## **Optimal Bang-Bang Control in Coupled Dynamical Systems**

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## Abstract

In many dynamical systems the behavior can be changed or even optimized by a suitably chosen external parameter. However, the complexity of the dynamics grows significantly if coupled systems are considered. As an example we examine the coupled dynamics of cancer cells and healthy cells during fractionated radiation therapy. In this method the iterative sequence of radiation and breaks leads to a destruction of tumor cells while keeping the destruction in the surrounding healthy cells at a low level. Thereby one takes advantage of the fact that the healthy cells recover to a higher degree in the time interval between the radiation treatments. The resulting optimum control problem is solved using bang-band control, i.e. the doses rate switches between a lower and an upper bound.

Finding the optimum switching times requires a suitable numerical approach. We here present results of optimized control of the cancer growth on the basis of the COMSOL Multiphysics® Optimization Module.

The model equations describe the dynamics of cancer cells and healthy cells (e.g. in the environment of a tumor) during radiation treatment in the linear response regime. They take into account, in particular, the partial recovery of cells affected by the radiation, a different sensitivity of tumor and healthy cells to radiation and the tumor growth rate. The simulations using the Optimization Module finds, for a given parameter, the radiation treatment that minimizes the damage to healthy cells by significant reducing the cancer cells.

## Figures used in the abstract



**Figure 1**: Coupled dynamics of healthy (h) and tumor cells in the intact state (1) and the affected state (2).