Transient Electromagnetic-Thermal FE-Model of a SPICE-Coupled Transformer Including Eddy Currents

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Outline

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1. Introduction

Current Transformers

- Used to measure high currents in power grid systems
- Primary winding:
  - Normally only one turn (the power line)
- Secondary windings:
  - Some hundreds up to thousands,
  - Close to short-circuit condition

\[ i_2 = \frac{1}{N_2} i_1 \]

Quelle: Bienzle (Wikimedia)
Bar-type Current Transformer (Low Voltage)

Pole mounted Current Transformer (High Voltage)

Quelle: ABB Stotz S&J

Quelle: ABB
2. Modelling Approach

**Coupled Model**
- Electromagnetic FE model of the transformer
- Network models of the primary and secondary circuitry
- Thermal FE model of the transformer
Electromagnetic FE model

Thermal FE model

\[ T(r) \]

\[ q(r) \]
3. Electromagnetic FE Model

- Parametric geometry
- Ampère's circuital law and Faraday's law of induction
- \( mf \) mode (magnetic field only) for time-dependent simulation
- Non-linear magnetic material behavior in \( \mathbf{H} = f(\mathbf{B})\mathbf{e}_B \) form
Modelling of Eddy Currents

- **Power Line (Primary winding):**
  - Sinusoidal primary current \( i_1(t) \) is modeled as a total current density \( J_z(r,t) \) inside of the bus bar
  - \( J_z(r,t) \) can not be imposed directly as an external current density
    \[
    J_z(r,t) = J_{ez}(r,t) + J_{iz}(r,t)
    \]
  - A global equation (ge mode) determines \( J_{ez} \) inside of the bus bar by
    \[
    i_1 - I_{prim} = 0
    \]
  - \( J_z(r,t) \) in the primary conductor is calculated from
    \[
    I_{prim} = \frac{1}{L_{prim}} \int J_z \, dV
    \]
**Eddy Current in the Power Line**

- Simulated z-component of the total current density with skin effect in the primary conductor ($i_{1\text{peak}} = 1000 \text{ A}$, $R_{\text{secExt}} = 20 \Omega$, $t = 0.23 \text{ s}$)
Modelling of Eddy Currents

- Secondary windings:
  - Modeled as bulk material (eight prismatic bodies)
  - Conductivity is set to 10 S/m to suppress eddy current effects
  - Secondary current $i_2(t)$ is modeled as an external current density, derived from the induced voltage
**Eddy Current in the Power Line**

- Simulated z-component of the total current density with skin effect in the primary conductor ($i_{1\text{peak}} = 1000 \text{ A}$, $R_{\text{secExt}} = 20 \Omega$, $t = 0.23 \text{ s}$)
Non-linear Magnetic Behavior

- $H = f(|B|)e_B$ form avoids circular variable definitions in constitutive relations
- Several approximation approaches with remarkable influence on solution time

<table>
<thead>
<tr>
<th>Approximation approach</th>
<th>Relative solution time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piecewise cubic interpolation</td>
<td>1.0</td>
</tr>
<tr>
<td>Global rational function</td>
<td>1.3</td>
</tr>
<tr>
<td>Linear interpolation</td>
<td>4.0</td>
</tr>
<tr>
<td>Cubic spline interpolation</td>
<td>$\approx 100$</td>
</tr>
<tr>
<td>Nearest neighbour</td>
<td>no convergence</td>
</tr>
</tbody>
</table>
Non-linear Magnetic Behavior

- $H = f(|B|)e_B$ form avoids circular variable definitions in constitutive relations
- Several approximation approaches with remarkable influence on solution time
- Cubic spline interpolation may lead to a non-monotonic curves
Coupling with SPICE Components

- \textit{cir} mode
- Sinusoidal current source at the primary side
- External load resistor at the secondary side coupled to the secondary windings (External I vs. U element)
- Variable

\[
V_{\text{sec}} = R_{\text{coil}} i_2 - V_i
\]

\[
V_i = \frac{N_2 (V_1 + V_2 + \ldots + V_8)}{A_{\text{sec}}}
\]
4. Thermal Model

- Temperature-dependent electrical conductivity of the conductors
- Heat conduction in solids applying the *ht* mode
  - Heat sources (mean value over a period of the losses field)
  - Heat conduction in solids and narrow air gaps,
  - Thermal contact resistances between solids which are in mechanical contact
  - External convection on solid-air interfaces applying empirical correlations
- Time-average of the local power loss density in the time interval $[0, t_i]$

$$\overline{q}(r, t_i) = \frac{1}{t_i} \int_0^{t_i} \frac{[J(r, \tau)]^2}{\sigma} d\tau$$
Thermal Model
5. Coupled Time-dependent Simulation

- Time-dependent simulation
- Time scales of the electromagnetic and the thermal model are very different
- Bi-directionally coupling of the electromagnetic and the thermal model
- Iterating alternate solutions:
  - Stationary study steps of the thermal model
  - Time-dependent study steps of the electromagnetic and circuit model
Convergence

\[ i_{\text{peak}} = 1000 \, \text{A}, \quad R_{\text{secExt}} = 20 \, \Omega \]
**Currents**

- Simulated primary current $i_1$ and secondary current $i_2 \cdot N_2$ ($i_{1\text{peak}} = 1000 \, \text{A}$, $R_{\text{secExt}} = 25 \, \Omega$)
- Imperfect transformer coupling due to the air gaps in the core
Currents

- $R_{\text{secExt}} = 1 \, \text{k}\Omega$
- Deformation of the sinusoidal current due to magnetic saturation in the core
Current density

- $i_{1\text{peak}} = 1000 \text{ A}, \ R_{\text{secExt}} = 25 \ \Omega$
**Flux density**

- \( i_{1\text{peak}} = 1000 \, \text{A}, \ R_{\text{secExt}} = 25 \, \Omega \)
6. Summary

**Transient Electromagnetic-Thermal FE-Model of a SPICE-Coupled Transformer Including Eddy Currents**

- Time-dependent simulation of a transformer coupled to an external circuitry
- Non-linear magnetic material properties based on experimental data
- Eddy current effects are included using a global equation
- Time-averaged power loss density distribution
- The bi-directionally coupled thermal model considers the influence of the temperature on electrical material properties
- Future work will focus on
  - consideration of the transformer core lamination,
  - the anisotropic material behaviour inside of the coils
Thank you very much for your attention.