How Finite Element Analysis Revolutionized a 100-Year Old Equation

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Abstract

In our view, finite element modeling (FEM) is a major invention in the history of science and technology — as significant at the microscope or telescope. Like the microscope and telescope, FEM extends our senses and instrumentation to new realms of the universe where it was previously impossible or difficult to observe phenomena. Here is an example.

In 1901 Weiss proposed a simple equation to predict the activation of nerve fibers by external electrical stimulation.

$I_{th} = I_{rh} (1+pw/\tau_{ch})$

His original equation and variations have been used for over a century in neuroscience and in the last two decades in neuromodulation, the set of protocols applying electromagnetic fields to modulate neural circuits in humans. Neuromodulation became one of medicine's fastest-growing fields because it works on patients whose conditions are drug-resistant. Now organizations such as DARPA and SmithKline Glaxo have 'electroceutical' initiatives to find stimulation protocols for the entire human nervous system. Consequently, the Weiss equation has become increasingly important in neuroscience.

The y-axis of Fig 1 shows the stimulation amplitude (voltage or current) emitted at the electrode. Then the firing threshold of the nerve fiber ('axon') is measured via a recording electrode. At long pulse widths, less amplitude is required than at short pulse-widths, and the amplitude required if the pulse width is infinite, i.e. the lowest amplitude that will fire the axon, is called the rheobase.

The pulse width that will fire the axon at twice the rheobase amplitude is called the chronaxie, which is the corner point of the strength-duration curve and the point of minimum energy required to fire the axon.

The amplitude used in the Weiss equation and strength-duration curves was always measured at the electrode emitting the signal. What stimulus amplitude actually reaches the axons is unknown and difficult to measure in vivo. Therefore strength-duration curves were always relative, not absolute, since the amplitude required at the electrode to stimulate the axon was subject to geometry (e.g. the separation between electrode and

fiber), material conductivities, the configuration of electrodes if an array were used, and waveform. In the model built using the COMSOL Multiphysics® software (Fig 2), four different different electrode configurations are applied to the spinal cord with and without electrically resistive scarring under the electrodes.

However, finite element modeling can examine the microcosm of the axon and the electric field along its surface. FEM thus renders a physical quantity that was previously unobservable, observable, like Galileo's observation of the surface of the moon with a telescope, or van Leuwenhoek's observation of microbes with a microscope.

Using the COMSOL® software, we measured what is known as the 'activating function' of the fiber (AF) and used that value instead of the electrode amplitude as the rheobase quantity in the Weiss equation. Thus we created absolute, rather than relative, fiber strength-duration curves (Fig 3). We corroborated our theoretical model against clinical data for 8.7 and 11.5 μm fibers and its predictions were within 10% of empirical values.

Reference

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Figures used in the abstract

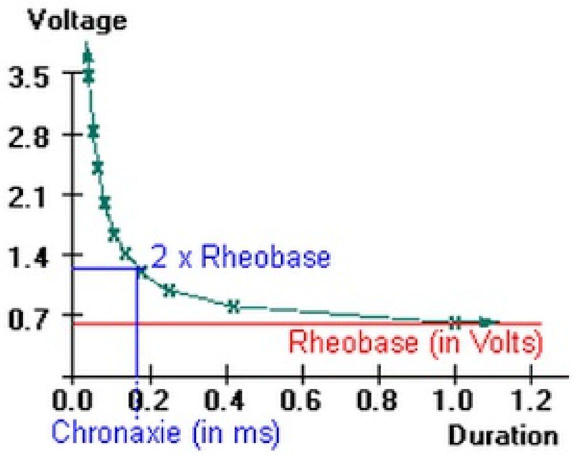

Figure 1: Relative nerve fiber strength-duration curve showing rheobase and chronaxie.

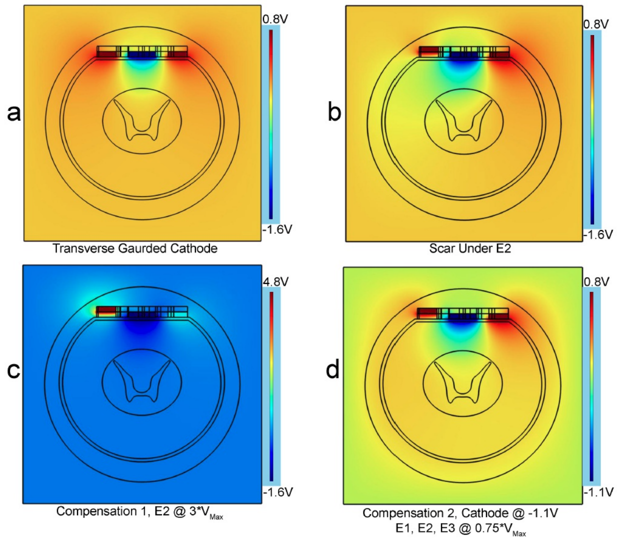

Figure 2: COMSOL model of spinal cord with different electrode array patterns and degrees of electrically resistive underlying scar formation.

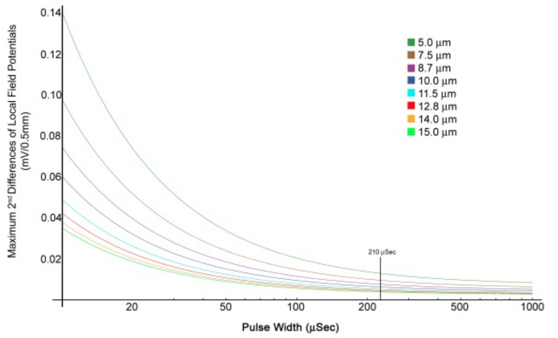

Figure 3: Absolute nerve fiber strength-duration curve derived from electric potential gradient measured in COMSOL model.