

# Thermal Stresses in Functionally Graded Metal-Ceramic Composite Plates

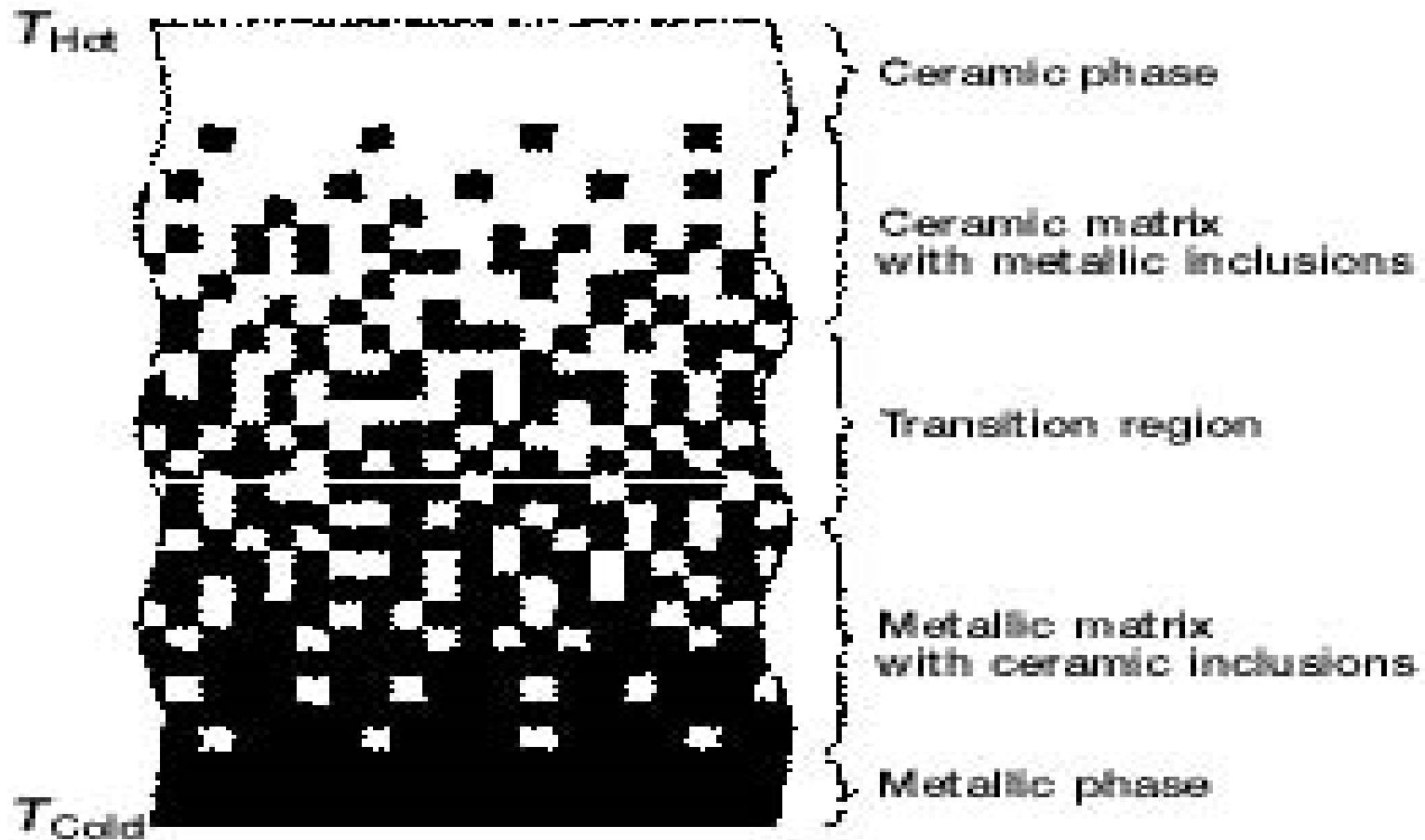
**Ernesto Gutierrez-Miravete**  
**Rensselaer at Hartford**

COMSOL  
CONFERENCE  
2015 BOSTON

# Motivation and Objective

- Functionally graded ceramic-metal composites have significant potential application in high temperature technology.
- To demonstrate the use of the Finite Element Method (FEM) in COMSOL Multiphysics to perform linear thermo-elastic stress analysis of functionally graded materials.

# Functionally Graded Material



(a) Continuously graded microstructure.

# FGM - Characteristics

- FGMs are defined as anisotropic materials whose physical properties vary throughout the volume, either randomly or strategically, to achieve desired characteristics or functionality
- FGMs differ from traditional polymer matrix composites in that their material properties vary continuously, whereas in the composite the properties change at each laminate interface.
- FGMs accomplish this by gradually changing the volume fraction of the materials which make up the FGM.
- FGMs can be readily produced through 3D Printing

# Thermal Stress Analysis

- Uni-directional coupled analysis requiring first the solution of the differential thermal energy balance equation to then incorporate the calculated temperature field values into the constitutive linear elastic model and finally compute the displacement, strain and stress fields.

# Mori-Tanaka Method

The Mori-Tanaka Method is used to estimate the material properties of the FGM (density  $\rho$ , bulk modulus  $K$  and shear modulus  $\mu$ ) at any point in the plate as functions of the volume fractions and material properties of the constituent materials

$$\rho_{FGM} = \rho_M V_M + \rho_C V_C$$

$$K_{FGM} = K_M + \frac{(K_C - K_M)V_C}{1 + \frac{(1 - V_C)(K_C - K_M)}{K_M + \left(\frac{4}{3}\right)\mu_M}}$$

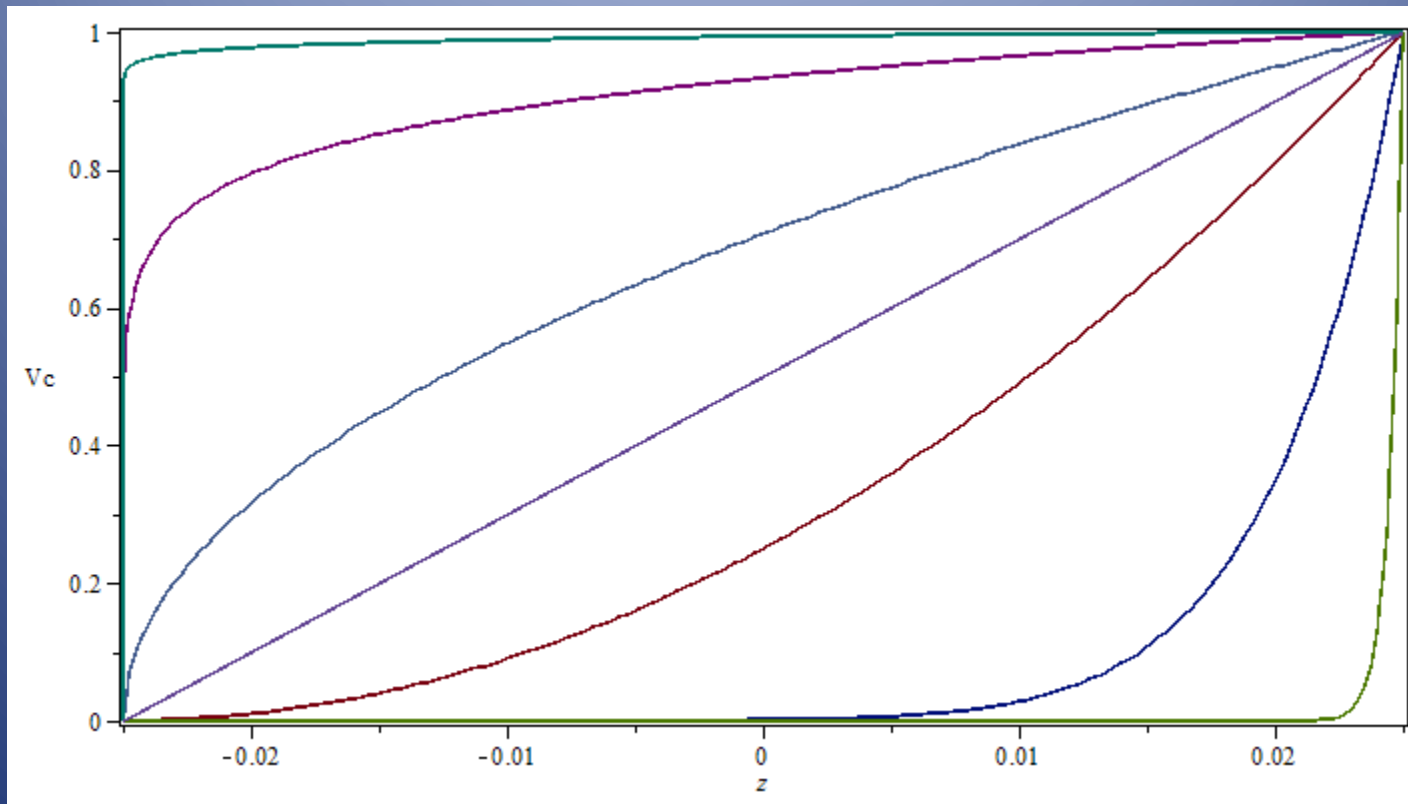
$$\mu_{FGM} = \mu_M + \frac{(\mu_C - \mu_M)V_C}{1 + \frac{(1 - V_C)(\mu_C - \mu_M)}{\mu_M + f_1}}$$

$$\lambda = K - (2/3) \mu$$

$$\nu = [2(1 + \mu/\lambda)]^{-1}$$

$$E = 3 (1 - 2 \nu) K$$

# Volume Fraction of Ceramic through Plate Thickness



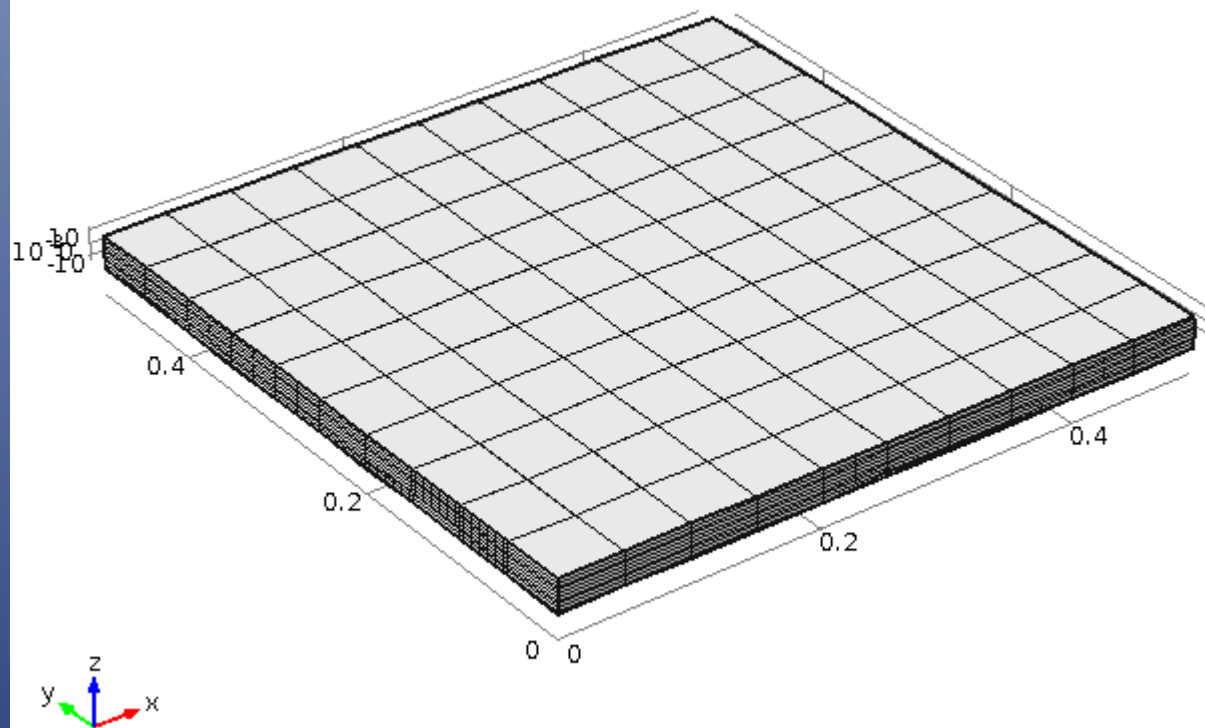
# Material Properties

Property	Ni	ZrO <sub>2</sub>
$\rho$ (kg/m <sup>3</sup> )	8908	6000
E (Pa)	2e11	2.1e11
$\nu$ (-)	0.31	0.3
k (W/m K)	91	2
$\alpha$ (1/K)	13.4e-6	10.3e-6

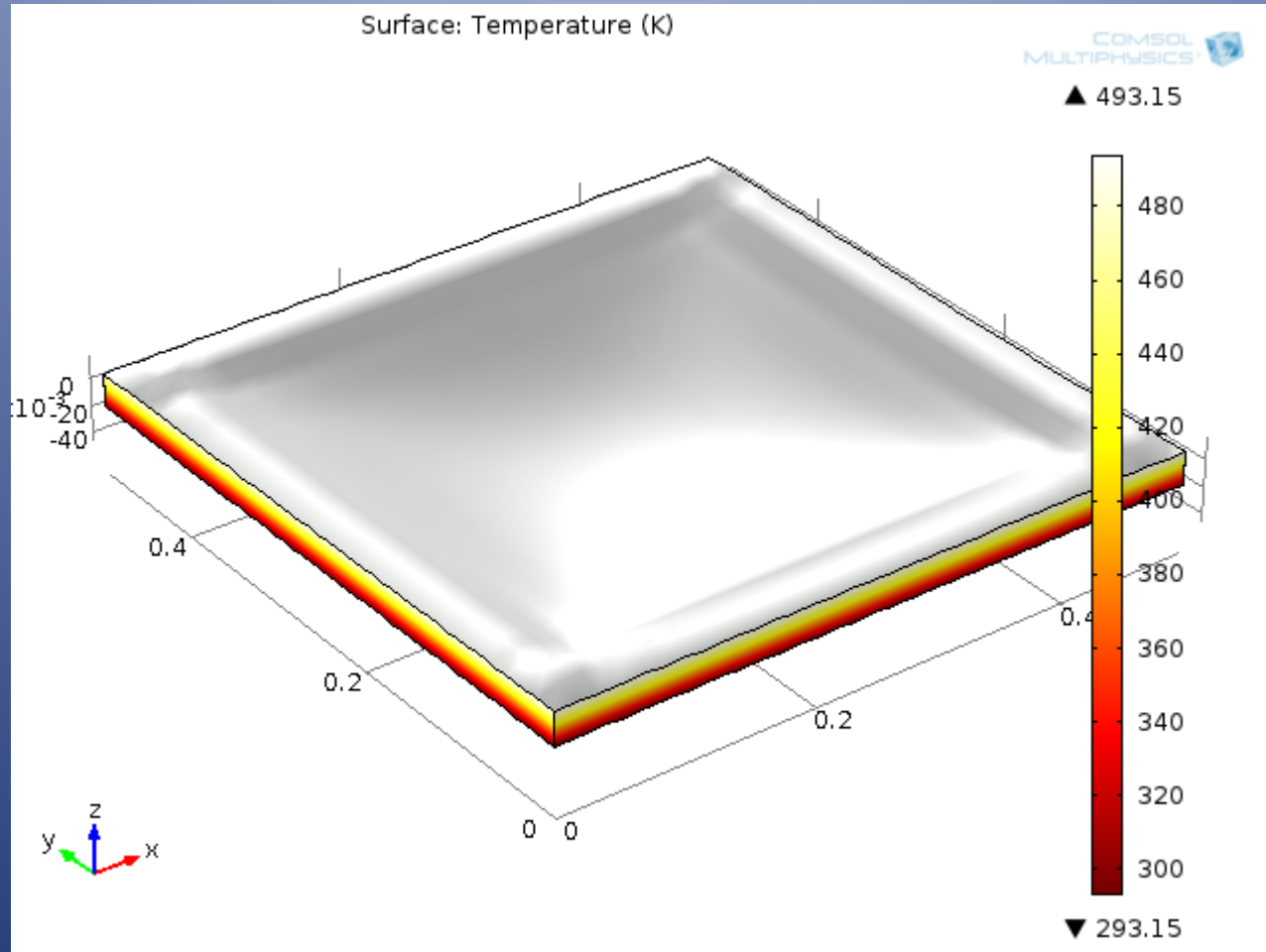


# Mesh

COMSOL  
MULTIPHYSICS®

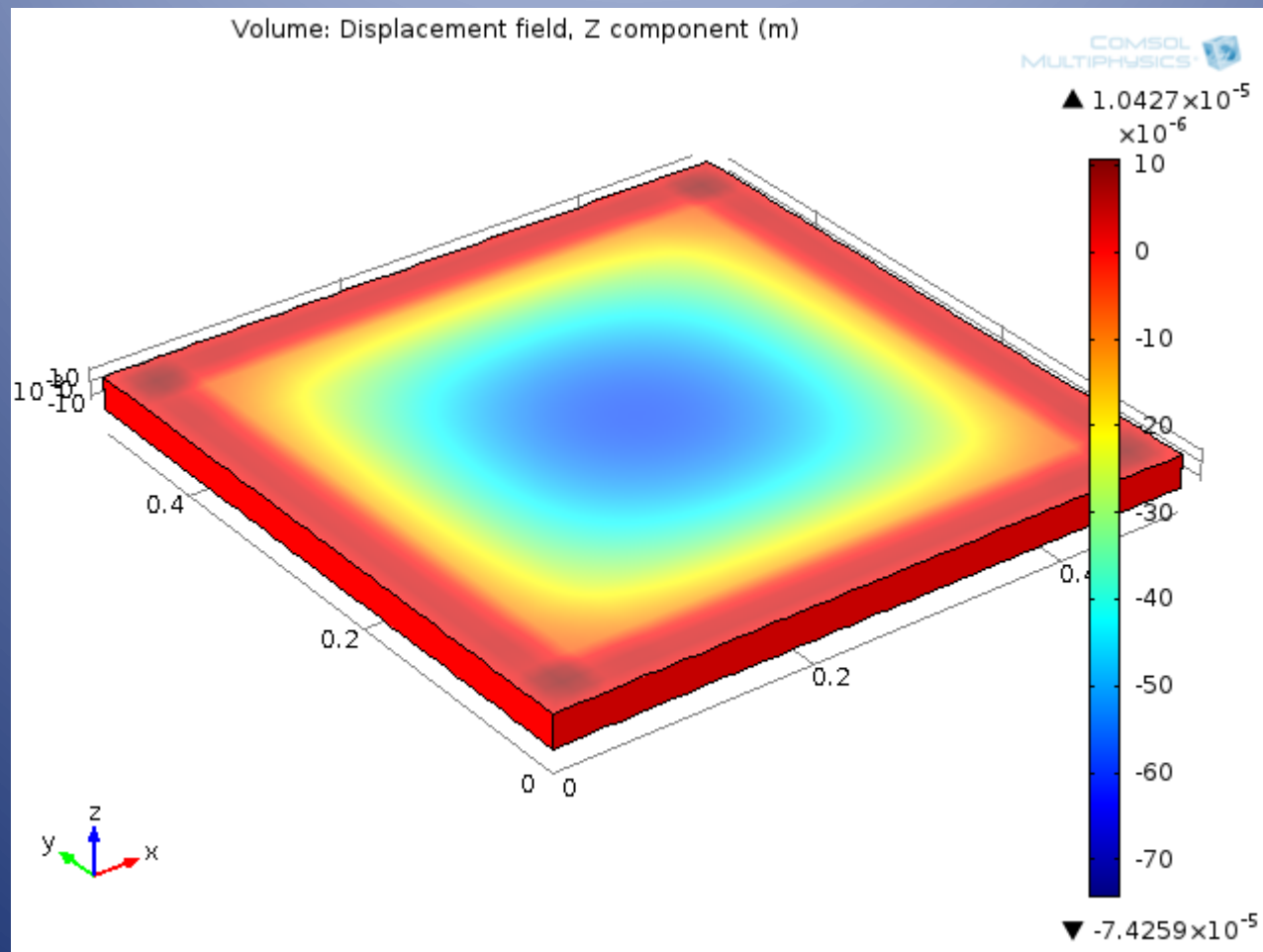


# Temperature



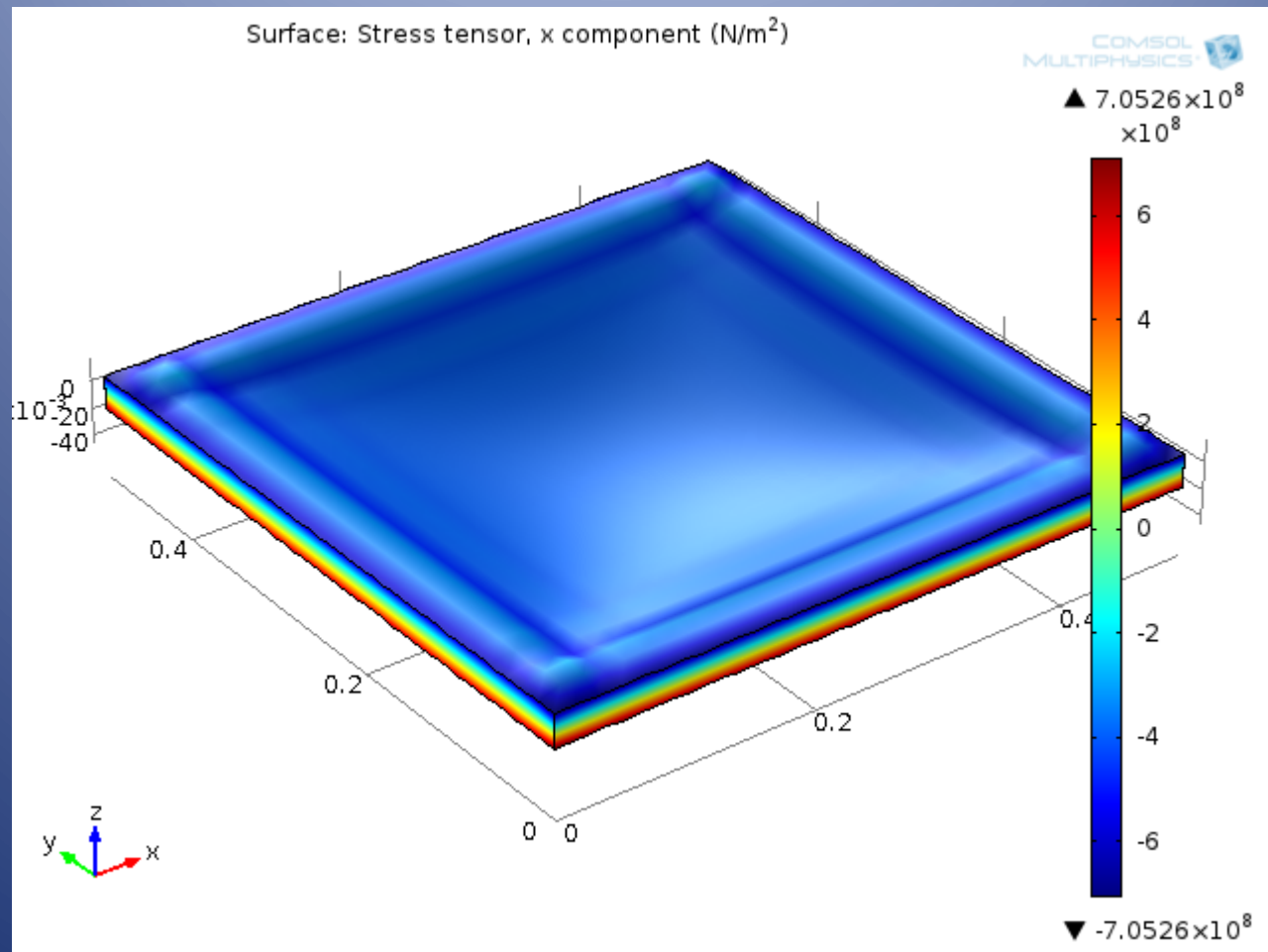
# Transverse Displacement

All Metal - Four Sides Clamped



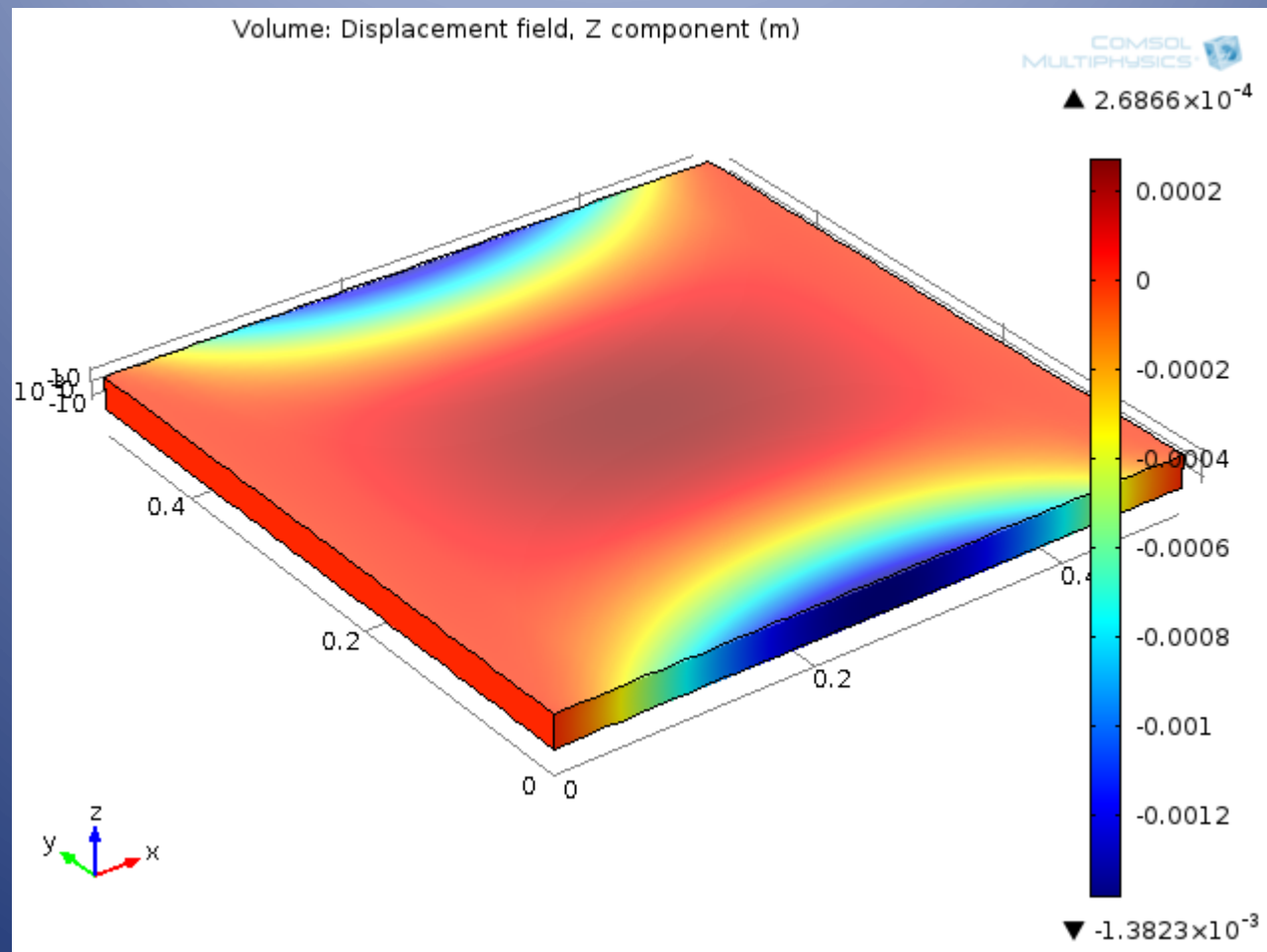
# x-Stress

All Metal - Four Sides Clamped



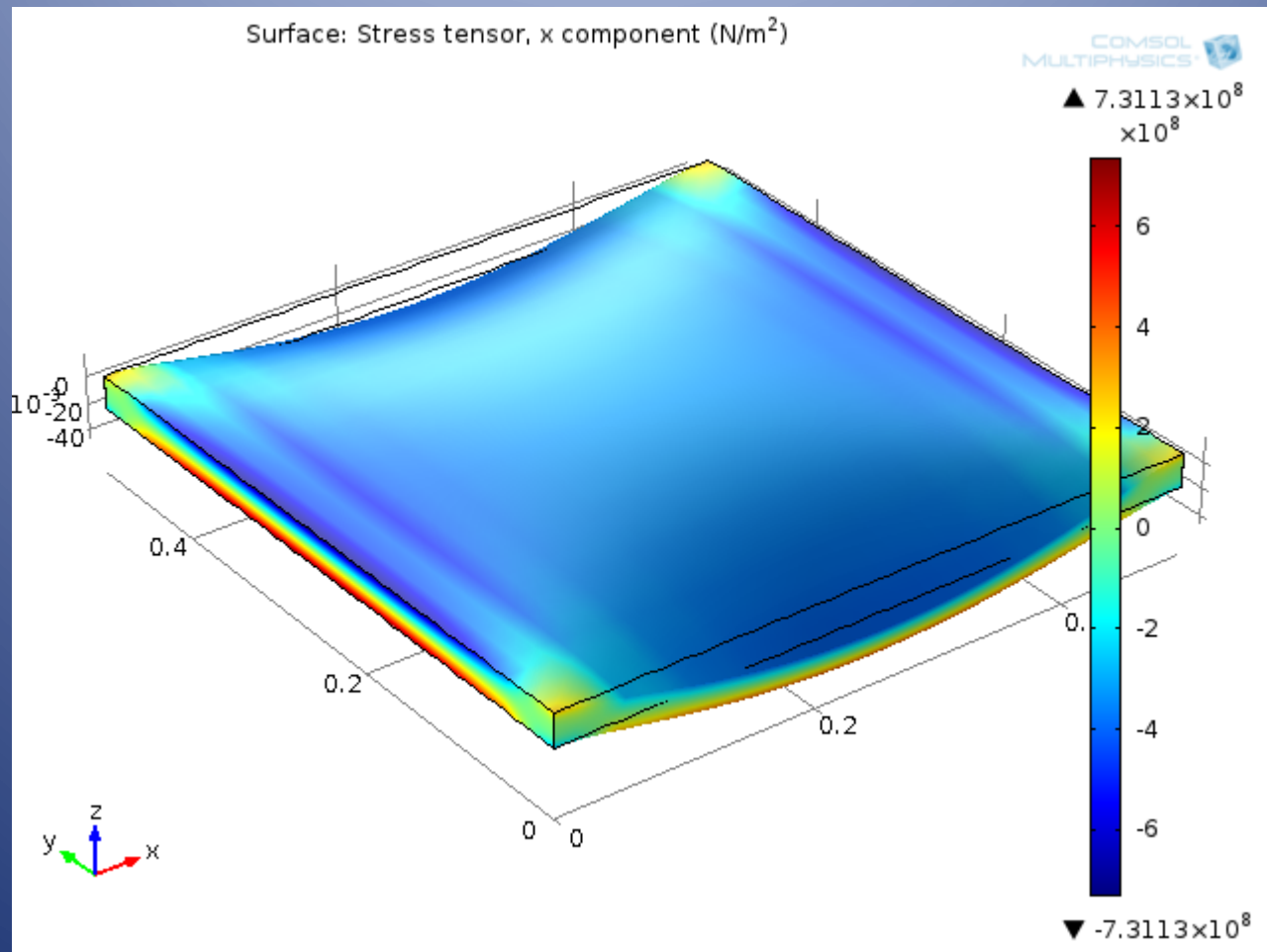
# Transverse Displacement

All Metal – Two Opposite Sides Clamped

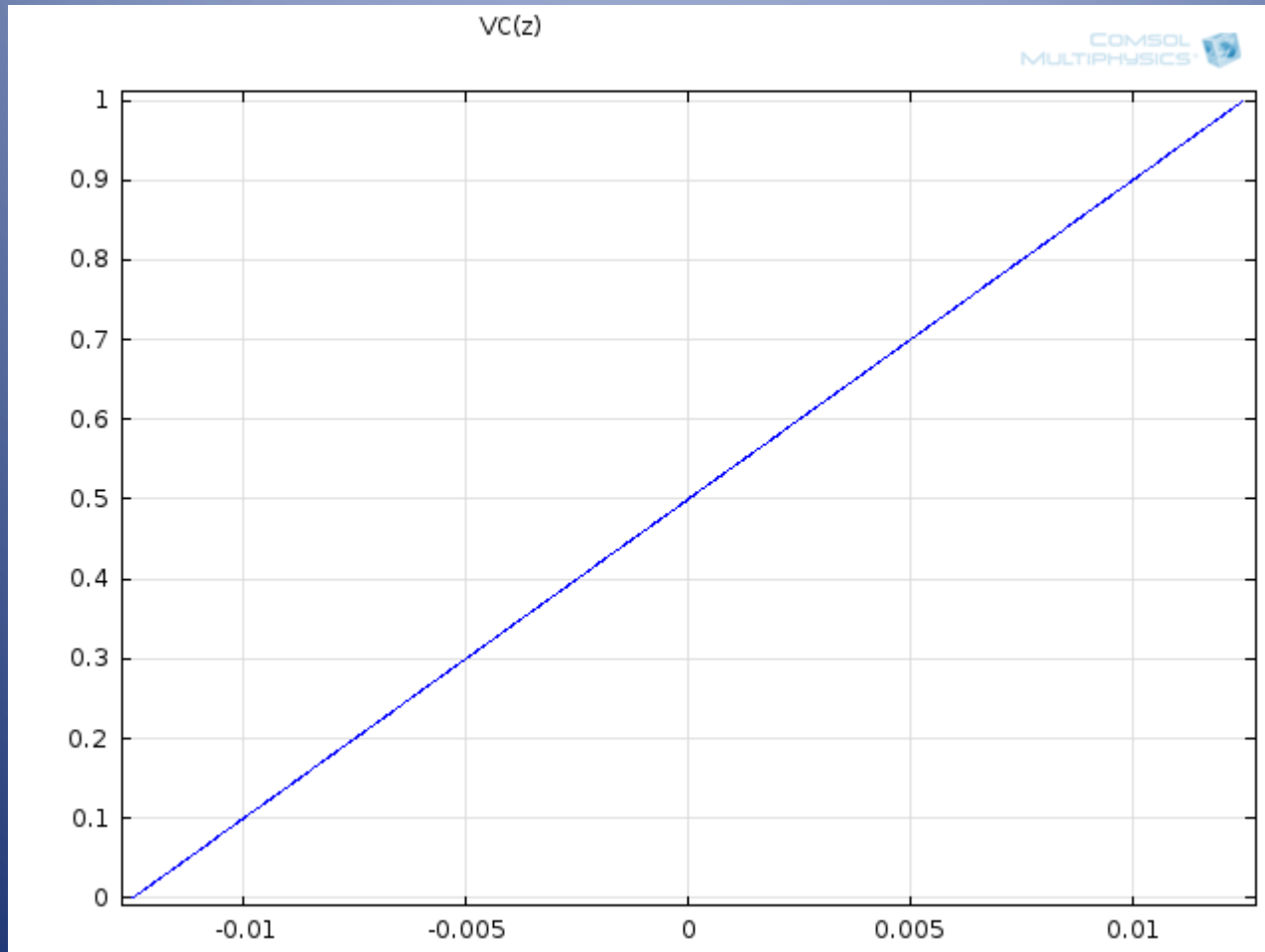


# x-Stress

All Metal – Two Opposite Sides Clamped

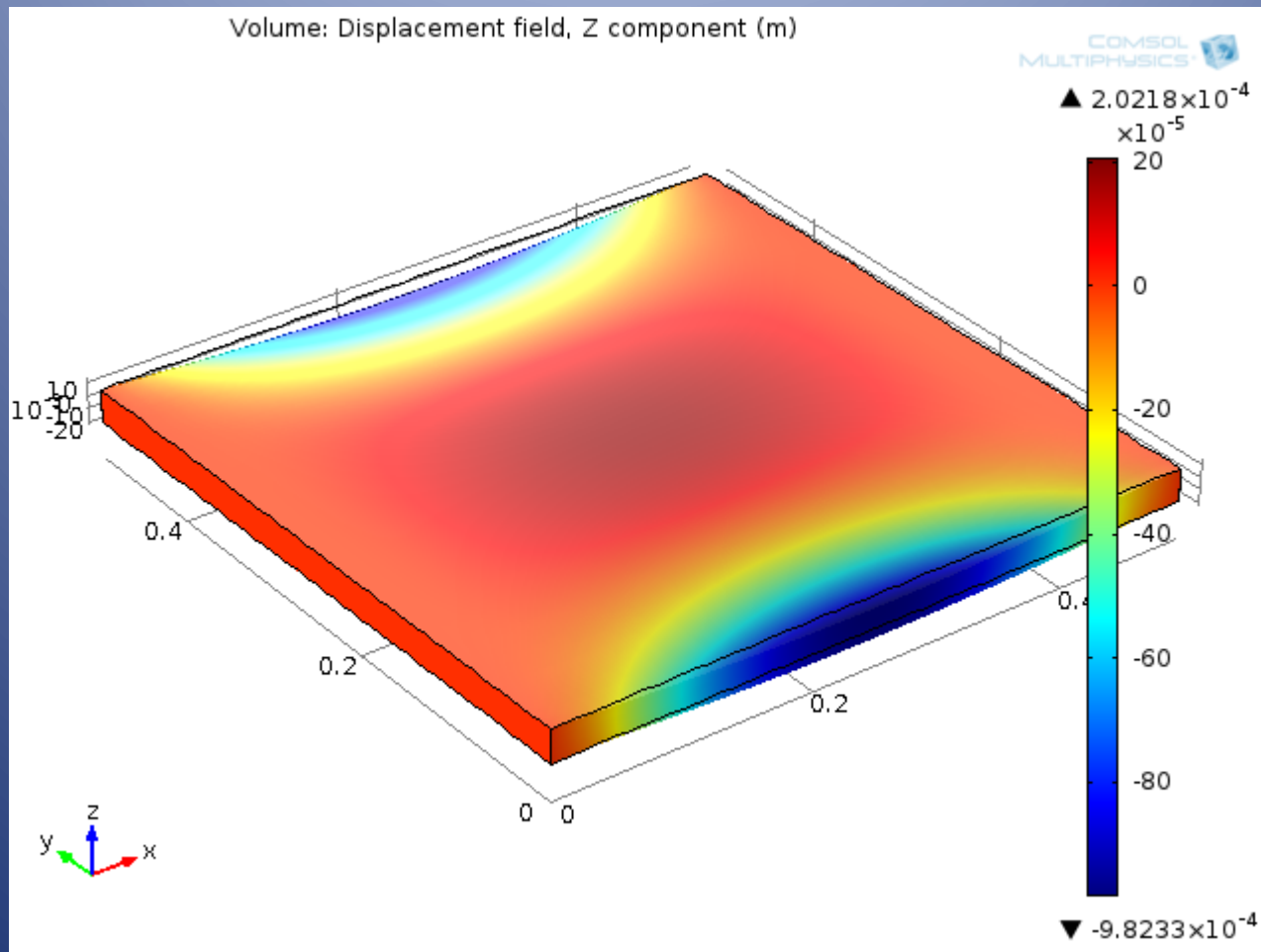


# Volume Fraction of Ceramic through Plate Thickness for $p = 1$



# Transverse Displacement

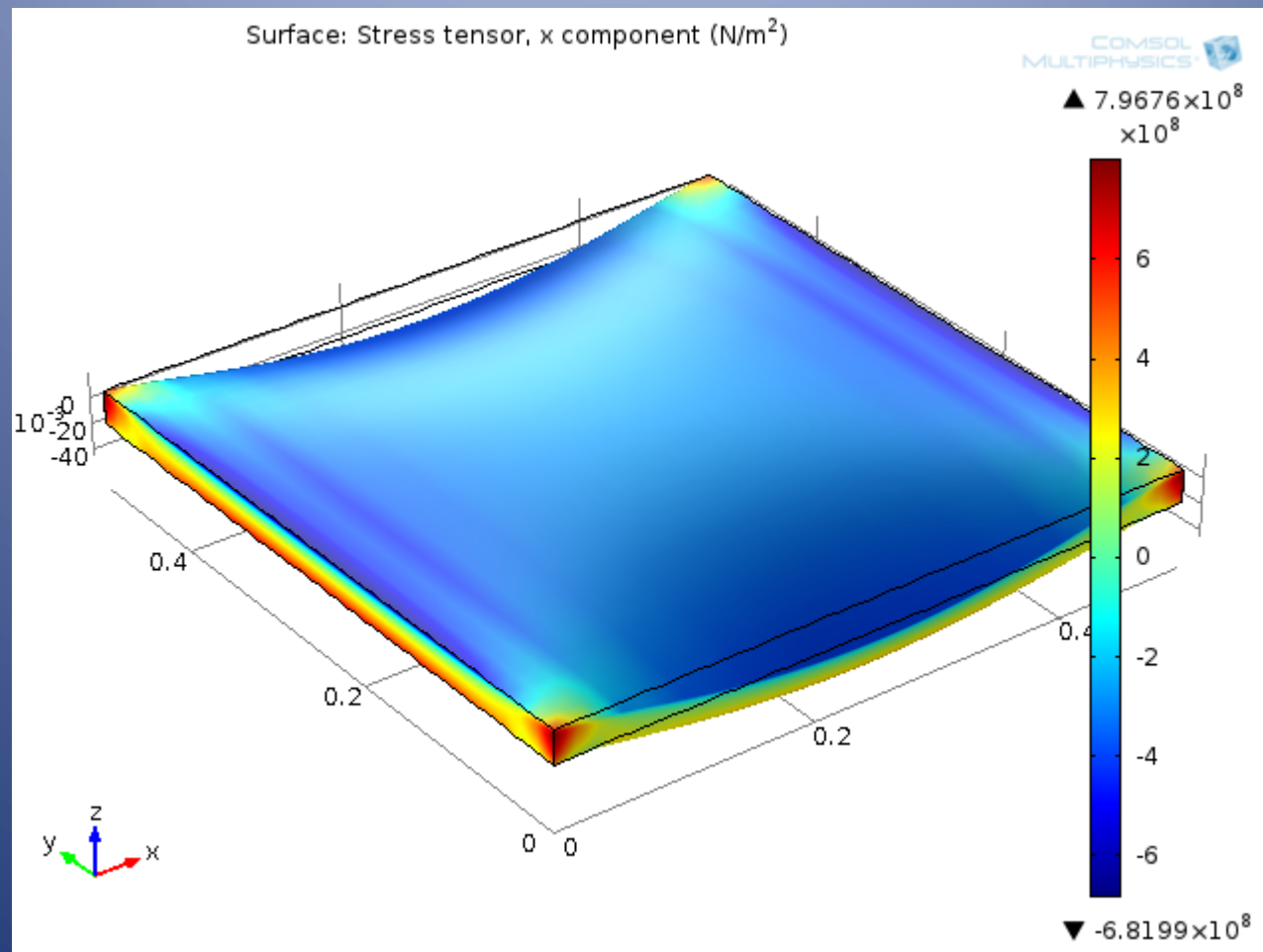
FGM ( $p=1$ ) – Two Opposite Sides Clamped



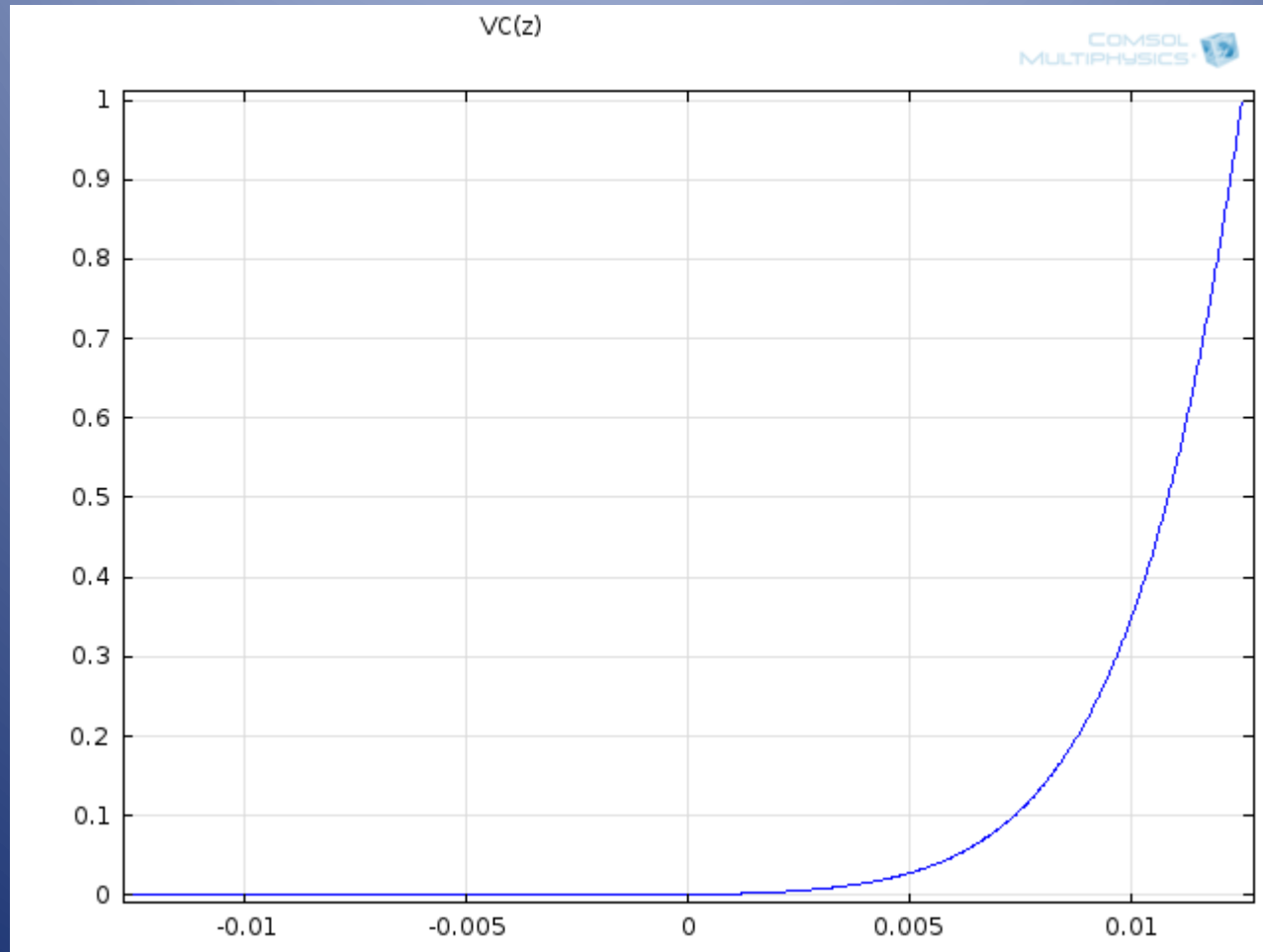


# x-Stress

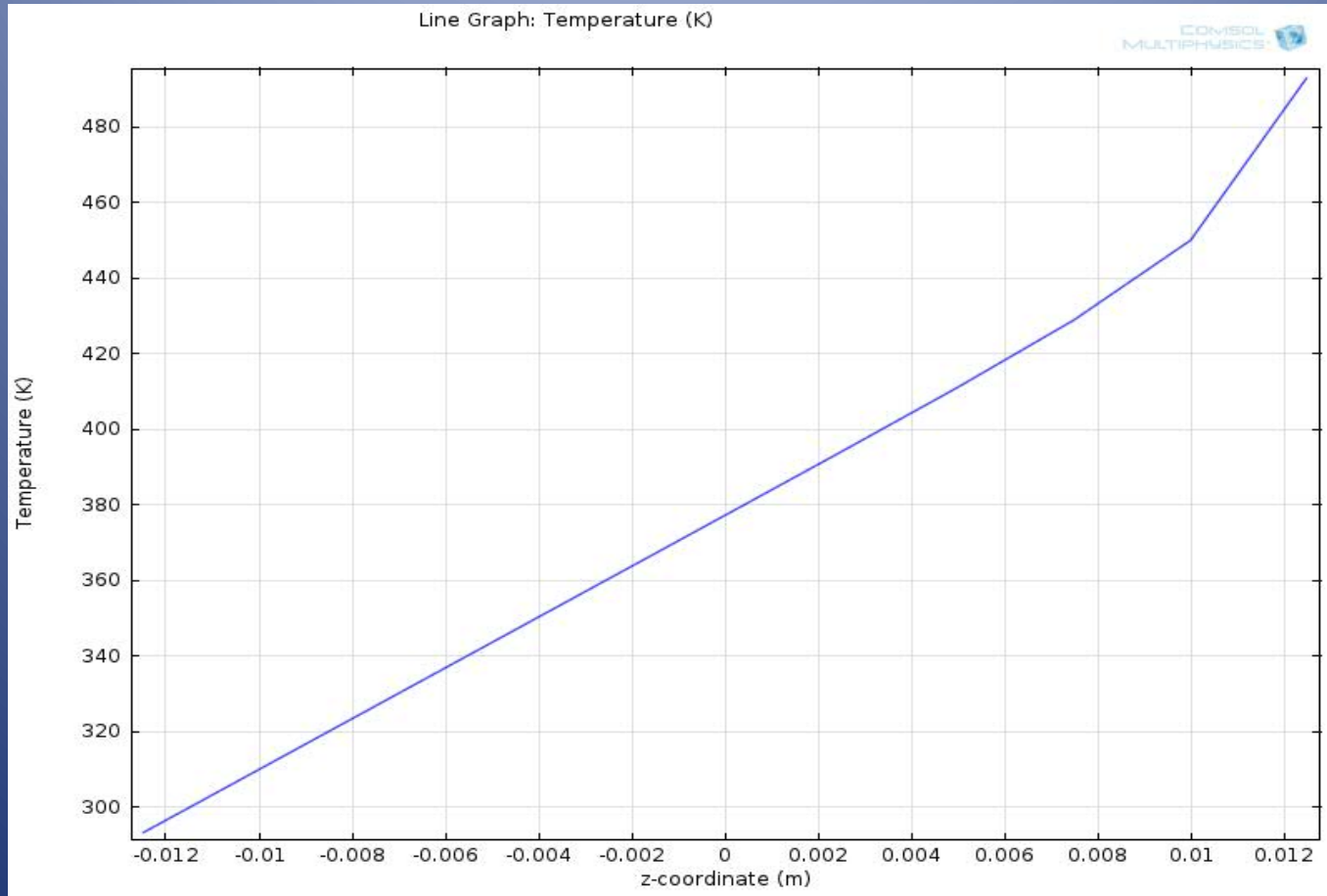
FGM (p=1) – Two Opposite Sides Clamped



# Volume Fraction of Ceramic through Plate Thickness for $p = 10$

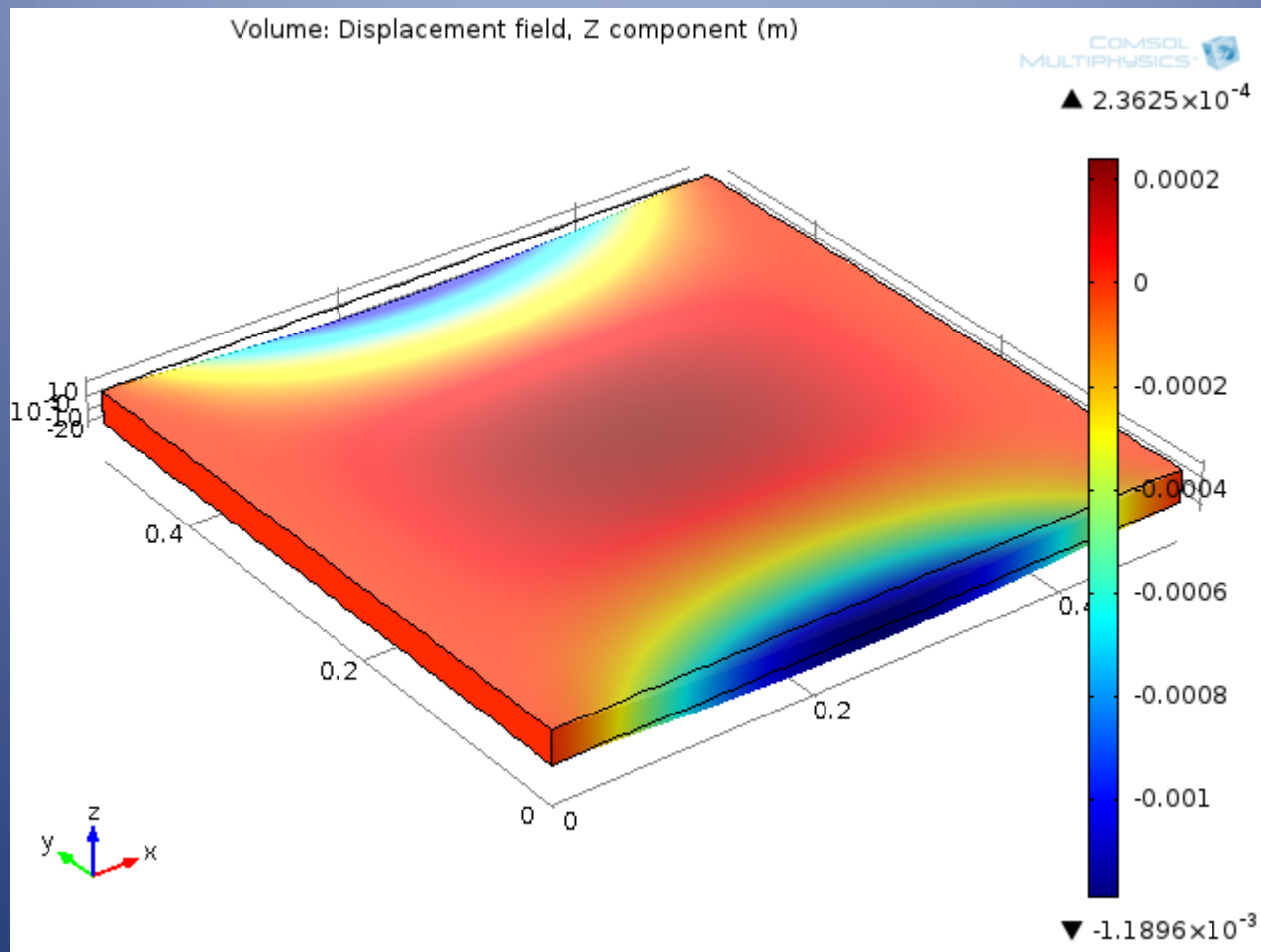


# Computed Temperature through Plate Thickness for $p = 10$



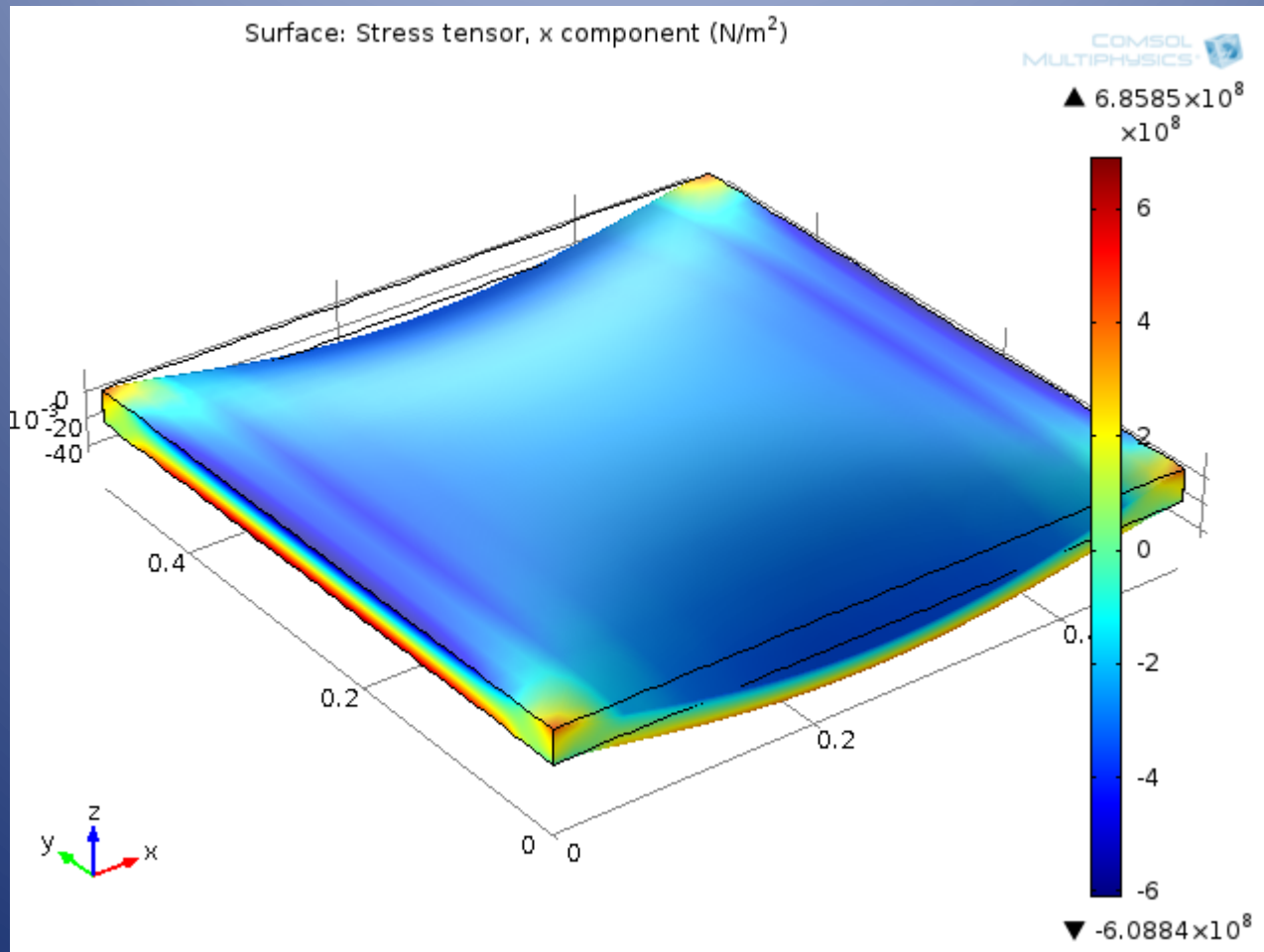
# Transverse Displacement

FGM ( $p=10$ ) – Two Opposite Sides Clamped



# x-Stress

FGM (p=10) – Two Opposite Sides Clamped



# Conclusions

- Linear thermo-elastic analysis of isotropic plates is easily performed using COMSOL Multiphysics
- Linear thermo-elastic analysis of isotropic functionally graded plates is also easily performed using COMSOL Multiphysics when the Mori-Tanaka method is used to compute the material properties.
- Computed results seem to vary smoothly and continuously with changes in the parameter  $p$  controlling the volume fraction of ceramic through the thickness of the plate