Importance of Assembly Discontinuity Factors In Simulating Reactor Cores Containing Highly Heterogeneous Fuel Assemblies

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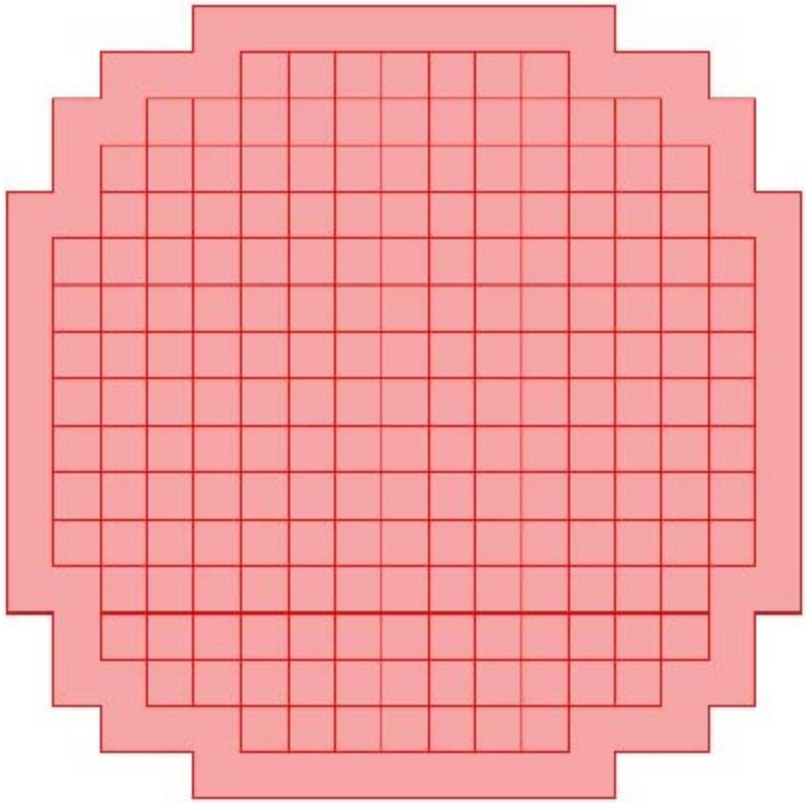
Introduction

- **Most modern reactor core physics codes rely on a solution of the few-group neutron diffusion equation.**
- **To make the computational time manageable, domains over which the equation is solved are often homogenized as much as possible, typically at the fuel assembly or lattice cell level.**
- **Because homogenization affects the solution, various techniques have been developed to counter the effect of homogenization. One technique is through the use of what are now commonly known as assembly discontinuity factors (ADF).**

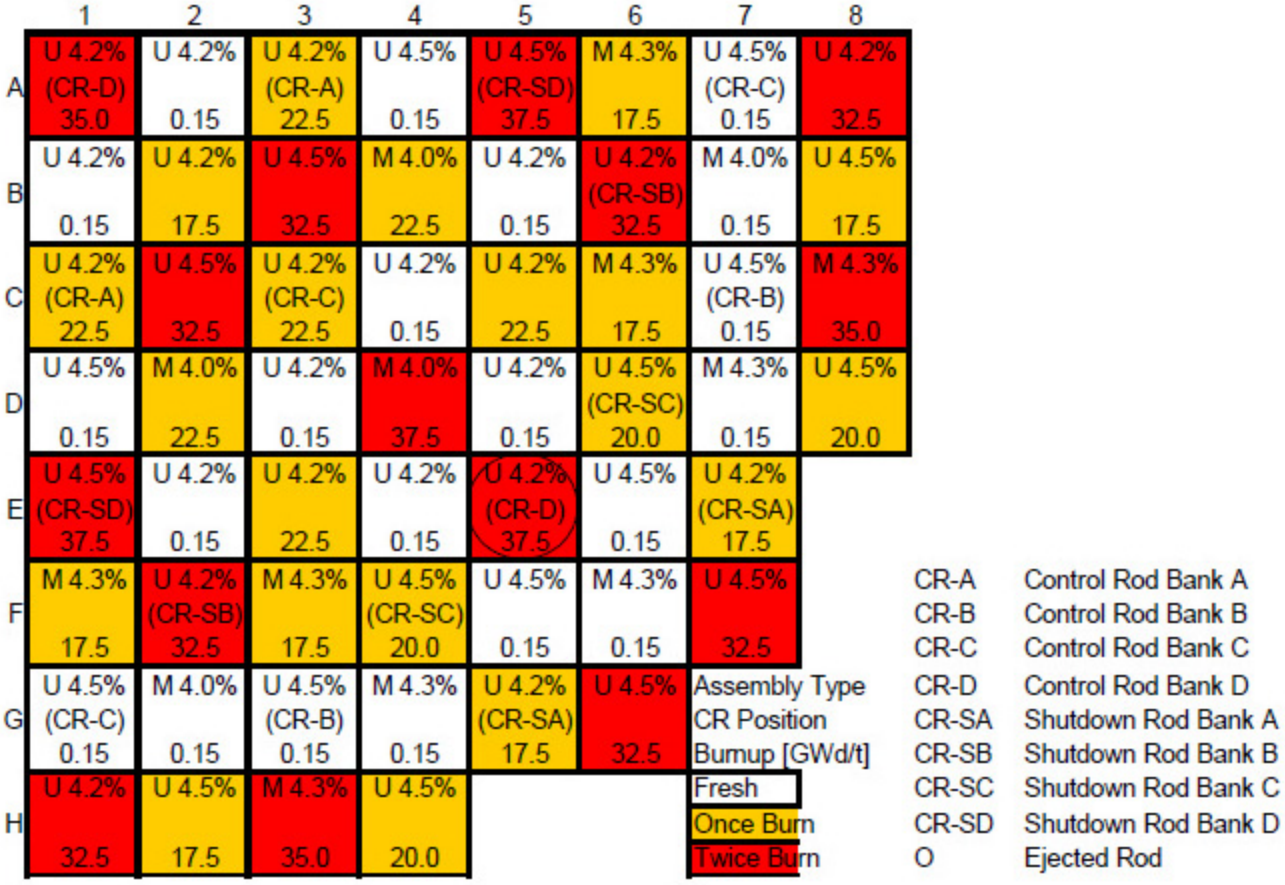
Introduction …continued

- **An analysis was performed, using COMSOL, to examine errors in assembly powers when ADF are not implemented in the analysis of a relatively heterogeneous reactor core.**
- **Such a core is the four-loop Westinghouse (Pressurized Water Reactor) PWR, similar to the reactor chosen for plutonium disposition in the US.**
- **An information package for an international benchmark study was obtained from the (The Nuclear Energy Agency) NEA, containing data for the simulation as well as results obtained by participants in the benchmark.**
- **Assembly powers and k_{eff} (effective multiplication factor) obtained with COMSOL were compared with those from two other codes; PARCS a participant in the benchmark, and the benchmark standard, DeCART.**

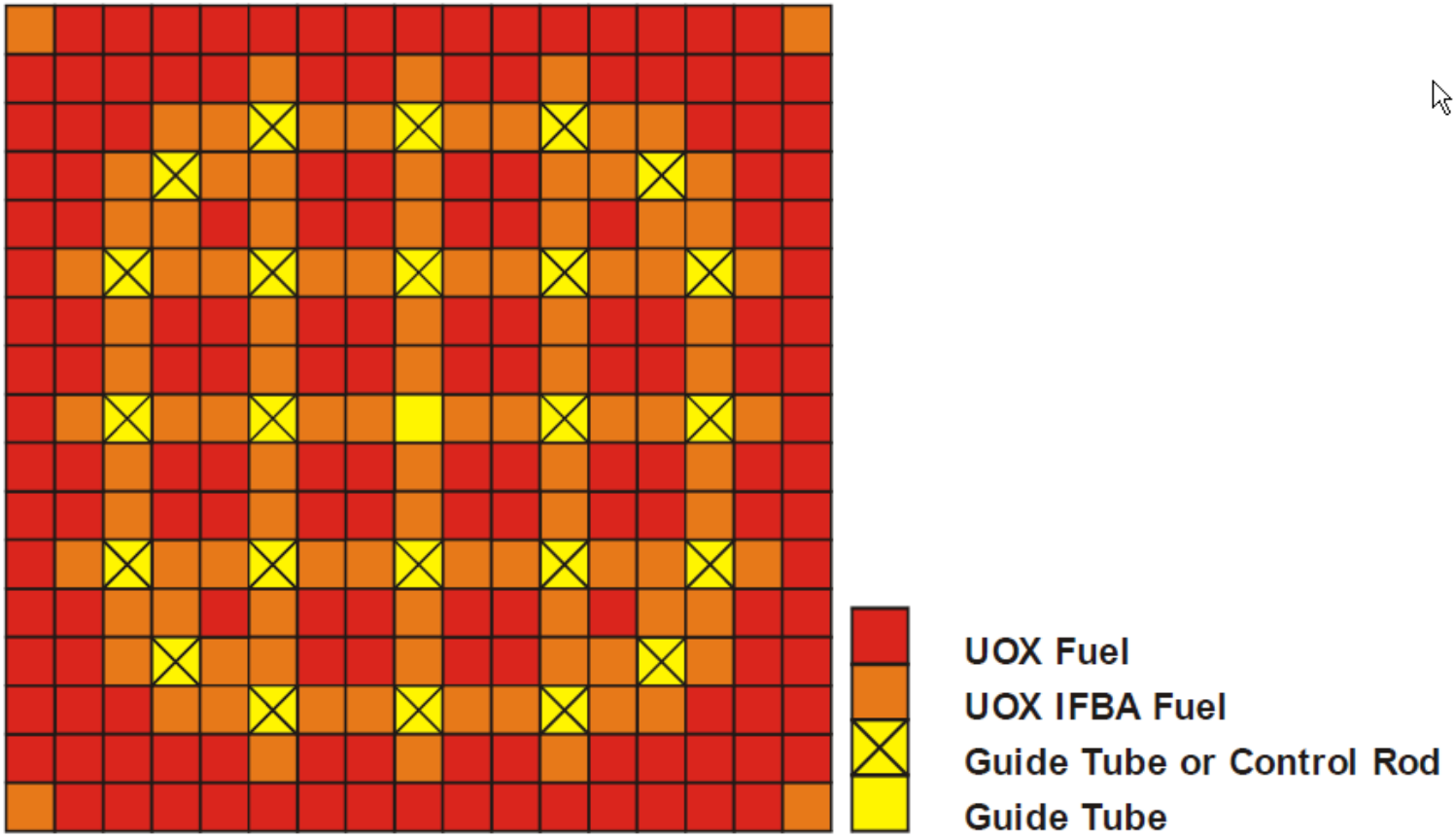
Geometry analyzed



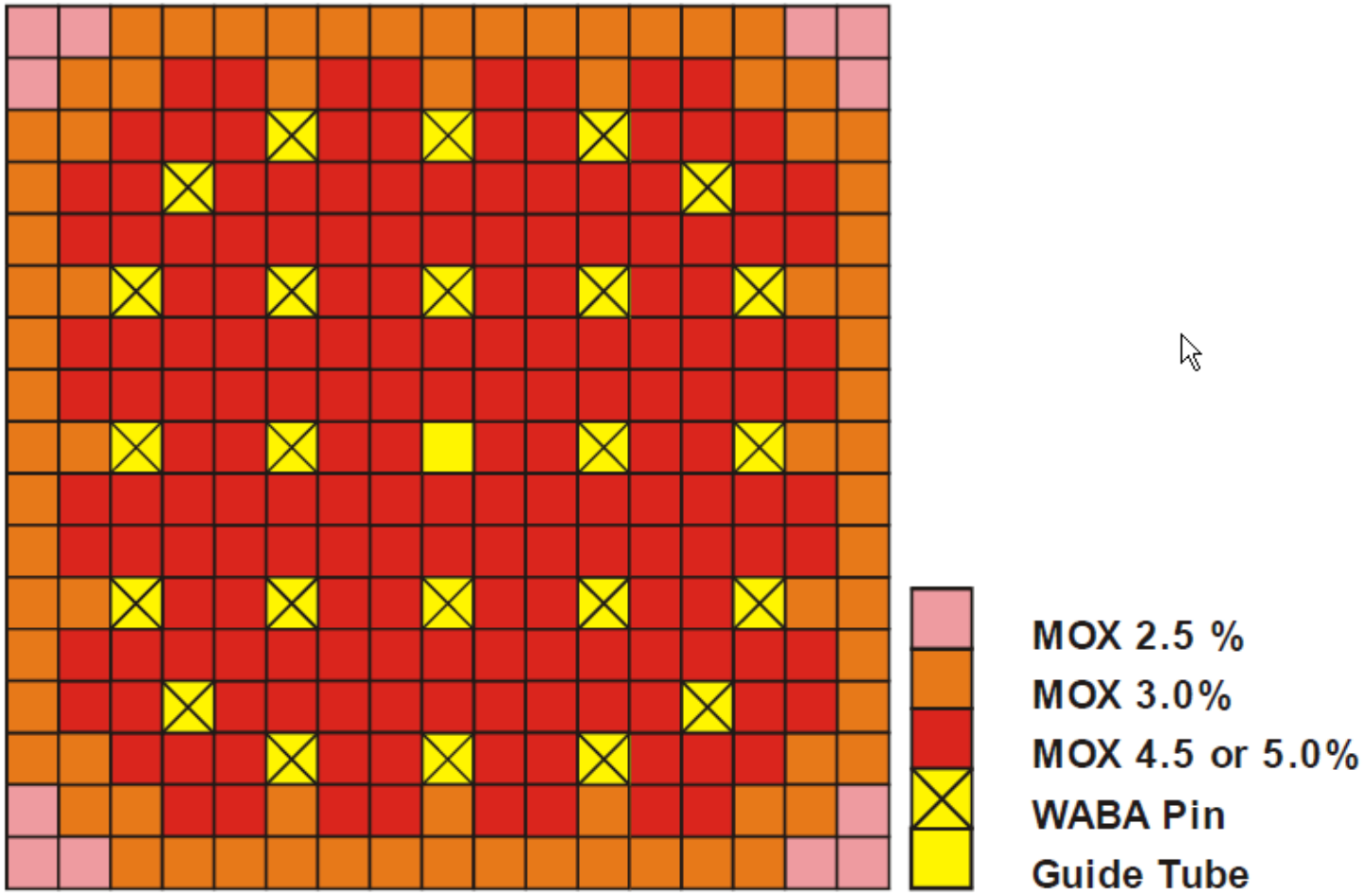
Fuel loading scheme



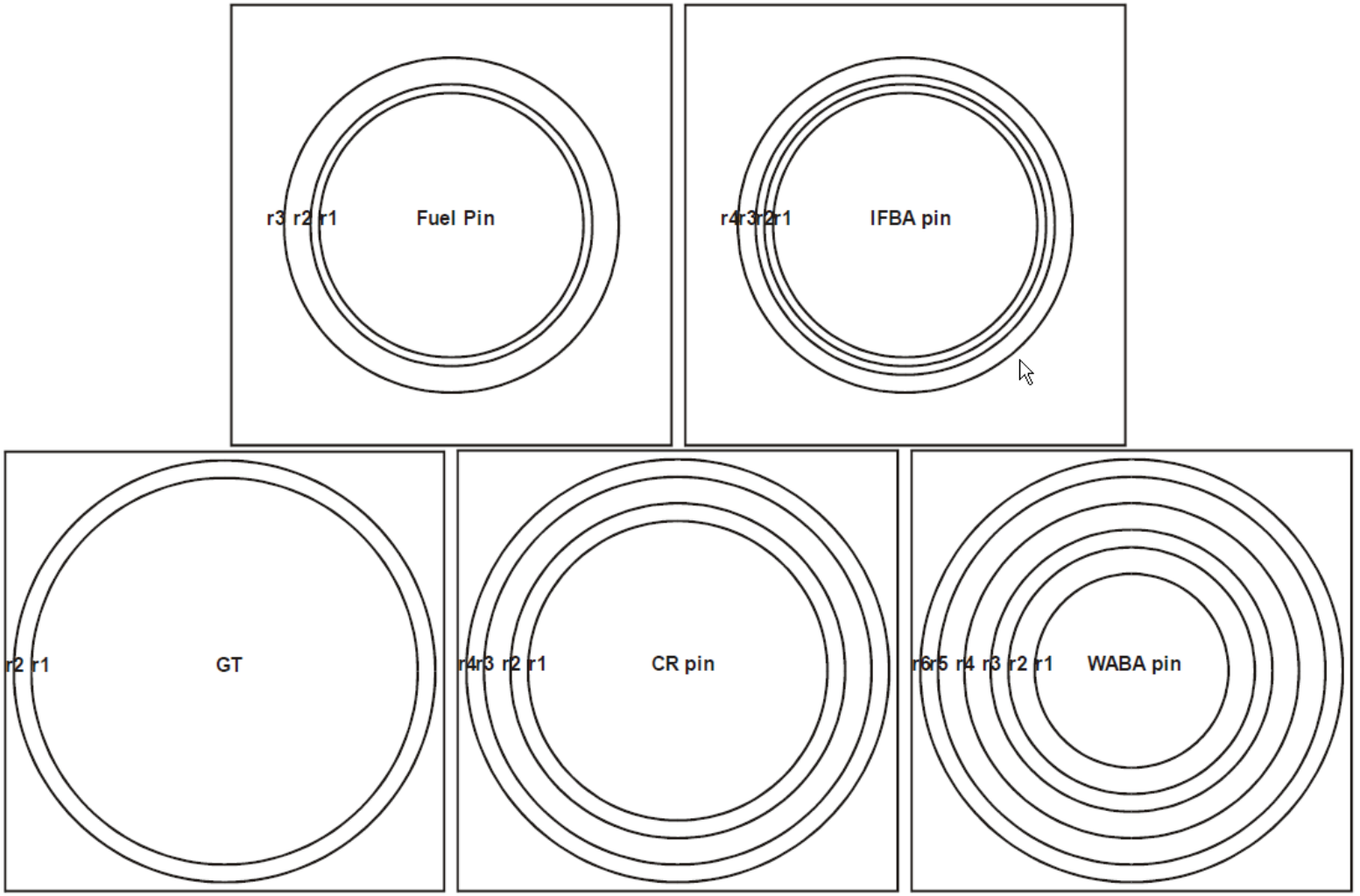
Fine detail within each UO_2 assembly – 17 x 17 pin assembly



Fine detail within each MOX assembly – 17 x 17 pin assembly



Fine detail within each pin



Neutron Diffusion

- **The mathematical problem being solved on the 2D surface is the two-group neutron diffusion equation.**

$$-D_1\nabla^2\phi_1 + \Sigma_{R1}\phi_1 = \frac{1}{k_{eff}}\left[\nu\Sigma_{f1}\phi_1 + \nu\Sigma_{f2}\phi_2\right] + \Sigma_{s21}\phi_2$$

$$-D_2\nabla^2\phi_2 + \Sigma_{R2}\phi_2 = \Sigma_{s12}\phi_1$$

where the removal cross sections are defined as

$$\Sigma_{R1} = \Sigma_{a1} + \Sigma_{s12}$$

$$\Sigma_{R2} = \Sigma_{a2} + \Sigma_{s21}$$

Neutron Diffusion …continued

Equations can be written in matrix notation

$$\begin{pmatrix} -D_1\nabla^2+\Sigma_{R1} & -\Sigma_{S21} \\ -\Sigma_{S12} & -D_2\nabla^2+\Sigma_{R2} \end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}=\frac{1}{k_{eff}}\begin{pmatrix} \nu\Sigma_{f1} & \nu\Sigma_{f2} \\ 0 & 0 \end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$$

This is a generalized eigenvalue equation of the form

$$M\phi = \lambda F\phi$$

Solved using COMSOL's built-in PDE Coefficient Form Eigenvalue mode

ADF theory

- **The principal error with homogenization is that the node-integrated reaction rates in the assembly using a homogeneous model do not match those obtained using a heterogeneous model, which contains the fine detail within each assembly.**
- **An improvement to standard homogenization, widely used for light water reactors, is Generalized Equivalence Theory (GET).**
- **According to GET the node-integrated reaction rates of the fine and coarse models can be matched by allowing for discontinuities in the surface multi-group flux at assembly boundaries.**
- **This is accomplished by a suitable multiplier on each side of a surface called an assembly discontinuity factor (ADF).**

ADF theory …continued

- **The assembly discontinuity factor f is defined as the face-averaged flux heterogeneous flux, to the face-averaged homogeneous flux :**

$$f = \frac{\overline{\psi}}{\overline{\phi}}$$

- **From the requirement for continuity of heterogeneous flux at the face between neighboring assemblies one gets:**

$$\overline{\phi^+} f^+ = \overline{\phi^-} f^-$$

which can be rewritten as

$$\overline{\phi^+} = \overline{\phi^-} \frac{f^-}{f^+}$$

• This shows that the face-averaged flux in neighboring assemblies is dependent on the ratio of the respective assembly discontinuity factors.

Expected Results

- **The last equation suggests that in highly heterogeneous cores, where the ratio of neighboring discontinuity factors deviates significantly from 1, homogenized reaction rates should differ significantly from reaction rates using a detailed or heterogeneous model.**
- **A metric created to gauge the effect of ADF on assembly power was the average ratio of the ADF on all four sides of a fuel assembly to that of its neighbors:**

$$\overline{ADFR}_{(i,j)} = \frac{(ADF_{(i-1,j)} + ADF_{(i,j+1)} + ADF_{(i+1,j)} + ADF_{(i,j-1)})}{4ADF_{(i,j)}}$$

where i and j designate row and column respectively.

Expected Results …continued

- **If the ADF are indeed the cause of a large power error then one would expect a correlation between the metric and power error.**
- **Moreover, since most of the the energy release is from thermal fission, one would expect the correlation to hold only for thermal ADF.**
- **Normalized assembly power in the COMSOL model was computed as**

$$\frac{\iint_{assembly} (\kappa\Sigma_{f1}\phi_1 + \kappa\Sigma_{f2}\phi_2)}{\iint_{core} (\kappa\Sigma_{f1}\phi_1 + \kappa\Sigma_{f2}\phi_2)}$$

Analysis

- **In order to quantify errors, the benchmark study used two metrics for comparison of the results; the power-weighted error (PWE) and the error-weighted error (EWE).**

$$PWE = \frac{\sum_i |e_i| ref_i}{\sum_i ref_i}$$

$$EWE = \frac{\sum_i |e_i| |e_i|}{\sum_i |e_i|}$$

where

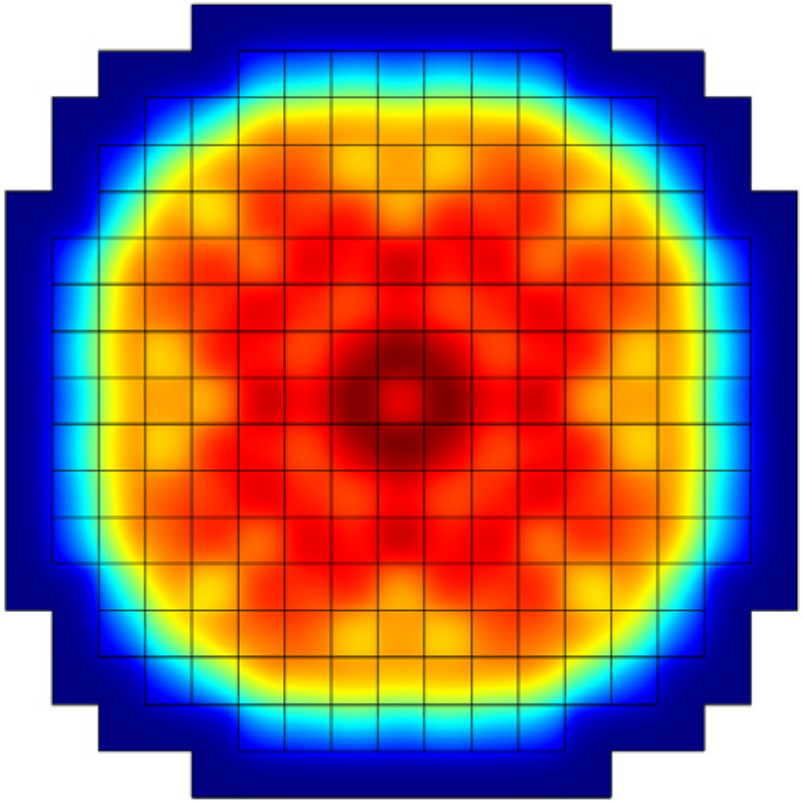
$$e_i = \frac{calc_i - ref_i}{ref_i}$$

and where e_i is the assembly power error and $calc_i$ designates an assembly power from either COMSOL or PARCS and ref_i designates an assembly power from DeCART, the benchmark standard.

Analysis …continued

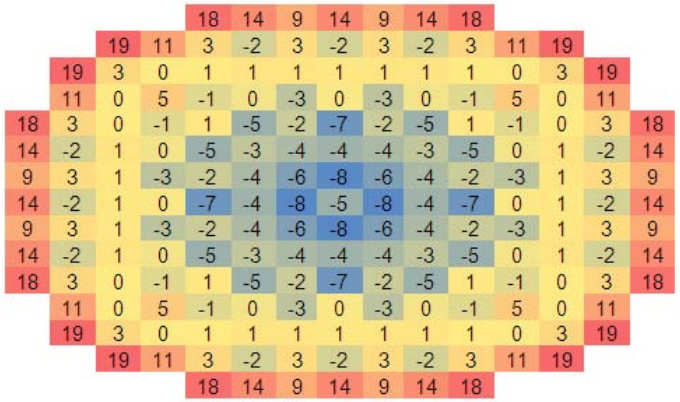
- **The PWE diminishes the importance of the error in the low power region and amplifies the importance in the high power region. The EWE is not linked to the power distribution but only to the magnitude of the error and can therefore magnify the effect of large errors in low power regions.**

Results

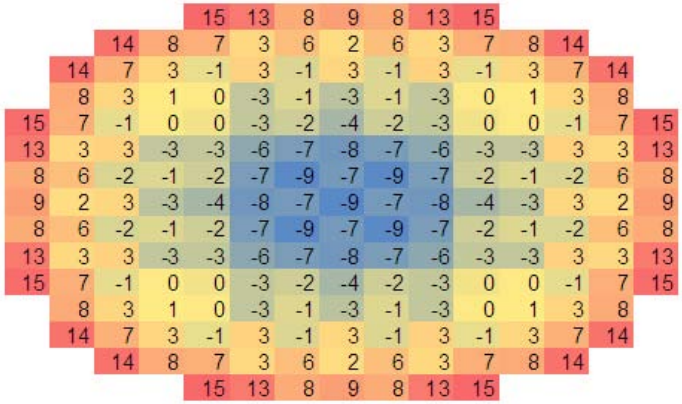


2D surface plot of fast flux

Results …continued



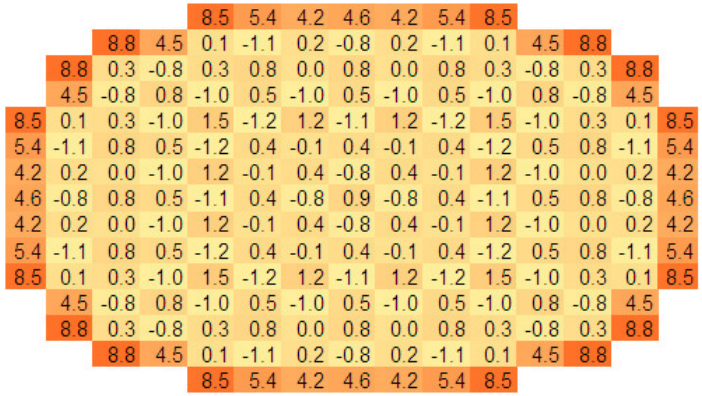
Assembly power errors (%) - COMSOL with no ADF



Assembly power errors (%) - PARCS with no ADF

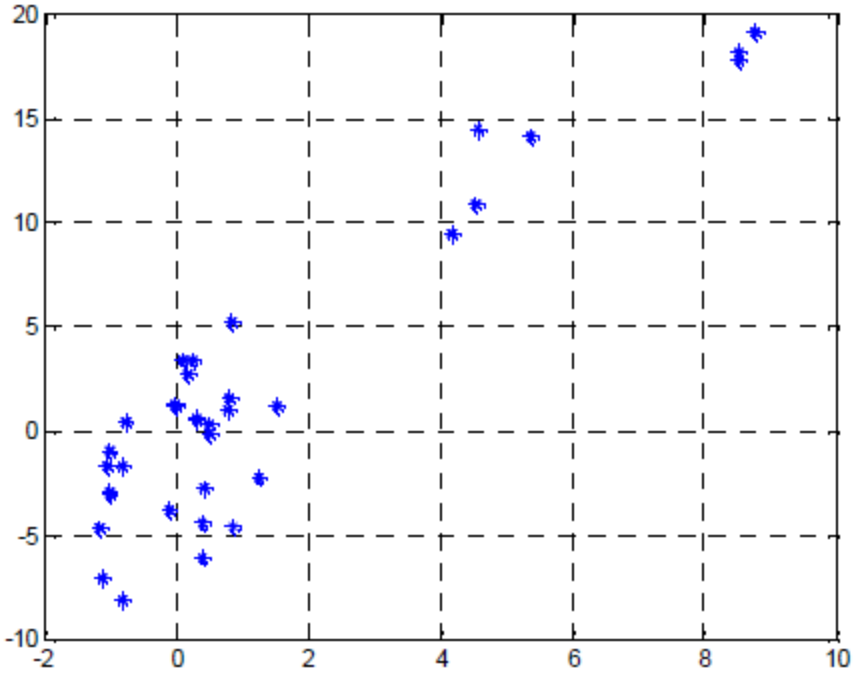
Results …continued

Assembly power errors (%) – PARCS with ADF



Distribution of $\overline{(ADFR-1)*100}$

Results …continued



Plot of COMSOL assembly power error (%) versus $(\overline{ADFR} - 1) * 100$

Results …continued

Assembly Power and Eigenvalue comparison

Code	k_{eff}	Δ mk	%PWE	%EWE
DeCART	1.05852	ref	ref	ref
PARCS with ADF	1.06379	5.27	0.96	1.63
PARCS without ADF	1.06501	6.49	4.33	3.99
COMSOL	1.06500	6.48	3.76	11.0

Conclusion

- **For highly heterogeneous cores such as one with MOX and LEU fuel assemblies, the use of techniques that counter the effect of homogenization, such as ADF, are necessary to reduce power errors to acceptable levels.**