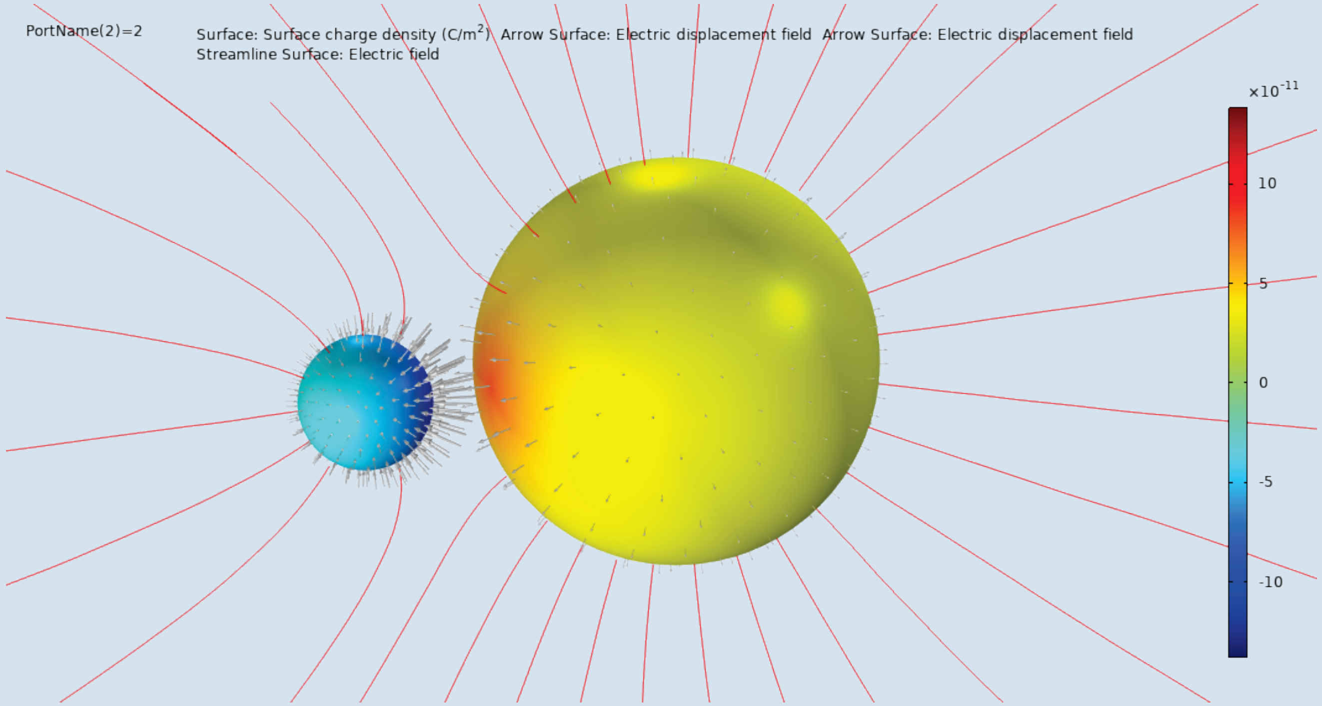
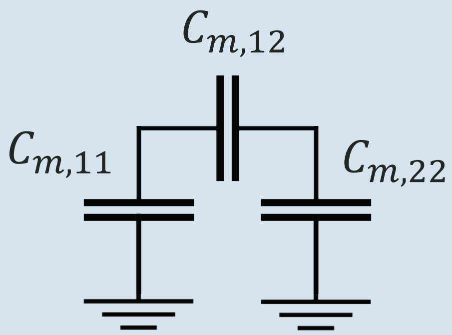
Capacitance Matrix of Two Spheres

Background

- This model describes a benchmark for computing the capacitance matrix for a system of 2 non-concentrical spheres.
- Numerical results of COMSOL are compared to an analytical solution (de Queiroz, 2003, Lekner, 2011)
- The model discusses also the relation between Maxwell capacitance matrix and mutual capacitance matrix.
- Full discussion in blog post.





COMSOL

Analytical solution

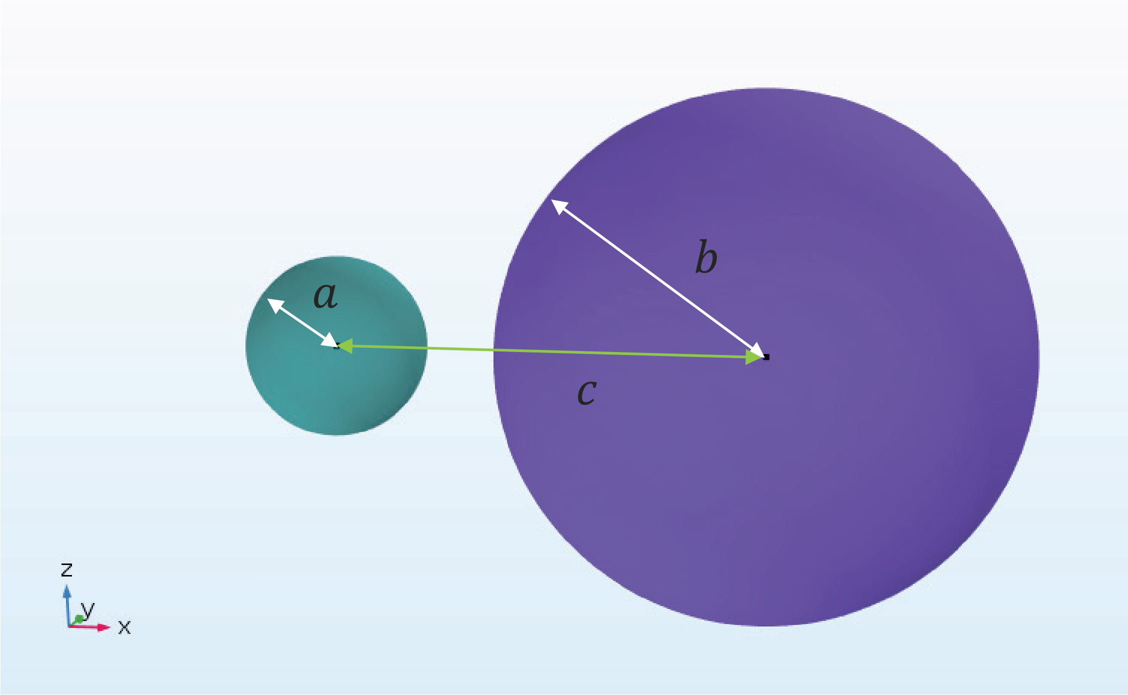
$$C_{11} = Fab \sinh u \sum_{n=0}^{\infty} [\mathrm{a} \sinh nu + b \sinh(n+1)u]^{-1}$$

$$C_{22} = Fab \sinh u \sum_{n=0}^{\infty} [\mathrm{b} \sinh nu + a \sinh(n+1)u]^{-1}$$

$$C_{12} = -F \frac{ab}{c} \sinh u \sum_{n=1}^{\infty} [\sinh nu]^{-1}$$

$$\cosh u = \frac{c^2 - a^2 - b^2}{2ab}$$

$$F = 4\pi\varepsilon_0 \cdot 1[m]$$

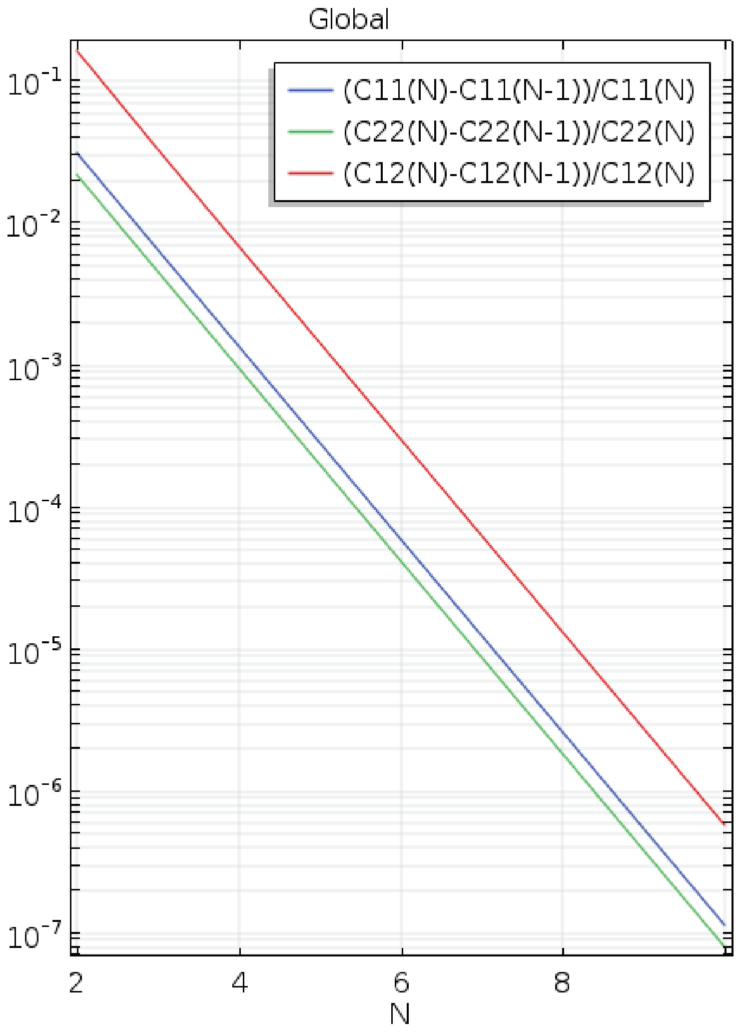


Analytical solution in COMSOL

- A parametric sweep over N shows that the series converge rapidly and N=10 is a very reasonable choice for the parameter set a,b, and c chosen in the model.

Variables

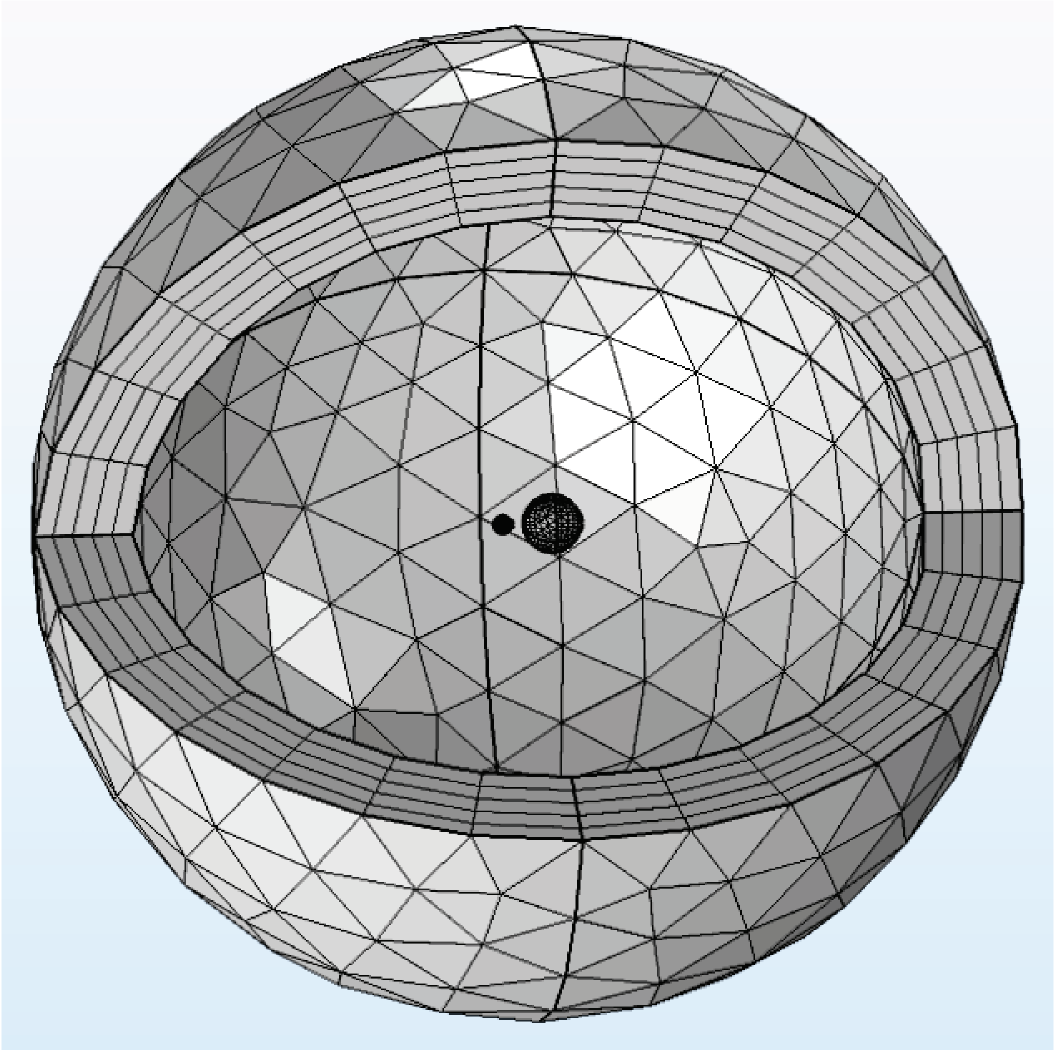
Name	Expression	Unit
coshu	(c^2-a^2-b^2)/(2*a*b)	
u	acosh(coshu)	
F	4*pi*epsilon0_const*1[m]	F
C11	F*a*b*sinh(u)*sum(1/(a*sinh(n*u)+b*sinh((n+1)*u)),n,0,N)	F
C22	F*a*b*sinh(u)*sum(1/(b*sinh(n*u)+a*sinh((n+1)*u)),n,0,N)	F
C12	-(F*a*b/c*sinh(u)*sum((1/sinh(n*u)),n,1,N))	F



Numerical Settings and Mesh

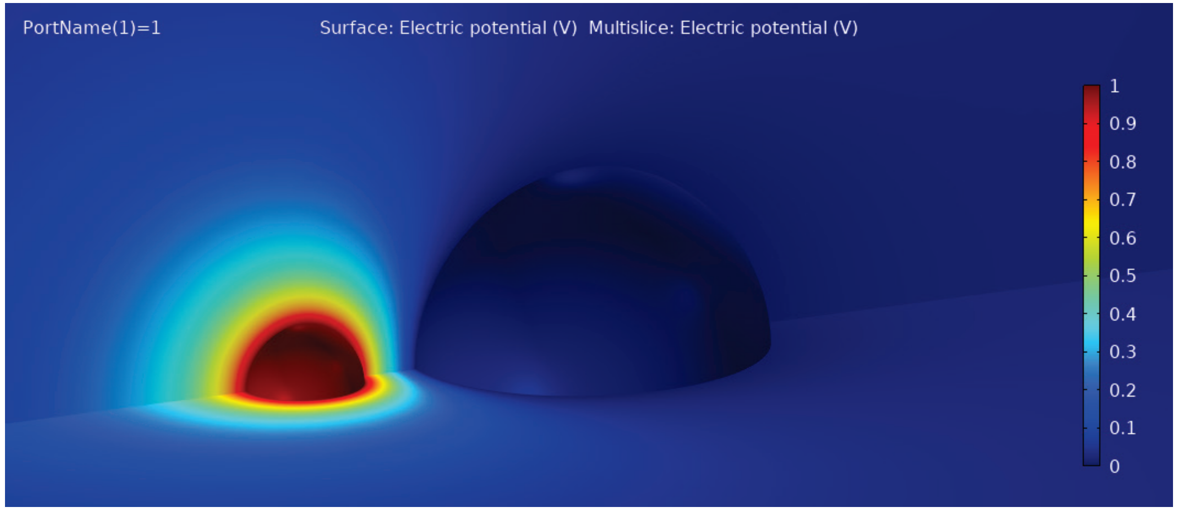
- Outer BNDs set to Ground
- Each sphere is set on a terminal with V=1
- There are two options to realize a study:
  - Use a ***Parametric Sweep*** for PortName, the Maxwell Capacitance Matrix will be calculated first.
  - Use a ***Stationary Source Sweep*** (or ***Frequency Domain Source Sweep)***, the inverse Maxwell capacitance matrix will be calculated first.

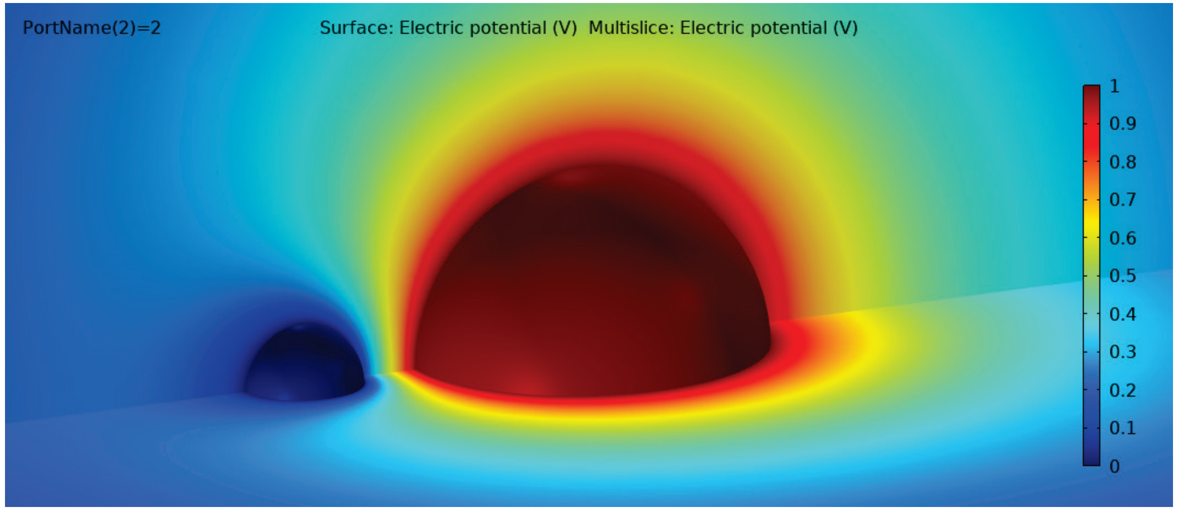
For both options, the Maxwell- and Mutual Capacitance matrices can be extracted by ***Data Series Operations.***



Results Parametric Sweep

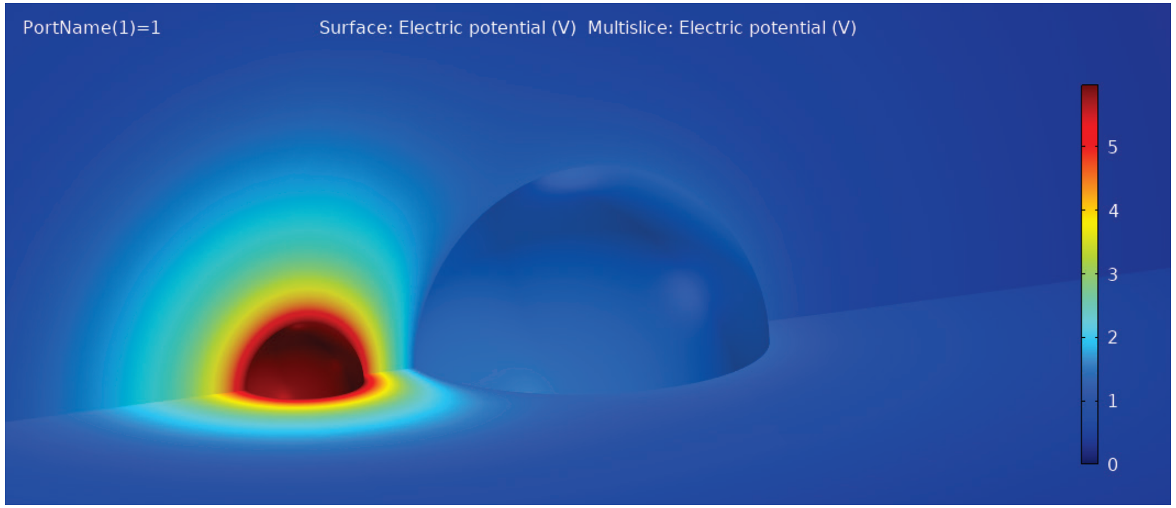
- In the Parametric Sweep study the chosen terminal to 1 V and all others to GND
- One additional conductor or a shell at infinity must additionally be set to GND.

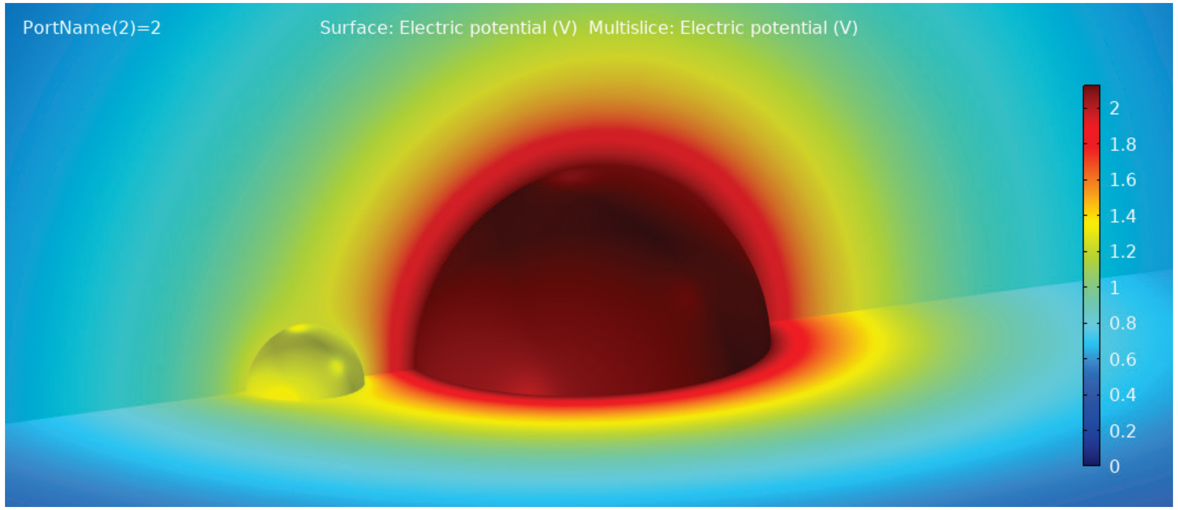




Results Source Sweep

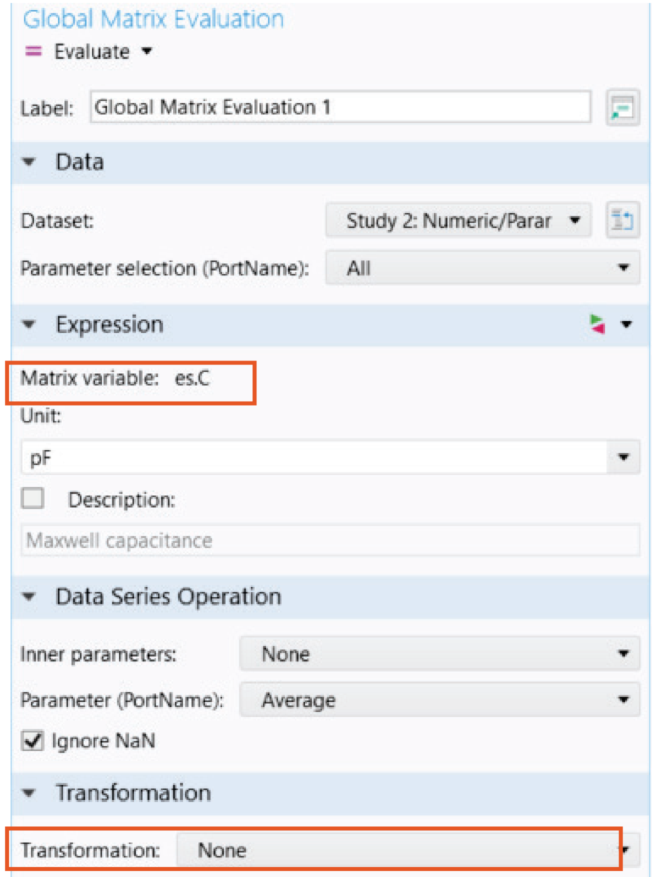
- In the Source Sweep study the chosen terminal is set to 1 C. Hence the voltage on the terminal will not be 1 V and GND.
- Only the additional conductor or a shell at infinity will remain at GND.





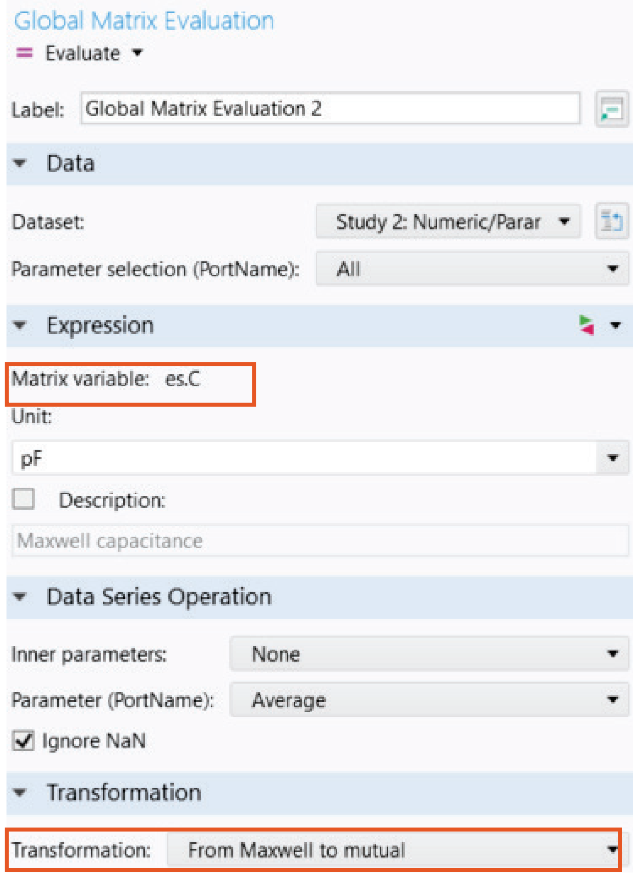
Matrix Extraction for Parameter Sweep

Maxwell Capacitance



Maxwell capacitance (pF)	
13.772	-8.3704
-8.3704	38.663

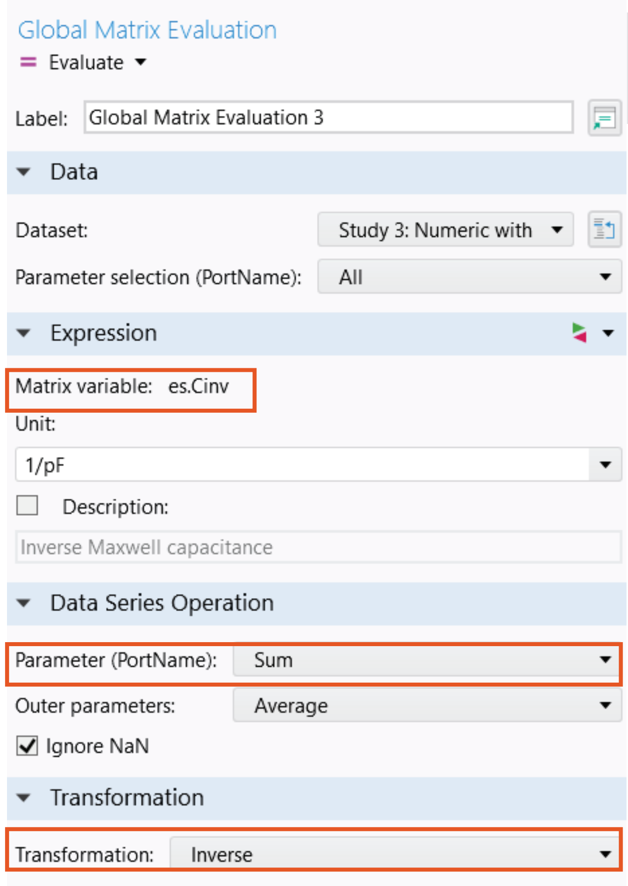
Mutual Capacitance



Maxwell to mutual:	Maxwell capacitance (pF)
5.4019	8.3704
8.3704	30.292

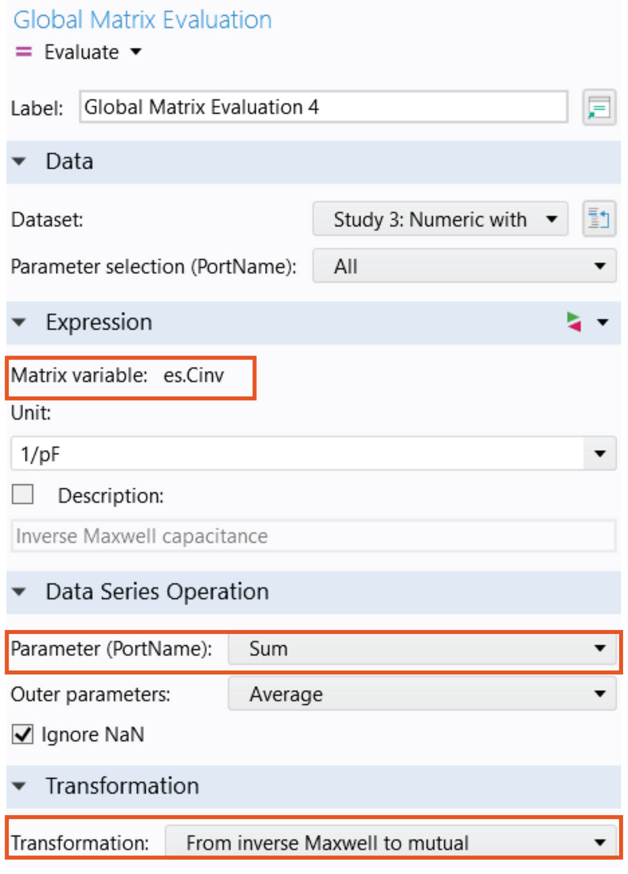
Matrix Extraction for Source Sweep

Maxwell Capacitance



Maxwell capacitance (pF)	
13.772	-8.3704
-8.3704	38.663

Mutual Capacitance

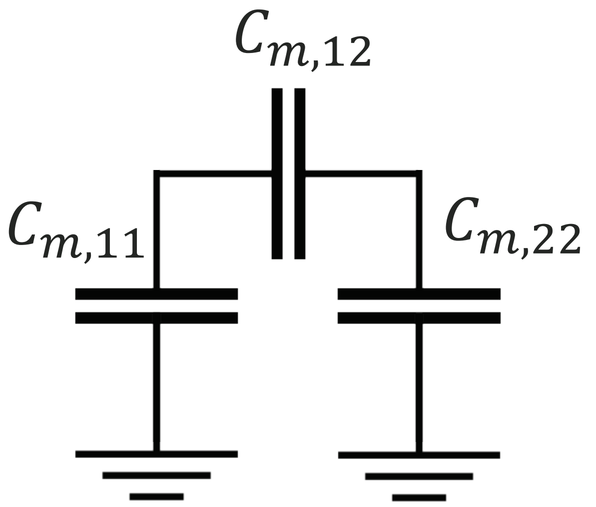


Mutual capacitance (pF)	
5.4019	8.3704
8.3704	30.292

Results

- Maxwell capacitance matrix: $$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$
- Mutual capacitance matrix: $$\begin{bmatrix} C_{m,11} & C_{m,12} \\ C_{m,21} & C_{m,22} \end{bmatrix} = \begin{bmatrix} C_{11} + C_{12} & -C_{12} \\ -C_{21} & C_{22} + C_{21} \end{bmatrix}$$

Coefficient	de Queiroz (2003)	COMSOL analytic	rel. Error	COMSOL numerical	rel. Error
C11	13.76053840	13.76053797	3.10E-08	13.77227480	8.53E-04
C22	38.63340410	38.63340333	2.00E-08	38.66272810	7.59E-04
C12	-8.36260590	-8.36260467	-1.47E-07	-8.37041736	-9.34E-04
Cm,12	8.36260590	8.36260467		8.37038717	9.30E-04
Cm,11	5.39793250			5.401887631	7.33E-04
Cm,22	30.27079820			30.29231074	7.11E-04



References

De Queiroz, A. C. M., 2003, Capacitance Calculations, Link

Lekner, J. (2011), Capacitance coefficients of two spheres, Journal of Electrostatics 69, 11-14. Link