Propagation of Tsunamis over Large Areas using COMSOL

G. Bellotti[1*] and C. Cecioni[1]
[1]DSIC University of Roma TRE, Rome, Italy
*Corresponding author: via Vito Volterra 62, 00146 Rome, Italy, bellotti@uniroma3.it

Abstract: This paper presents a numerical model based on the mild-slope equation (MSE for short) suitable to reproduce the propagation of small amplitude tsunamis in the off-shore field. The model solves the governing equations in the frequency domain and allows the reproduction of the frequency dispersion for broad banded spectrum sea states. The model application to reproduce laboratory experiments and real life tsunamis in the south Tyrrhenian sea (Mediterranean Sea) are described, showing its applicability to relative large geographical areas. The proposed method is suitable to support a real time tsunami early warning system.

Keywords: Tsunamis, wave modeling, elliptic PDE.

1. Introduction

Tsunamis are long water waves generated by a sudden disturbance of the sea/ocean floor due to submarine earthquakes or landslides or volcanic eruptions. These surface waves, travelling at high celerity, can cross the ocean and inundate the coasts producing disasters.

In the recent decades, the research interest on tsunamis waves has risen enormously, due in large part to the devastating consequences of some tragic events. It can be cited the more recent Indian Ocean tsunamis of December 26, 2004, which caused the loss of about 230, 000 human lives. Regarding the Mediterranean Sea, it is worth to mention the event occurred at the Stromboli volcano, on December 30, 2002 [5].

Tsunamis, during their propagation across large geographical areas, are very long waves with a wave height of the same order of magnitude of that of wind generated waves. Therefore the tsunamis amplitude is much smaller than the water depth where they propagate (typically 1 m over 100-1000 m), and the steepness of the waves is also extremely small, since their length is of the order of the thousand of meters. This suggests that nonlinear terms that appear in the model equations suitable to reproduce these waves, may be neglected when reproducing tsunamis propagation.

In this paper we describe a model, partially presented in [2], able to reproduce the waves propagation over large areas, based on the MSE. The model's ability of reproducing the full frequency-dispersion of small amplitude tsunamis generated by landslides, makes it an attractive tool for the simulation of tsunamis in the far field. The purpose of this paper is to present the validation of the model against available laboratory experiment results, and a sample application to a large geographical scale phenomenon. For the latter the attention is focused on a specific tsunami scenario in the south Tyrrhenian sea (Mediterranean sea), i.e. the waves generated by a landslide at the island of Stromboli.

2. Governing Equations

The MSE, in terms of free surface elevation ζ, is able to reproduce transient waves over gently varying topography. It can be written as follows:

$$\frac{\partial^2 \zeta}{\partial t^2} - \nabla\left(cc_g \nabla \zeta\right) - \left(k^2 cc_g - \omega^2\right)\zeta = \frac{1}{\cosh(kh)} \frac{\partial^2 f}{\partial t^2} \qquad (1)$$

where c and c_g are respectively the phase and the group velocities, k is the wave number and ω is the angular frequency. The right hand side of eq. (1) represents a source term introduced in order to incorporate the wave generation inside the numerical domain. For the applications presented in this paper the wave source is represented by the motion of a landslide of given properties and kinematics. Thus $f(x, y, t)$ is the elevation of the moving submerged landslide. Eq. (1) is a time-dependent version of the MSE for nearly harmonic waves. In the present model a spectral approach is used to solve the MSE: by taking the Fourier transform of eq. (1), we obtain a frequency-dependent version of the MSE [1]

$$\nabla \cdot \left(cc_g \nabla\right) + \omega^2 \frac{c_g}{c} Z = -\frac{1}{\cosh(kh)} F \qquad (2)$$

where the complex variables $Z(x,y,\omega)$ and $F(x,y,\omega)$ are the Fourier transform of ζ and $\partial^2 f/\partial t^2$ respectively. Eq. (2) is used in engineering for the reproduction of harmonic waves, and represents a formidable tool for studying the propagation of small amplitude waves into harbours and over coastal areas. It can be viewed as a sort of Helmoltz equation for which the group and phase wave celerity varies over the domain, in view of the varying water depth. The numerical model here described solves eq. (2) for all those frequencies ω, which contain relevant wave energy. The linearity of the problem allows the superposition of the effects of all the monochromatic solutions of eq. (2). Finally the inverse transform of Z is carried out to get the free surface elevation in the time domain, $\zeta(x,y,t)$.

The boundary conditions here employed are that of full reflection along solid boundaries, of radiation at the offshore border of the computational domain, and the wave maker condition if waves are generated at a boundary of the domain. At the reflection boundary the derivative of the Fourier transform of the free surface elevation along the normal to the boundary should be zero

$$\frac{\partial Z}{\partial n} = 0 \tag{3}$$

The radiation boundary condition can be obtained by using a mathematical formulation that allows the waves that propagate toward the open boundaries to freely exit the computational domain. This condition can be easily formulated for progressive outgoing waves, and here is expressed in the frequency domain as

$$\frac{\partial Z}{\partial n} + ik\cos(\theta_n)Z = 0 \tag{4}$$

where θ_n is the angle the wave direction forms with the outgoing normal to the considered boundary. Since θ_n depends on the solution itself, we have in the past [6] proposed to apply iterative methods to solve the indeterminacy.

The wave-maker boundary condition is conveniently formulated in terms of the velocity potential at z = 0 as follows

$$\frac{\partial \varphi}{\partial n} = u^I \tag{5}$$

where u^I is the velocity at z = 0 of the desired wave field orthogonal to the wave-maker boundary. In order to obtain a mathematical expression involving ζ and consequently Z, we make use of the usual relationship between φ and ζ (Φ and Z in the frequency domain) which allows rewriting of the wave-maker condition as

$$\frac{\partial Z}{\partial n} = -\frac{i\omega}{g}U^I \tag{6}$$

being U^I the Fourier Transform of the desired time series of velocity at z = 0.

3. Use of COMSOL Multiphysics

The model is solved using the PDE mode of the software COMSOL Multiphysics.

The MSE and the boundary conditions are inserted into the model by assigning the coefficient into the PDE. These coefficients refer to the wave features, i.e.: k, ω, c and c_g, and to the bathymetry and the sea floor motion. The model solves the mathematical problem for one frequency, ω, of the wave spectrum, and it makes use of the linear stationary solver Direct PARDISO. The reproduction of the frequency dispersion is reached by running other simulations for all the desired frequencies. In the computational procedure, we have written a MatLab code which read from COMSOL the fem structure and repeats the computations by changing each time the wave frequency.

The mesh element size is chosen in order to ensure a minimum of 10 points for one wave length, therefore it vary with the water depth and the wave period. The post-processing of the data is done using MatLab code.

4. Numerical Model Validation

New physical experiments which aims at studying the tsunami wave field generated by landslide have been used in order to validate the tsunami generation in the numerical model. The physical model was built in a large wave tank at the Research and Experimentation Laboratory for Coastal Defence (LIC) of the Technical University of Bari, Italy, in cooperation with the Environmental and Maritime Hydraulics Laboratory Umberto Messina (LIAM) of the University of L'Aquila, Italy. The laboratory experiments [3] simulate a landslide body falling on the flank of a conical island, built in order to approximately reproduce in scale 1:1000, the Stromboli island, south Tyrrhenian Sea, Italy [3].

The physical model consists in a wave tank, 30.00 *m* wide, 50.00 *m* long and 3.00 *m* deep; at the centre of the tank is placed a conical island, built using PVC sheets (thickness 0.01 *m*) and sustained by a steel frame, with a radius of 4.47 *m* at the tank bottom level. The slope of the flanks of the island is 1:3 (1 vertical, 3 horizontal).

Figure 1. Picture of the experimental model of the conical island with the landslide body.

Experiments have been carried out varying the water depth, and consequently the shoreline curvature radius, and by varying the initial distance of the landslide from the undisturbed shoreline. The landslide model is a rigid body, with the shape of an half of the ellipsoid described by the equation $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$, where $a = 0.2$ *m*, $b = 0.4$ *m* and $c = 0.05$ *m*, for a total volume $V = 0.0084\ m^3$. The landslide is constrained to slide down the inclined surface by means of rails.

Traditional resistive gauges were employed to register the instantaneous vertical displacement of the free surface. All the signals have been acquired simultaneously at a frequency of 1000 *Hz*. The relative positions of all the gauges can be found in figure 2.

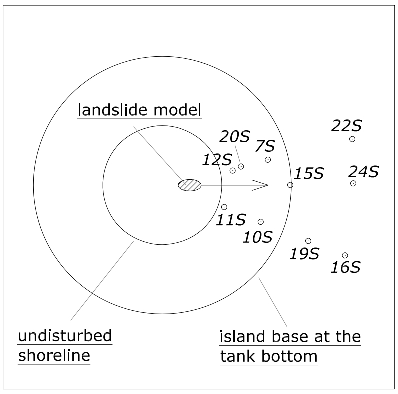

Figure 2. Layout of the sea-level gauges position in the 3D tests at LIC..

The numerical computations have been carried out on a two-dimensional domain. The numerical simulation here presented reproduces just one experimental case, defined by the off-shore constant water depth of 0.80 *m*, and the shoreline radius of 2.07 *m*, and characterized by an aerial landslide which falls from a distance of $\zeta = 0.30$ *m* from the undisturbed shoreline. At the internal circular border (the undisturbed shoreline) the reflection condition (eq. 3) is imposed. Along the external circular boundary the waves are allowed to freely exit the computational domain (eq. 4). The water depth function *h(x,y,t)*, which takes into account the sea floor motion, due to the landslide, is calculated by knowing the landslide shape and movement. The second derivative in time of the function, *f(x,y,t)*, is carried out using a numerical approximation and its Fourier transform is applied, in order to insert it into the field eq. (2). The numerical simulation has been carried out using triangular linear elements, whose maximum size is 0.05 *m*. Figures 3 and 4 show the comparison between the laboratory measurement of the water surface elevation (red dashed lines) and the numerical simulations (solid black lines).

As it can be seen from the figures, the model gives reliable results; this is evident especially for the gauges located in the off-shore area (refer to figure 2 for gauges position). The comparison shows that the model is not able to exactly reproduce the water level oscillations close to the shoreline, see the numerical results at gauges S12 and S11.

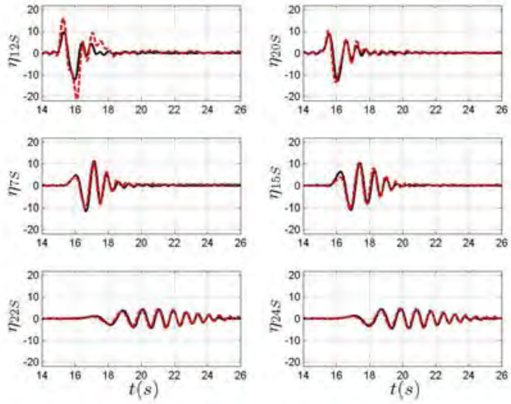

Figure 3. Comparison of the free surface elevations (measures are in *mm*) at six gauges as measured in the physical model (red dashed line) and as obtained from the numerical model (black solid line).

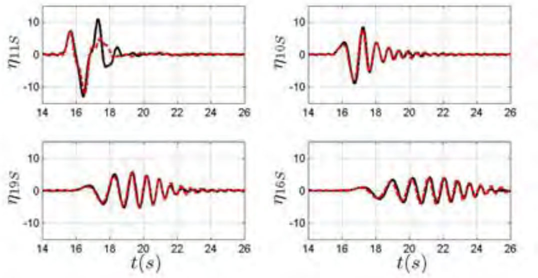

Figure 4. Comparison of the free surface elevations (measures are in *mm*) at the other four gauges as measured in the physical model (red dashed line) and as obtained from the numerical model (black solid line).

This behaviour can be explained by the fact that the numerical model equations are valid for submerged landslides, because they reproduce the effect of the moving seafloor on the water surface. No reproduction of the landslide impact phase is given which, as seen in the laboratory experiment, has a piston-like effect (especially if it has a steep front), that induces a deformation of the shoreline and is responsible of part of the generation of waves propagating along the coast. Another point to be considered is that the numerical model does not reproduce any dissipation at the interface between water waves and the island.

5. Large scale Application

The model is applied to reproduce a possible tsunami scenario in the Aeolian archipelago (Italy). The Aeolian Islands are located in the south Tyrrhenian Sea to the west of Calabria and to the North of Sicily and constitute a volcanic islands arc, depicted in Fig. 5.

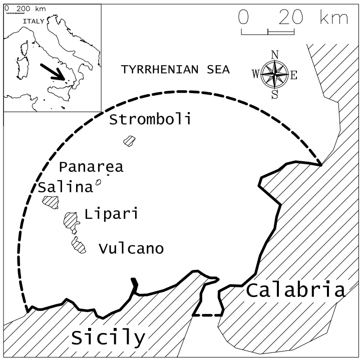

Figure 5. Sketch of the Aeolian islands and boundaries of the numerical domain.

Figure 6. Photo of Stromboli island.

Stromboli is the island located at north east of the archipelago and it is one of the most active volcanoes in the Mediterranean Sea.The volcanic edifice of Stromboli is a broadly regular cone, with very steep slope: around 15% in the less steep submarine portion of the edifice.

The volcano is characterized by a persistent activity with moderate rhythmic explosions (every 15 minutes) that are named “strombolian” by volcanologists, and by occasional more energetic paroxysmic phases also with lava effusion and fountains; pyroclastic flows occur at intervals of several years or decades. Volcanic eruptions of huge dimensions happened with a return period of thousands of years. The last episode is dated to less than 5,000 years ago, and caused the partial collapse of the edifice, creating a scar, named Sciara del Fuoco, on the north-west flank. This flank is characterized by very steep slope (38% in the subaerial island’s portion) and thus represented a preferential lane for the lava and landslide fall, as it can be seen from the picture in Figure 6.

The most relevant volcanic activities have been accompanied by local tsunamis. In some cases the tsunami generation is directly caused by the pyroclastic flows entering into the sea, in some others it can be caused indirectly by the failure of aerial or underwater landslides. The generation of tsunami waves was due to submarine landslides or hot avalanches and caused several damages in the Stromboli coastal zone: inundation depth of the order of hundred meters, boats carried inland and building damages. The most recent tsunami event occurred on the 30th December 2002 [5].

We will present the preliminary model results applied to the Aeolian islands. The numerical domain is that included in the dashed line of fig. 5. Only the fluid domain is modeled through the MSE. The coastline is assumed to completely reflect the incoming waves, therefore eq. (3) is imposed at these boundaries. Eq. (4) is applied at the boundaries represented by the dashed line (fig. 5) in order to allow wave to freely exit the domain.

The modelled time series is 20,000 s long, with a Δt equal to 5 s, leading to 4,000 time steps, which correspond to the same number of frequencies to be solved. By inspection of the Fourier Transform of the reproduced wave field, the frequencies at which the energy content is significantly larger than zero and that consequently are worth to be reproduced are identified as those in the interval $0.00005 < f < 0.04$ *Hz*, resulting in 800 components of the spectrum. Thus 800 equations like (2) are solved, one for each frequency component. The computational time is in total 4 hours and 30 minutes in a AMD Opteron 246 2 GHz computer equipped with 4 GB of RAM. At the boundary of Stromboli island along the coast of the Sciara del Fuoco, a wave-maker condition as equation (6) is imposed, which generates a N-shaped wave with period of 100 s and height of 60 *m*.

The results, in terms of water surface elevations in the time domain, are shown at five points in front of the five islands (see figure 5), all the points are located at a water depth of about 100 m. In figure 7 are shown the results of the present model. As it can be seen from the free surface elevations, the first wave at each points are not the highest ones. From the plots it can be noted a strong reduction of the maximum wave height registered in front of Panarea, Salina, Lipari and Vulcano islands.

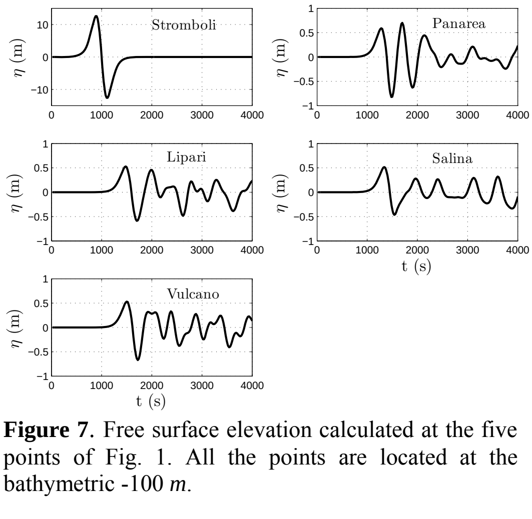

Figure 7. Free surface elevation calculated at the five points of Fig. 1. All the points are located at the bathymetric -100 *m*.

Note that the vertical scale of the plots varies. This is mainly due to the fact that the wave energy is spread into circular wave fronts as it moves away from the source. The times of arrival of the first waves, from Stromboli to the other islands, are in good agreement with those obtained by [4]. The wave which firstly reaches the island of Panarea (the closest one) arrives after about 380 *s*, while it takes about 430, 550 and 570 *s* to reach the other islands, respectively Salina, Lipari and Vulcano.

5. Conclusions

In this paper we have presented how COMSOL can be applied to reproduce the propagation of tsunamis over large geographical areas. Two sample computations have been include, the first considers landslide tsunamis around a conical island reproduced in the laboratory, the second a real life case of tsunamis in the Aeolian Islands.

The model can serve as support to tsunami early warning systems, that are based also on extensive numerical modeling.

8. References

1. Berkhoff J.C.W., Computation of combined refraction-diffraction, *Proceedings of the 13th International Conference of Coastal Engineering,* Vancouver, Canada (1972).

2. Bellotti G., Cecioni C. & De Girolamo P, Simulation of small amplitude frequency-dispersive transient waves by means of the mild-slope equation, *Coastal Engineering,* **55**, 447-458 (2008).

3. Di Risio M., De Girolamo P., Bellotti G., Panizzo A., Aristodemo F., Molfetta M.G., Petrillo A.F., Landslide-generated tsuanmis runup at the coast of a conical Island: New physical model experiments, *Journal of Geophysical Research,* **114** (2009)**.**

4. Tinti S., Pagnoni G., Zaniboni F., Bortolucci E., Tsunami generation in Stromboli island and impact on the south-east Tyrrhenian coasts, *Natural Hazard and earth System Sciences*, **3**, 299-309, (2003).

5. Tinti S., Maramai A., Armigliato A., Graziani L., Manucci A., Pagnoni G., Zaniboni F., Observation of physical effects from tsunamis of December 30, 2002 at Stromboli volcano,

southern Italy, *Bullettin of Volcanology*, **68(5)**, 450-461, (2006).

6. Beltrami G.M., Bellotti G., De Girolamo P. & Sammarco P., Treatment of wave-breaking and total absorption in a mild-slope equation FEM model. *Journal of Waterway, Port, Coastal and Ocean Engineering-ASCE*, Vol. **127(5)**, pp. 263-271, (2001).

9. Acknowledgements

This research has been carried out thanks to many research projects, funded by Italian governmental agencies. The most recent ones have been funded by the Italian Ministry of Research (MIUR) and are listed here below:

-"Tsunami waves generated by landslides in water: mechanics of wave generation and propagation, development of forecasting tools and real-time warning systems based on tidal measurements" (PRIN-2004);

-"Development and validation of hydraulic and geologic tools for supporting a Tsunami Early Warning System. Implementation to the Stromboli (Eolie) landslide case" (PRIN-2007);

-"Design, construction and operation of the SMO-Submarine Multidisciplinary Observatory-experiment" (FIRB 2008).

Thanks are due to Dr. Marcello Di Risio and prof. Paolo De Girolamo (University of L'Aquila) and prof. Leopoldo Franco (University of Roma TRE).