

Figure 2. Sketch of tube-in-tube cross section.

As the water inlet temperature is given the aim is to find out the required flow rate to obtain the specified cooling power. As it is difficult to estimate how the water is cooled on the way through the tube at different speeds, a FE simulator is an appropriate tool for finding trustworthy temperatures.

A conventional 3D model is compared with the new suggested 2D model. For simplicity, in the following the conventional 3D model is called the 3D model and the new suggested 2D model simply as the 2D model.

3.1 The 3D Model in Comsol

The 3D geometry is obtained by extrusion of the 2D cross sections sketched in Figures 1 and 2. The physics were adopted from a tutorial model: The MEMS heat exchanger with conductive and convective heat transfer with a defined water velocity. Turbular flow was assumed due to several orders higher water cross section as in the tutorial model. Using a constant water velocity in the whole tube cross section makes comparison with the 2D model easier. At the far end the water flow has to be redirected to the return tube. That was modeled by integrating the average temperature on the flow tube far end cross section and setting this temperature on the far end boundary surface of the return tube.

3.2 The 2D model in Comsol

For the 2D model only the cross section of the tube system was simulated. It was necessary to set the heat capacity of ground and of the tube to zero or close to zero. The inlet water was modelled to have regular positive heat properties whereas the oulet water has a negative heat property, absorbing cold instead of heat. In Comsol it does not work to insert a negative heat capacity under the material properties. It was necessary to use a Weak form contribution under *Physics: Global Equations: Subdomain settings*. The sign of the Weak term and of the Time-dependent weak term has to be inverted. A time-dependent model uses initial temperatures. As the solid materials were modeled without heat capacity their initial temperatures have no effect on the simulation. Setting the initial water temperature in both water tubes to the same value is valid at the tube end where the water the inlet tube is connected to the return tube.

After simulating 10s in the time-dependant model (corresponding to 10m being the location of the water pump) the temperature of the inlet tube increased to the water inlet temperature and the temperature in the return tube decreased to the water outlet temperature. In order to obtain the desired water inlet temperature the initial temperature at 0m has to be adjusted manually by running the model a few times until the final temperature in the inlet tube does not differ any more from the desired water inlet temperature. As the 2D model does not simulate convective heat flow the flow speed has to be modeled in another way. The water heat capacity is changed. Dividing the water heat capacity by e.g. 1000 translates to a water speed of 1mm/s instead of 1m/s.

3.3 Used model parameters

Simple geometrical and thermal properties were chosen, see Tab. 1 and 2.

Table 1: Thermal properties.

Part	k [W/K·m]	c_p [J/kg·K]	ρ [kg/m^3]	$T_{boundary}$ [ºC]
Ground	1	1000	1000	0
Steel tube	50	8000	1000	N/A
Water	1000	4200	1000	30

Table 2: Dimensions of tubes in ground.

Dimension	Value
Depth under ground surface	1m
Length	10m
Inner diameter	10cm
Water cross section	ca. 80cm^2
For loop arrangement: Distance between tubes.	2m
For tube-in-tube arrangement: Inner tube material thickness.	1cm

3.4 Simulation Results

This section compares the temperatures for both tube arrangements simulated with the 3D and the 2D model. Figures 3 and 4 show that the loop tube cools the water from 30ºC down to 9.5ºC. The water speed of only 1mm/s requires pumping of only 28 liters per hour but the heat exchange of 670W is below 1kW. For the 2D model in Figure 4 the time value of 10s corresponds to 0m, which gives water inlet and outlet temperature at the pump.

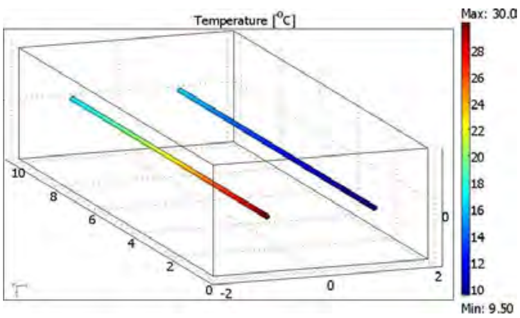

Figure 3. Temperatures in U-shaped tube loop for 1mm/s simulated by 3D steady-state model.

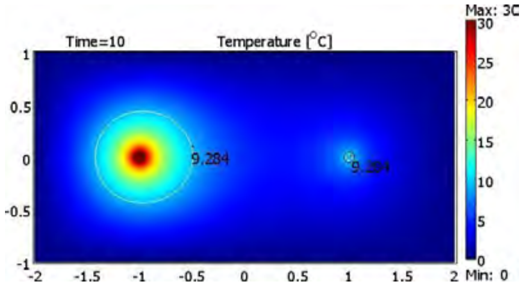

Figure 4. Water inlet and outlet temperature in tube loop at 0m (=10s) for 1mm/s simulated by 2D time-dependent model.

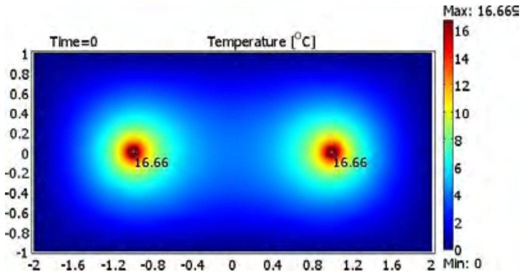

Figure 5. Water temperature in tube loop at return end 10m (=0s) for 1mm/s simulated by 2D time-dependent model.

In the tube-in-tube arrangement in Figure 6 the heat exchange is limited to the first meter. For a water speed of 10mm/s (Fig. 7), it becomes better, but according to Fig. 8 the outlet water (outer tube) is warm. Surface boundary integration gives 28.6ºC. The 2D model in Figures 9 and 10 agrees (28.9ºC). The resulting heat exchange of 430W is still moderate.

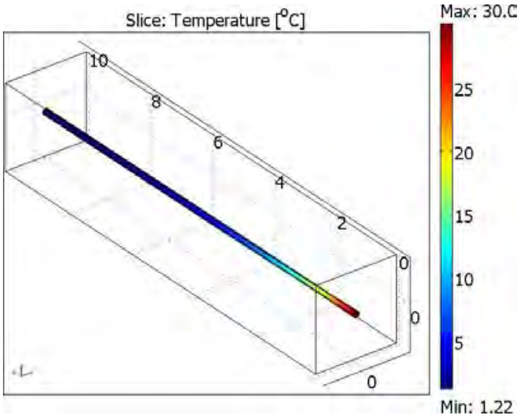

Figure 6. Temperatures in tube-in-tube cooling system for 1mm/s simulated by 3D steady-state model.

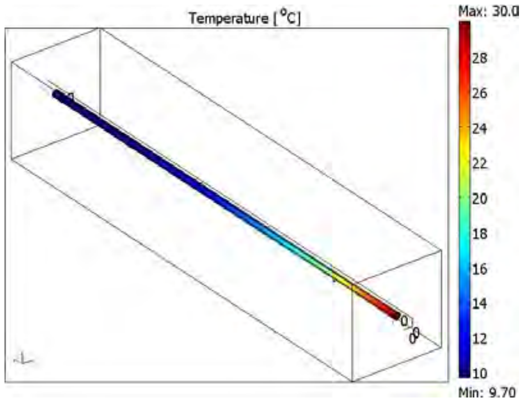

Figure 7. Temperatures in tube-in-tube cooling system for 10mm/s simulated by 3D steady-state model.

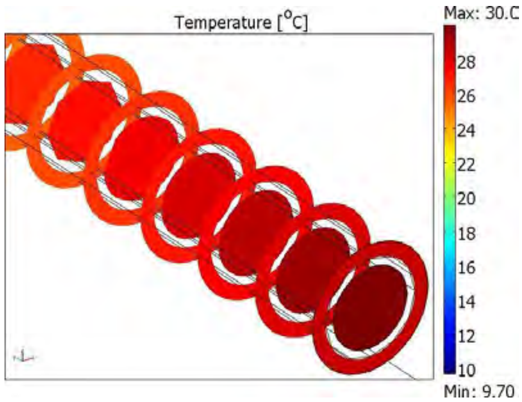

Figure 8. Inlet and outlet temperature in Fig. 7.

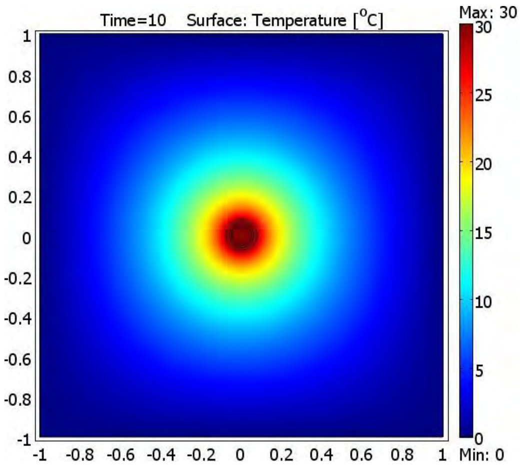

Figure 9. Water inlet and outlet temperature in tube-in-tube arrangement at 0m (=10s) for 10mm/s simulated by 2D time-dependent model.

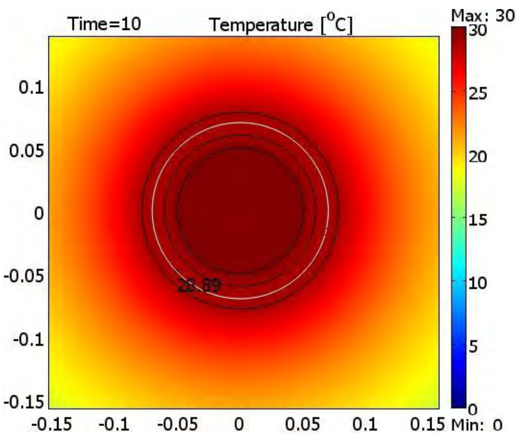

Figure 10. Zoom of Fig. 9 with temperature label of 28.89ºC on outlet water.

In the tube-in-tube loop the warm inlet water cools down from 0m to 10m, but is heated up again on the way back by warm incoming water in the inner tube. The steel tube between the incoming and the returning water acts as a thermal short-circuit as follows.

With the parameters in Tab. 1 and 2 the thermal time constant can be estimated by the thermal resistance *R* of the steel tube and the sum of the thermal capacity *C* of inner and outer water, both per meter.

$$R = \frac{1}{l} \cdot \frac{l}{A} = \frac{1}{50\frac{W}{K \cdot m}} \cdot \frac{1cm}{2p \cdot 5.5cm \cdot 1m} \quad (4)$$

$$C = 2 \cdot c_p \cdot r \cdot V = 2 \cdot 4.2 \frac{J}{g \cdot K} \cdot 10^3 \frac{kg}{m^3} \cdot 2p \cdot (5cm)^2 \cdot 1m \quad (5)$$

$$t = R \cdot C = 6.9m \frac{W}{m^2} \cdot 66 \frac{kJ}{m} = 0.45s \quad (6)$$

In simplified terms a temperature difference between inner and outer tube of 10K would decay to 1K within 1s. In real BHEs polyethylene tubes are used at least for the concentric configuration [4][5]. In addition, ref. [5] states that “the inner pipe is often thermally insulated to avoid thermal short-circuiting between the upward and downward flow channel”.

Tables 3 and 4 summarize the results by the 2D and the 3D model. In the 3D model heat absorbed through the tube surface was integrated for verification. For the loop tube a water flow of 280liters/hour gives more than the required 1kW, whereas the tube-in-tube system still does not work with 10 times that flow rate.

For the tube-in-tube model a fine mesh and strict time steps were necessary.

Table 3: Results for loop tube.

Water speed	Model	*T* outlet water [ºC]	*P* by ΔT [W]	*P* by heat flux through surface [W]
1mm/s	3D	9.5	676	600
	2D	9.3	682	N/A
10mm/s	3D	26.6	1120	1100
	2D	26.6	1120	N/A

Table 4: Results for tube-in-tube arrangement.

Water speed	Model	*T* outlet water [ºC]	*P* by ΔT [W]	*P* by heat flux through surface [W]
1mm/s	3D	28.8	40	278
	2D	N/A	N/A	N/A
10mm/s	3D	28.62	455	426
	2D	28.9	363	N/A
100mm/s	3D	29.79	690	713
	2D	29.8	660	N/A

The temperature profiles along the tubes in Figures 11 and 12 illustrate more clearly the above observations. The temperature variation in the third geometrical dimension *z* demonstrates how the 2D model can give 3D results, and in our example the variation is non-linear.

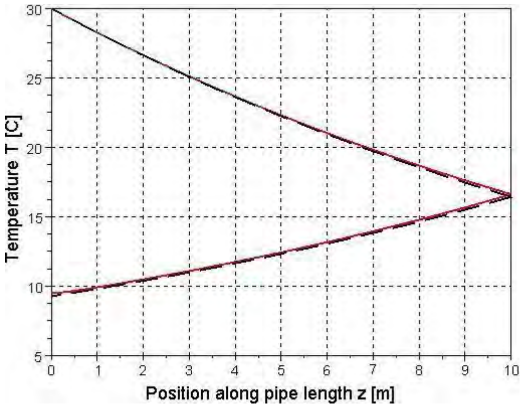

Figure 11. Loop tube: Temperature variation along flow and return tube for 1mm/s, calculated by 3D model (red, solid) and 2D model (black, dotted).

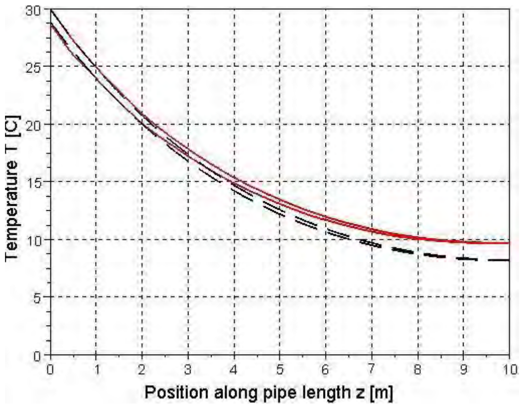

Figure 12. Tube-in-tube: Temperature variation along inner and outer tube for 10mm/s, by 3D model (red, solid) and 2D model (black, dotted).

7. Conclusions

The paper proofed the theory in the Introduction. A mathematical derivation was supported by a practical application in Comsol Multiphysics proofing that the modified 2D model gives the same three-dimensional results as a 3D model. Another conclusion is that the 2D model works for non-linear changes along the third geometrical dimension.

The 3D capability of the modified 2D model was successfully used in the practical application of a ground-source heat exchanger for selecting a feasible tube arrangement and for dimensioning of the required water flow rate of a water pump.

8. References

1. L. Schiavi, 3D Simulation of the Thermal Response Test in a U-tube Borehole Heat Exchanger, *Comsol Conference*, (2009).
2. A. Priarone, S. Lazzari, and E. Zanchini, Numerical Evaluation of Long-Term Performance of Borehole Heat Exchanger Fields, *Comsol Conference*, (2009).
3. Cenk Yavuzturk, *Modeling of vertical ground loop heat exchangers for ground source heat pump systems*, Ph.D. thesis, Oklahoma State University, (1999).
4. Ladislaus Rybach and Burkhard Sanner, Ground-source heat pump systems – The European Experience, *GHC Bulletin*, (2000).
5. Signhild Gehlin, *Thermal response test – Model development and evaluation*, Ph.D. Thesis, Luleå University of Technology, (2002).

9. Appendix

The tube loop was also implemented in an axial symmetric model. The circle tube diameter of 4.25m over 270º gives an arc length of 20m. Figures 13 and 14 confirm that this length gives an outlet temperature of 9ºC as in the U-shaped tube loop of 2x10m above.

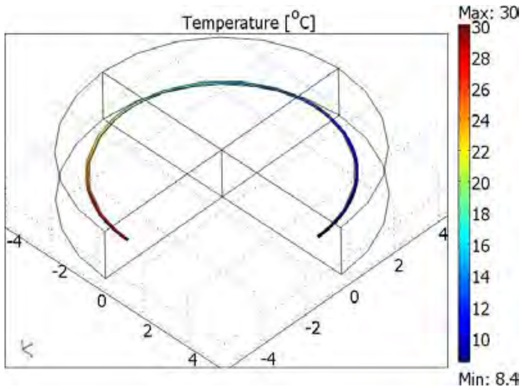

Figure 13. Temperatures for 1mm/s in circular tube loop simulated by 3D steady-state model. Compare with Fig. 3.

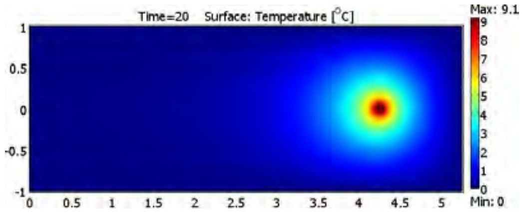

Figure 14. Water outlet temperature for 1mm/s in circular tube loop by 2D time-dependent model. Compare with Fig. 4.

Unlike the U-shaped loop the tube-in-tube system can be implemented in 2D axial symmetric coordinates. The result in Fig. 15 is the same as for the 3D model above. Fig. 15 includes a flow speed of 0.1m/s to show that this would cool the tube-in-tube system over the whole length making it more efficient.

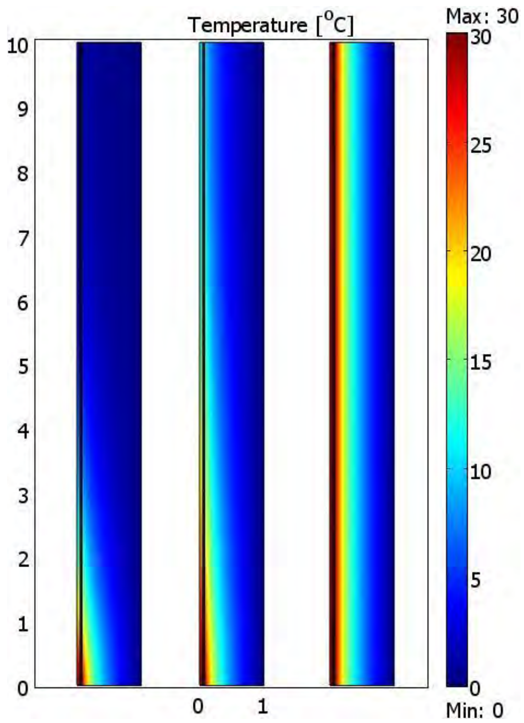

Figure 15. 2D axial symmetric model for tube-in-tube design for 1mm/s, 10mm/s and 100m/s. Compare with Figures 6 to 8.