ation time (ERT) approximation, $\Omega_i = -(f_i - f_i^{eq})/\tau$, has become the most popular form for the collision operator because of its simplicity and computational efficiency. However, the absence of a clear time scale separation between the hydrodynamic and nonhydrodynamic modes reduces the numerical stability [40,57], and can sometimes cause significant errors at solid-fluid boundaries [28]. Furthermore, it does not allow independent variation of the bulk and shear viscosities. Thus we employ the more flexible collision operator of Eq. (A1).

APPENDIX B: BREATHING-MODE OSCILLATIONS OF A SPHERICAL SHELL

Here we summarize the theoretical analysis of the oscillatory behavior of an elastic shell, both *in vacuo* and when filled with an inviscid fluid. We limit ourselves to the response of the system after an initial radial expansion. Following the initial expansion, the system will contract and expand in an oscillatory manner ("breathing mode") with a characteristic frequency. Because of the symmetry of the system the breathing mode is completely described by the radial component of the displacement vector $u_r(r,t)$ in the shell, and the radial component of the enclosed fluid velocity $v_r(r,t)$, which furthermore only depend on the radial coordinate r and time t. We denote the radius of the outer surface of the shell with a, that of the inner surface with b, while $R=(a+b)/2$ and $h=a-b$.

The displacement vector $\mathbf{u}(\mathbf{r},t)$ in the shell is determined by the equation of motion for an isotropic elastic medium (e.g., Ref. [58]),

$$\rho_s \frac{\partial^2 \mathbf{u}}{\partial t^2} = \frac{E}{2(1+\nu)} \nabla^2 \mathbf{u} + \frac{E}{2(1+\nu)(1-2\nu)} \nabla \nabla \cdot \mathbf{u}, \tag{B1}$$

with E the shell's Young's modulus, ρ_s its density, and ν the Poisson's ratio. Hence, for $\mathbf{u}(\mathbf{r},t)=u_r(r,t)\hat{\mathbf{r}}$, with $\hat{\mathbf{r}}$ the unit vector in the radial direction, Eq. (B1) for the elastic shell reduces to

$$\frac{1}{c_s^2}\frac{\partial^2 u_r}{\partial t^2} = \frac{\partial}{\partial r}\frac{1}{r^2}\frac{\partial}{\partial r} r^2 u_r, \tag{B2}$$

with c_s the longitudinal speed of sound in the shell

$$c_s = \sqrt{\frac{E(1-\nu)}{\rho_s(1+\nu)(1-2\nu)}}. \tag{B3}$$

The fluid velocity $\mathbf{v}(\mathbf{r},t)$ inside the shell is described by the equation of motion for an inviscid fluid (e.g., Ref. [59]),

$$\rho_f \frac{\partial \mathbf{v}}{\partial t} + \rho_f \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p, \tag{B4}$$

with ρ_f the fluid density and p its pressure. For $\mathbf{v}(\mathbf{r},t)=v_r(r,t)\hat{\mathbf{r}}$, and consequently $p=p(r,t)$ this reduces to

$$\rho_f \frac{\partial v_r}{\partial t} + \frac{1}{2}\rho_f \frac{\partial v_r^2}{\partial r} = -\frac{\partial p}{\partial r}. \tag{B5}$$

If we assume small amplitude oscillations, we can neglect the quadratic term in v_r, leaving

$$\rho_f \frac{\partial v_r}{\partial t} = -\frac{\partial p}{\partial r}. \tag{B6}$$

Furthermore, as long as the compression is small (as is the case for small amplitude oscillations)

$$\partial p/\partial t = -\kappa \nabla \cdot \mathbf{v} = -\kappa \frac{1}{r^2}\frac{\partial}{\partial r} r^2 v_r, \tag{B7}$$

with κ the bulk modulus. Differentiating both sides of Eq. (B7) with respect to time and using Eq. (B6), we then arrive at

$$\frac{1}{c_f^2}\frac{\partial^2 p}{\partial t^2} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial p}{\partial r} \tag{B8}$$

which is an equation for undamped, longitudinal compressional waves, with $c_f=\sqrt{\kappa/\rho_f}$ the speed of sound in the fluid.

The equations of motion can be solved by seeking solutions of harmonic waves, i.e., by assuming

$$u_r(r,t) = U(r)e^{i\omega t}, \quad v_r(r,t) = V(r)e^{i\omega t}, \quad p(r,t) = P(r)e^{i\omega t}. \tag{B9}$$

Equations (B2) and (B8) then reduce to

$$\frac{d}{dr}\frac{1}{r^2}\frac{d}{dr} r^2 U + \frac{\omega^2}{c_s^2} U = 0, \tag{B10}$$

$$\frac{1}{r^2}\frac{d}{dr} r^2 \frac{dP}{dr} + \frac{\omega^2}{c_f^2} P = 0, \tag{B11}$$

with the general solutions

$$U(r) = Aj_1(\omega r/c_s) + By_1(\omega r/c_s), \tag{B12}$$

$$P(r) = Cj_0(\omega r/c_f) + Dy_0(\omega r/c_f). \tag{B13}$$

Here, $j_n(x)$ and $y_n(x)$ are the nth order spherical Bessel functions of the first and second kind, and the coefficients A, B, C, and D are integration constants, that have to be obtained from the appropriate boundary conditions at the inner and outer surface of the shell.

For small deformations, the stress tensor σ in the shell is related to the displacement vector $\mathbf{u}(\mathbf{r},t)$ according to (e.g., [58])

$$\sigma = \mu(\nabla \mathbf{u} + \nabla \mathbf{u}^{\dagger}) + \lambda \nabla \cdot \mathbf{u}\mathbf{I}, \tag{B14}$$

with $\mathbf{I}$ the unit tensor and μ and λ the Lamé coefficients

$$\mu = \frac{E}{2(1+\nu)}, \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}. \tag{B15}$$

Using spherical symmetry, $\mathbf{u}(\mathbf{r},t)=u_r(r,t)\hat{\mathbf{r}}$, this gives

$$\sigma_{rr}=(\lambda+2\mu)\frac{\partial u_r}{\partial r}+2\lambda\frac{u_r}{r}. \quad (B16)$$

Balancing the radial stress with the pressure at the inner and outer surface of the shell gives $\sigma_{rr}(r,t)=-p(r,t)$ for $r=a$ and $r=b$. Hence, the boundary conditions for the stress are

$$(\lambda+2\mu)\left.\frac{dU}{dr}\right|_r+2\lambda\frac{U(r)}{r}=-P(r), \quad r=a,r=b. \quad (B17)$$

These equations are sufficient to solve for the displacement vector of an elastic shell *in vacuo* [in which case $P(a)=P(b)=0$].

For the fluid filled shell we need two more boundary conditions. The first is obtained by requiring the pressure to be finite at $r=0$, giving $D=0$. The second is obtained by requiring continuity of the normal velocity across the inner surface of the shell, i.e.,

$$v_r(b,t)=\left.\frac{\partial u_r}{\partial t}\right|_{r=b}. \quad (B18)$$

Using Eqs. (B6), (B7), and (B9), this reduces to

$$U(b)=\frac{1}{\rho_f\omega^2}\left.\frac{dP}{dr}\right|_{r=b}. \quad (B19)$$

1. An elastic shell *in vacuo*

First, we solve for the breathing mode oscillations in an elastic shell *in vacuo*. Substituting Eq. (B12) in Eq. (B17) and using $P(a)=0$ gives

$$A[s\alpha j_0(\alpha)-j_1(\alpha)]+B[s\alpha y_0(\alpha)-y_1(\alpha)]=0. \quad (B20)$$

Here, $\alpha=\omega a/c_s$, the constant s is defined as

$$s=\frac{\lambda+2\mu}{4\mu}=\frac{1-\nu}{2(1-2\nu)}, \quad (B21)$$

and we used the fact that $df_n(x)/dx=f_{n-1}(x)-(n+1)f_n(x)/x$, with f_n either j_n or y_n. The boundary condition at $r=b$ leads to an similar expression by substituting $\beta=\omega b/c_s$ for α in Eq. (B20). Eliminating A and B then results in

$$\frac{s\alpha y_0(\alpha)-y_1(\alpha)}{s\alpha j_0(\alpha)-j_1(\alpha)}=\frac{s\beta y_0(\beta)-y_1(\beta)}{s\beta j_0(\beta)-j_1(\beta)}, \quad (B22)$$

which can be simplified to [60]

$$\frac{\tan(\Omega\hat{h})}{\Omega\hat{h}}=\frac{1+s\Omega^2\hat{a}\hat{b}}{\Omega^2\hat{a}\hat{b}+(s\Omega^2\hat{a}^2-1)(s\Omega^2\hat{b}^2-1)}. \quad (B23)$$

Here, $\hat{h}=h/R, \hat{a}=a/R, \hat{b}=b/R$, and the dimensionless frequency Ω is defined as

$$\Omega=\frac{\omega R}{c_s}. \quad (B24)$$

Equation (B23) can be solved numerically for Ω. It has a single solution Ω_0 in the limit $\hat{h}\rightarrow 0$,

$$\Omega_0=\sqrt{\frac{2(1+\nu)(1-2\nu)}{(1-\nu)^2}}, \quad (B25)$$

which is Lamb's breathing-mode frequency for an infinitely thin elastic shell *in vacuo* [52]. At finite h, there is a hierarchy of solutions; the lowest frequency being a generalization of Lamb's breathing-mode frequency for shells with a finite thickness. The next higher frequency, which is proportional to R/h, is a mode where the shell's middle surface remains stationary in time and the shell itself becomes thinner (compressed) and thicker (expanded) in an oscillatory manner.

2. An elastic shell filled with an inviscid fluid

Next, we solve for the breathing mode oscillations in an elastic shell filled with an inviscid fluid. The general solution is given by Eqs. (B12) and (B13), where A,B, and C have to be obtained from Eqs. (B17) and (B19) (remember that $D=0$). Since, $P(a)=0$, as for the elastic shell *in vacuo*, the first boundary condition is identical to that of the empty shell [Eq. (B20)]. Using $P(b)=Cj_0(\omega b/c_f)$, the second boundary condition is obtained from Eq. (B20) by substituting $\beta=\omega b/c_s$ for α, and changing the right-hand side to $-(b/4\mu)Cj_0(\omega b/c_f)$. With Eqs. (B12) and (B13), and $dj_0(x)/dx=-j_1(x)$, the third boundary condition (B19) reduces to

$$Aj_1(\beta)+By_1(\beta)=-\frac{C}{\omega\rho_f c_f}j_1\left(\frac{\omega b}{c_f}\right). \quad (B26)$$

Eliminating A,B, and C and simplifying the result using the same notation as in Eq. (B23) gives

$$[\Omega^2\hat{a}\hat{b}+(s\Omega^2\hat{a}^2-1)(s\Omega^2\hat{b}^2-1)]\tan(\Omega\hat{h})-(1+s\Omega^2\hat{a}\hat{b})\Omega\hat{h}=\frac{s\Omega\hat{b}}{\gamma f}\frac{j_0(\gamma\Omega\hat{b})}{j_1(\gamma\Omega\hat{b})}[(s\Omega^2\hat{a}^2-\Omega^2\hat{a}\hat{b}-1)\tan(\Omega\hat{h})+\Omega\hat{h}+s\Omega^3\hat{a}^2\hat{b}], \quad (B27)$$

which can be solved numerically for given values of R,h,ν, and

$$\gamma=\frac{c_s}{c_f}, \quad f=\frac{\rho_s}{\rho_f}. \quad (B28)$$

Expanding Eq. (B27) in powers of h and keeping only the lowest order gives

$$\left[1-\frac{\rho_f R}{\rho_s h}\frac{j_0(\gamma\Omega)}{\gamma\Omega j_1(\gamma\Omega)}\right]\left(\frac{\Omega}{\Omega_0}\right)^2=1, \quad (B29)$$

with Ω_0 given in Eq. (B25). This result is identical to that obtained by Rand and DiMaggio [61] and Engin and Liu [62] for $n=0$, and reduces to that obtained from membrane theory for $\nu=1/2$ [63]. Note that for $\rho_f/\rho_s \to 0$, Eq. (B29) reduces to $\Omega=\Omega_0$, the correct result for an infinitely thin elastic shell *in vacuo*. For $\gamma \to \infty$, with c_f finite, it degenerates to

$$j_1\left(\frac{\omega b}{c_f}\right)=0, \tag{B30}$$

which is the frequency equation for radial oscillations of an inviscid fluid inside a rigid spherical cavity with radius b. The same result could have been obtained directly, by solving Eq. (B11), subject to the boundary condition (B19) for $U(b)=0$.

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