

# Theory of Proportional Solenoids and Magnetic Force Calculation

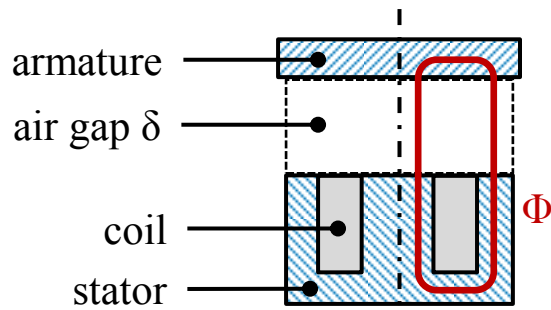
Presented at the COMSOL User Conference 2011 Ludwigsburg, Germany

# Agenda

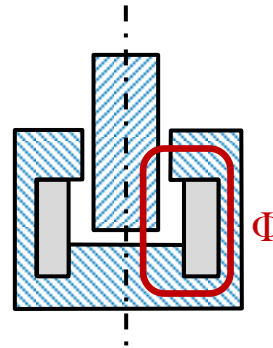
1. Common Electro-magnetic Actuator Types
2. Functional Principle of Proportional Solenoids
3. Analytic Approach
4. Finite Element Approach
5. Summary

# 1. Common Electro-magnetic Actuator Types

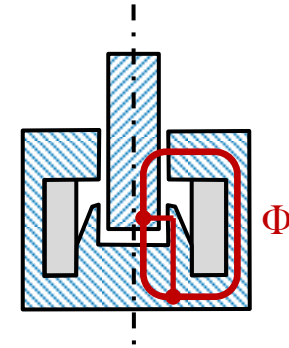
## 1. Magnetic Clamp



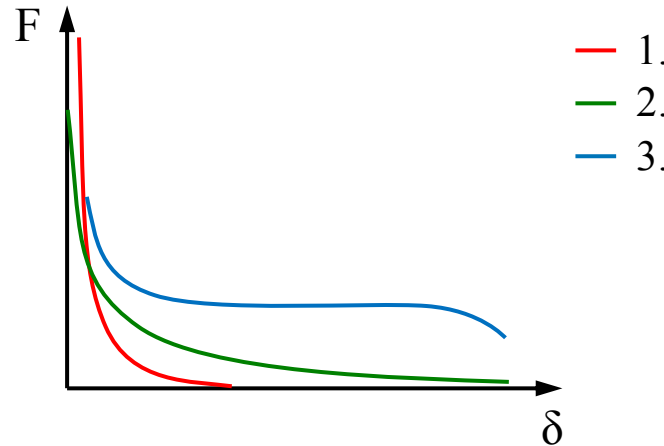
## 2. Common Solenoid



## 3. Proportional Solenoid

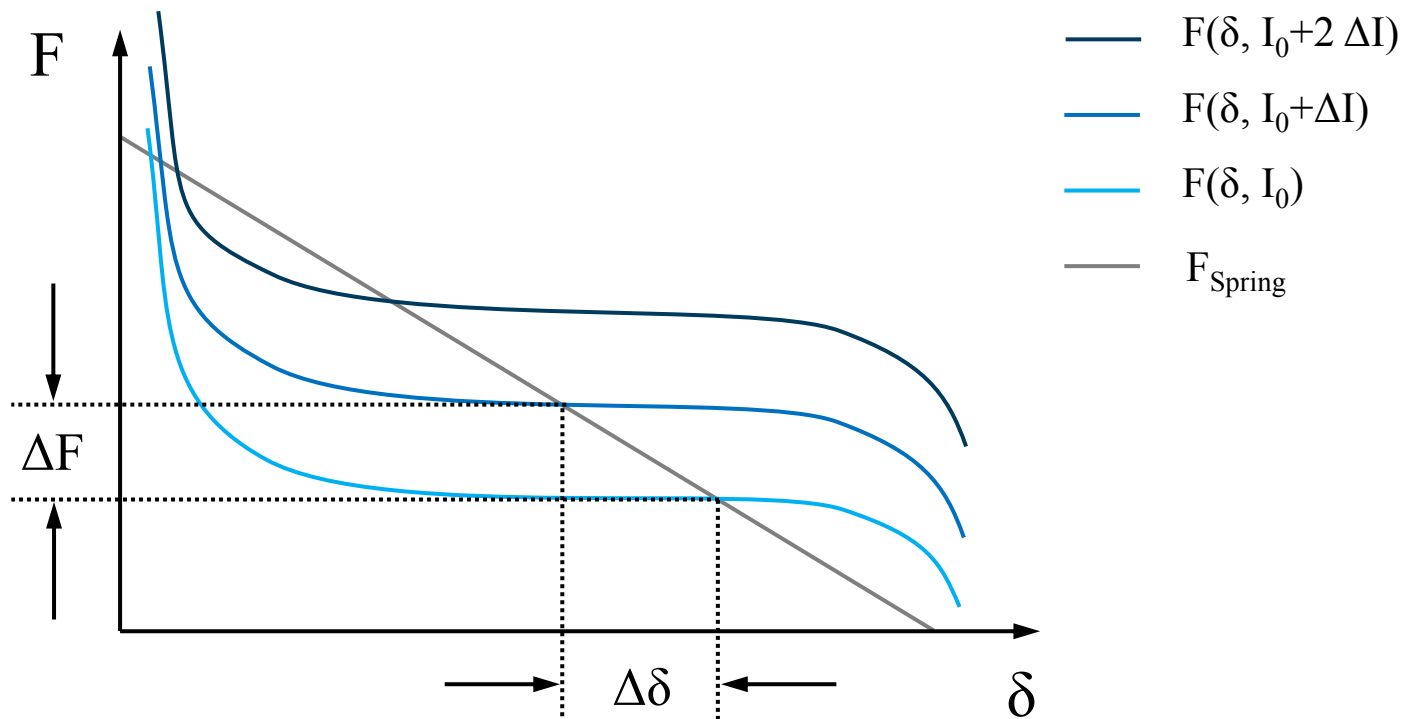


Characteristic  
force-stroke-curves:



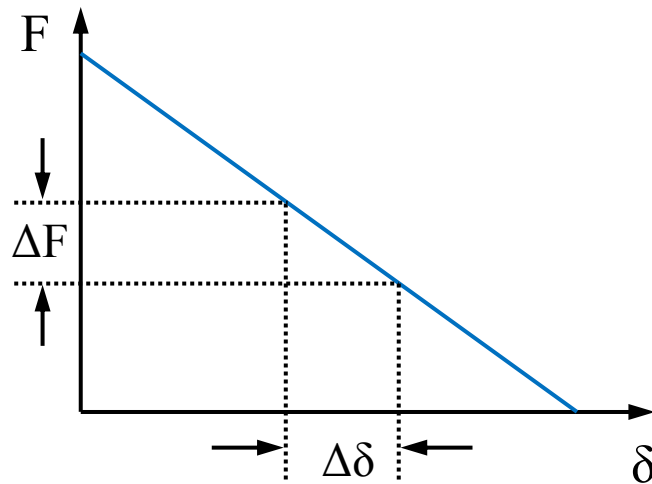
## 2. Functional Principle of Proportional Solenoids

### Interaction of a Proportional Solenoid and a Spring



## 2. Functional Principle of Proportional Solenoids

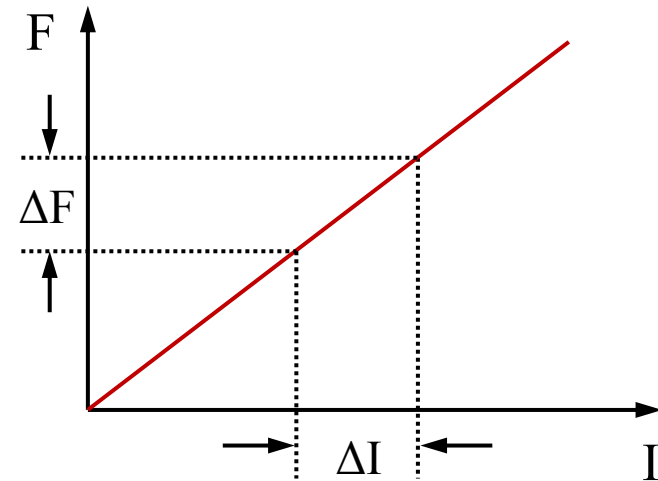
Force- $\delta$ -Curve



$$F \sim \frac{1}{\delta}$$

$$\rightarrow \delta \sim \frac{1}{I}$$

Force-Current-Curve



$$F \sim I$$

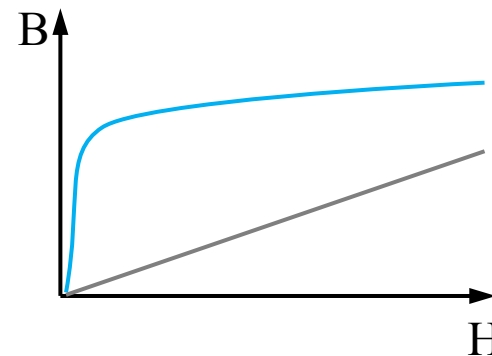
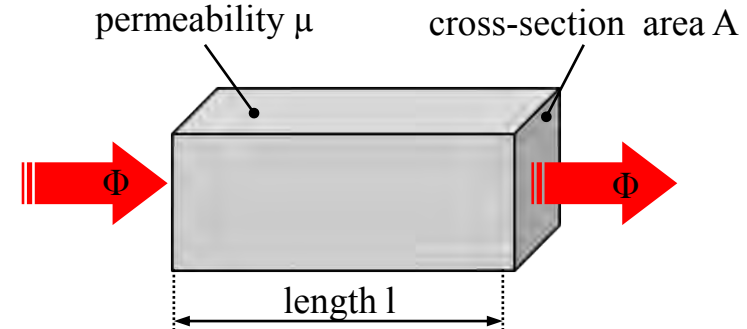
# 3. Analytic Approach

## Introduction of Reluctances

(Reluctances = magnetic resistors)

$$R_{\text{linear}} = \frac{l}{\mu_0 A}$$

$$R_{\text{nonlinear}} = \frac{H l}{B A}$$



— linear material  
— nonlinear material

## 3. Analytic Approach

### Magnetic clamp or common solenoid

Total reluctance:

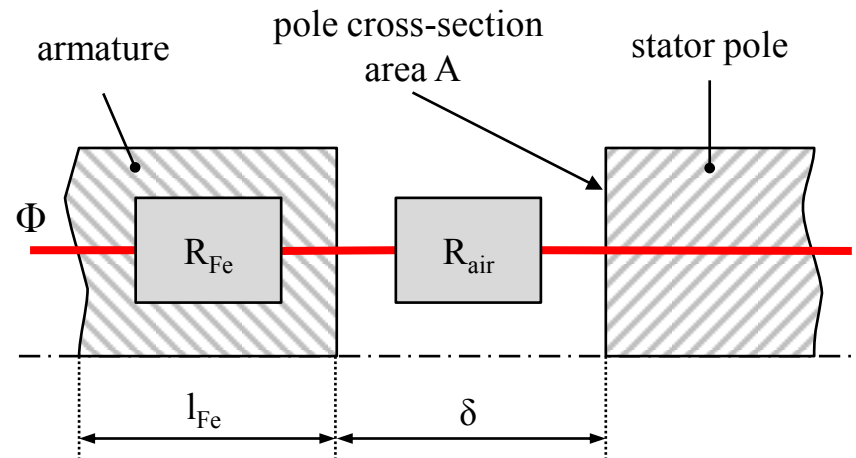
$$R_{\text{tot}} \approx R_{\text{air}} = \frac{\delta}{\mu_0 A}$$

Magnetic flux:

$$\Phi = \frac{\Theta}{R_{\text{tot}}} = \frac{\Theta \mu_0 A}{\delta}$$

Magnetic force (Maxwell Tensile Force):

$$F = \frac{\Phi^2}{2 \mu_0 A} = \frac{\Theta^2 \mu_0 A}{2 \delta^2}$$



### 3. Analytic Approach

#### Proportional Solenoid at Maximum Man Air Gap

Reluctance of main air gap and bypass at  $\delta_{\max}$ :

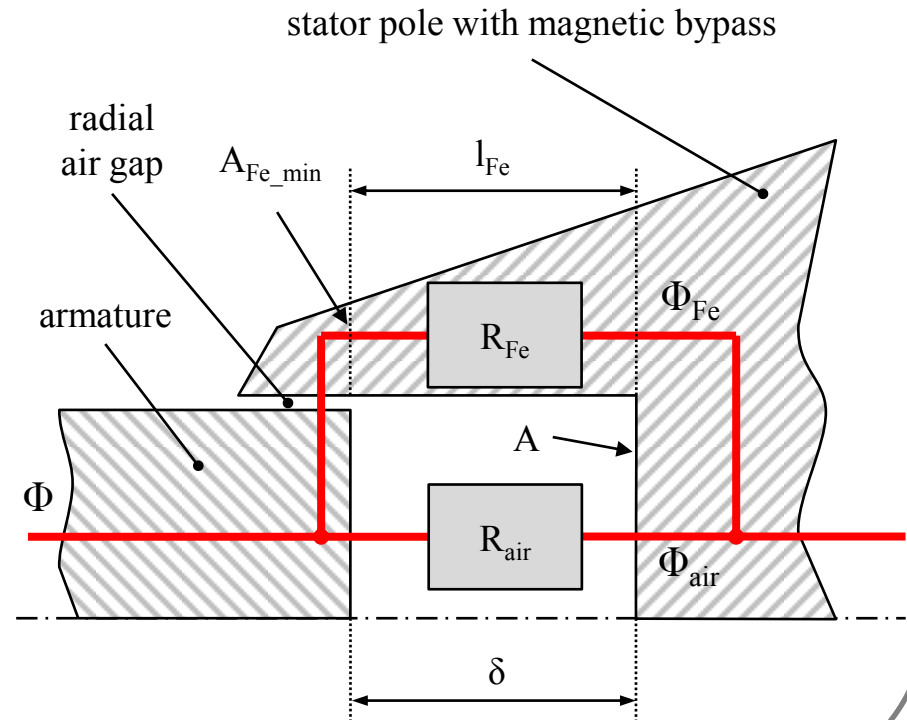
$$R_{\text{air}} = \frac{\delta_{\max}}{\mu_0 A}$$

(low permeability)  $R_{\text{Fe}} \approx \frac{l_{\text{Fe}}}{\mu_0 A_{\text{Fe}}} = \frac{\delta_{\max}}{\mu_0 A_{\text{Fe}_{\min}}}$

Reluctance ratio at  $\delta_{\max}$ :

$$\frac{\Phi_{\text{air}}}{\Phi_{\text{Fe}}} = \frac{R_{\text{Fe}}}{R_{\text{air}}} = \frac{A}{A_{\text{Fe}_{\min}}}$$

$A \gg A_{\text{Fe}_{\min}} \rightarrow \Phi_{\text{air}} \gg \Phi_{\text{Fe}}$





### 3. Analytic Approach

#### Proportional Solenoid at Minimum Main Air Gap

Reluctance of main air gap and bypass at  $\delta_{\min}$ :

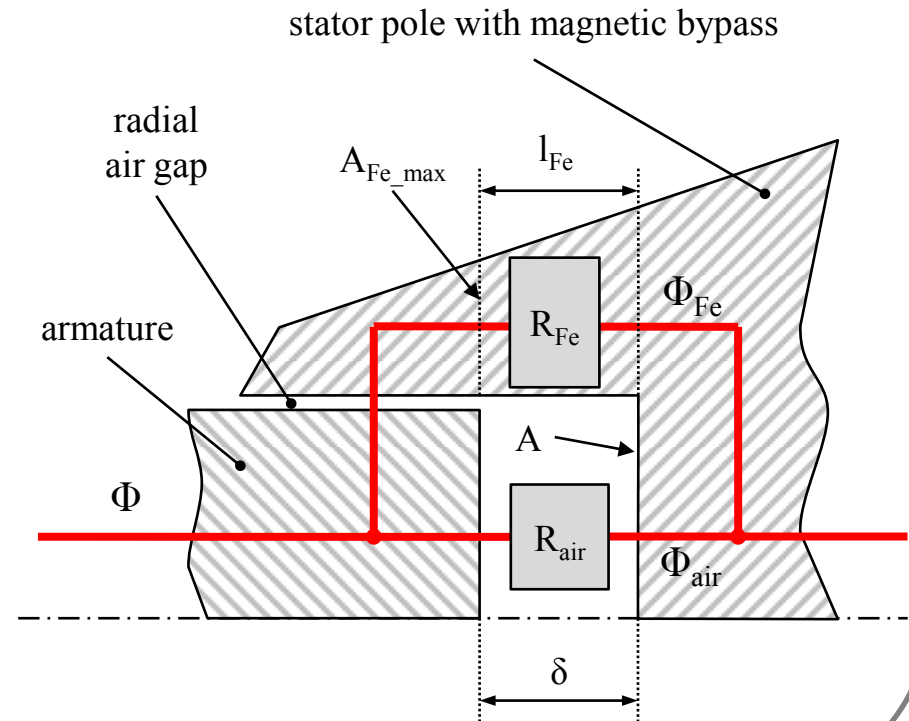
$$R_{\text{air}} = \frac{\delta_{\min}}{\mu_0 A}$$

(high permeability)  $R_{\text{Fe}} \approx \frac{\delta_{\min}}{100 \mu_0 A_{\text{Fe\_max}}}$

Reluctance ratio at  $\delta_{\min}$ :

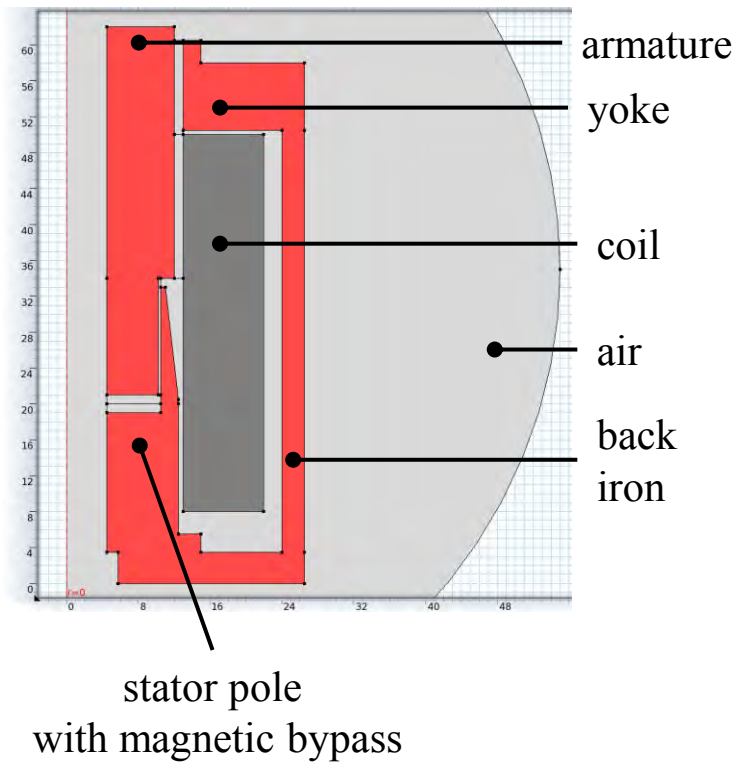
$$\frac{\Phi_{\text{air}}}{\Phi_{\text{Fe}}} = \frac{R_{\text{Fe}}}{R_{\text{air}}} = \frac{A}{100 A_{\text{Fe\_max}}}$$

$A < 100 A_{\text{Fe\_min}} \rightarrow \Phi_{\text{air}} < \Phi_{\text{Fe}}$

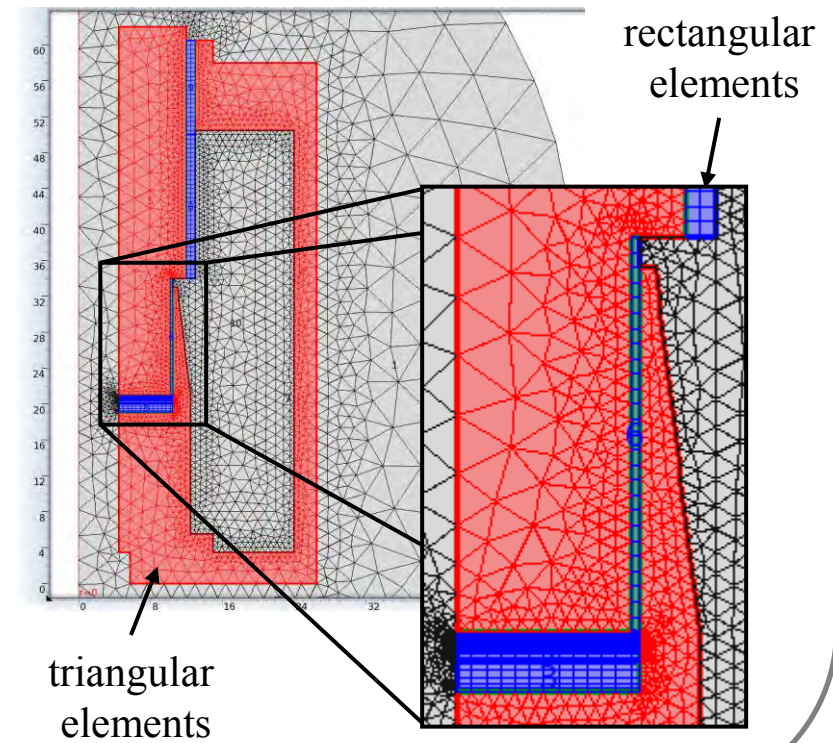


# 4. Finite Element Approach

## Geometry



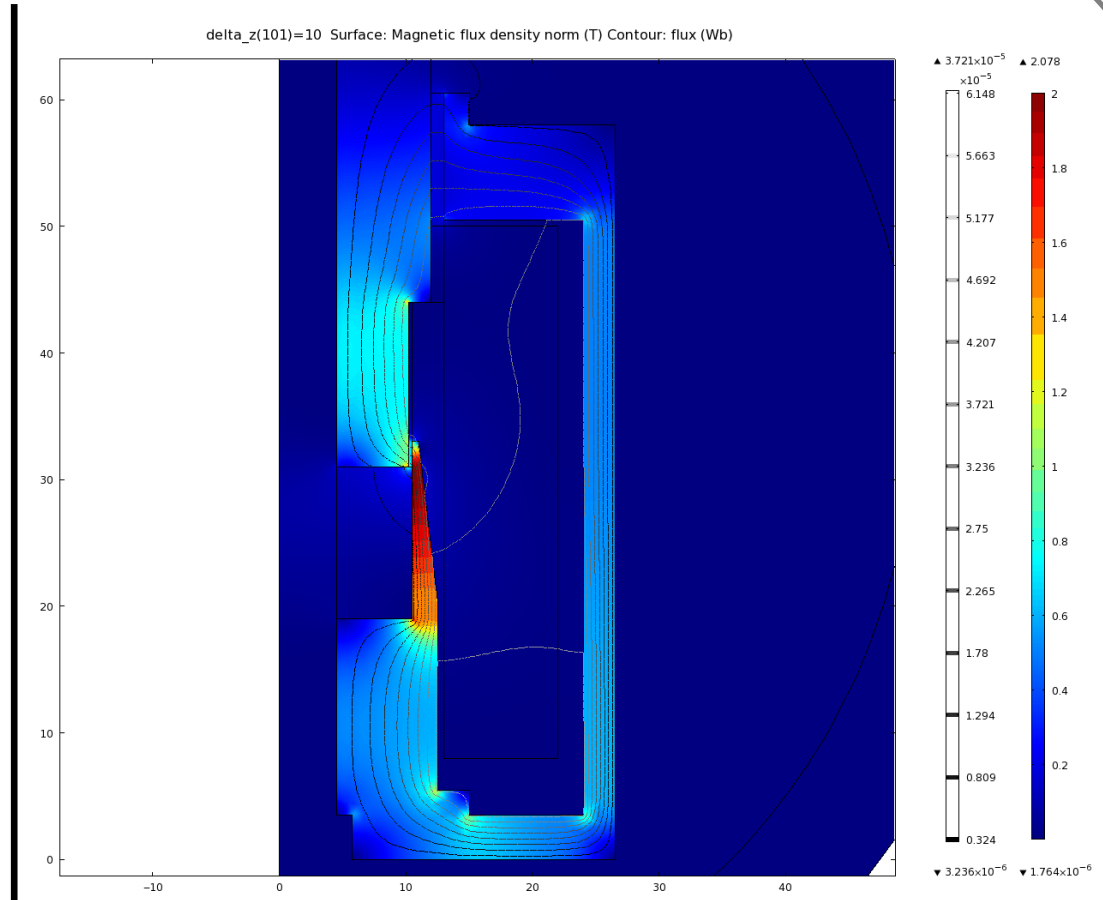
## Mesh



## 4. Finite Element Approach

### Simulation Results

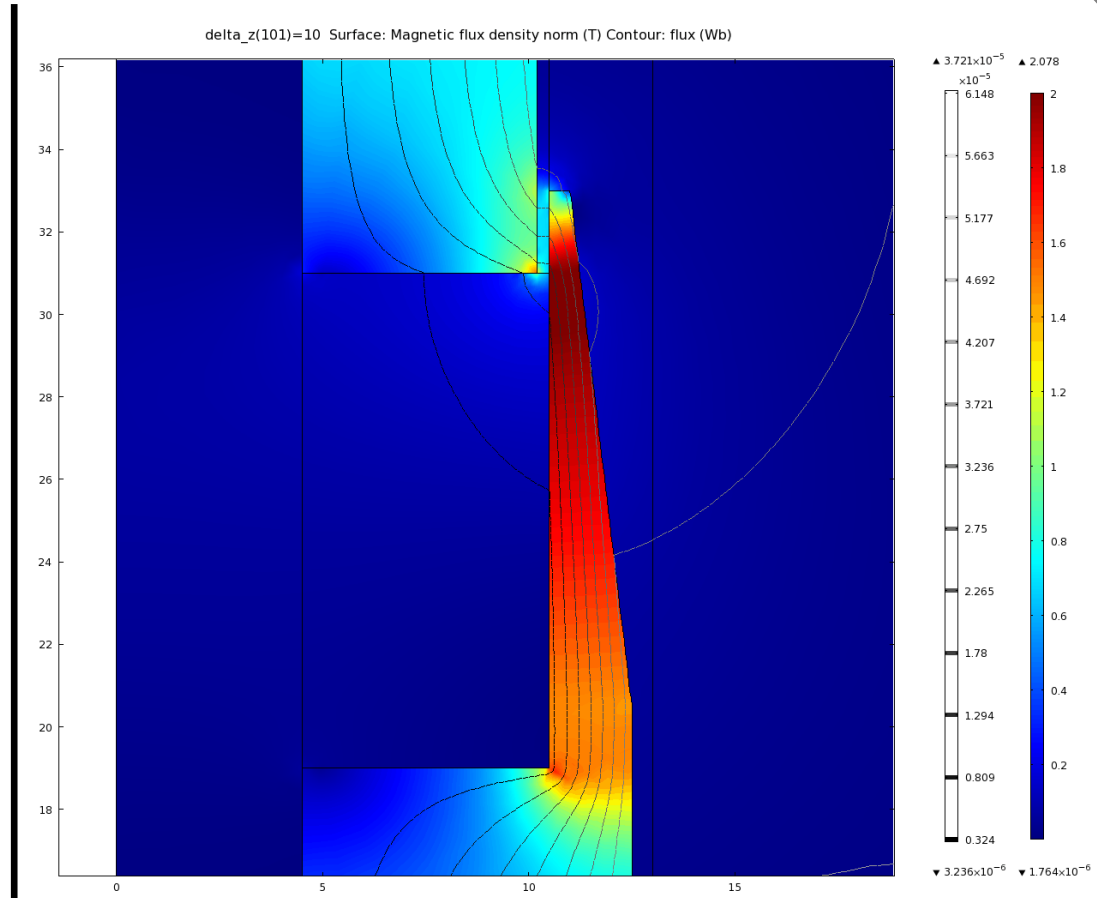
- ▶ Animated results of stationary study for different armature positions at constant coil current.
- ▶ Highly saturated regions in magnetic bypass due to radial flux component.



## 4. Finite Element Approach

### Simulation Results

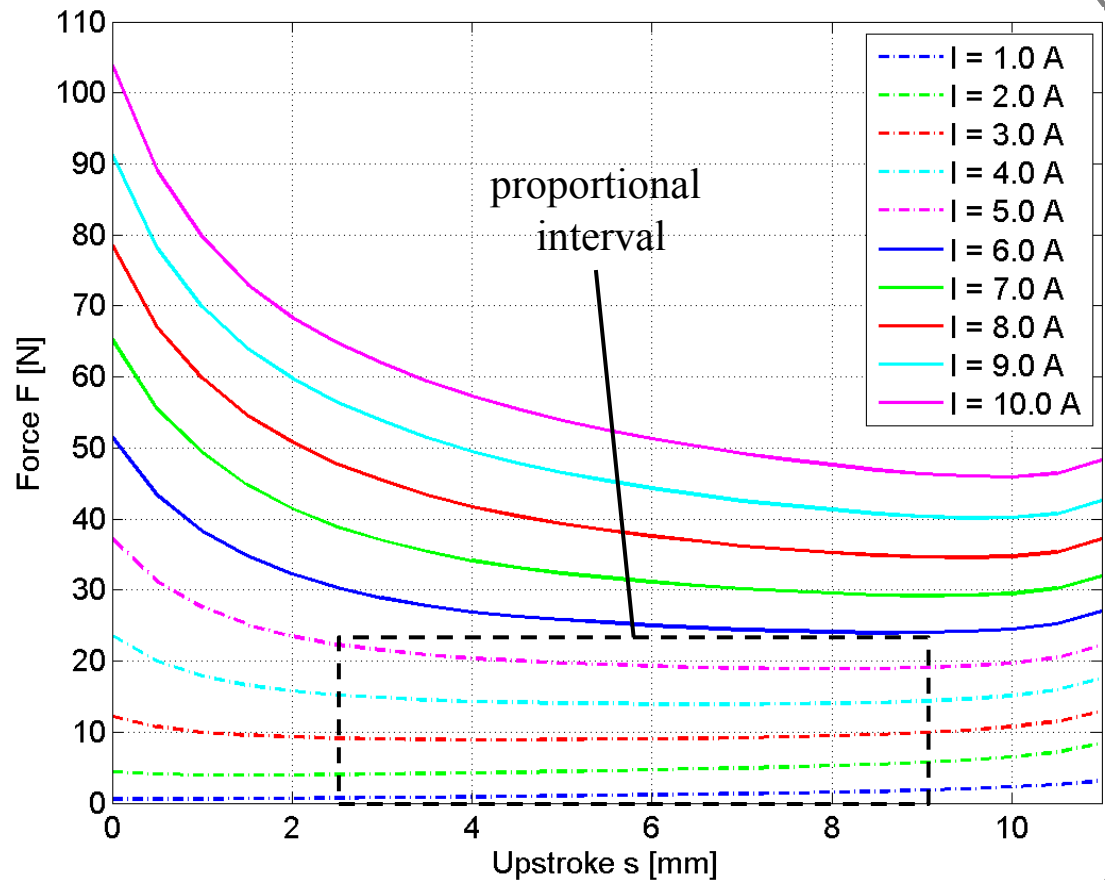
- ▶ Increasing bypass flux with decreasing main air gap.
- ▶ Highly saturated region (bottle neck) next to tip of armature.
- ▶ Dispersed flux passing by highly saturated region generating axial force.
- ▶ Direct flux between pole areas appears near minimum air gap.



## 4. Finite Element Approach

### Force-Stroke-Curves:

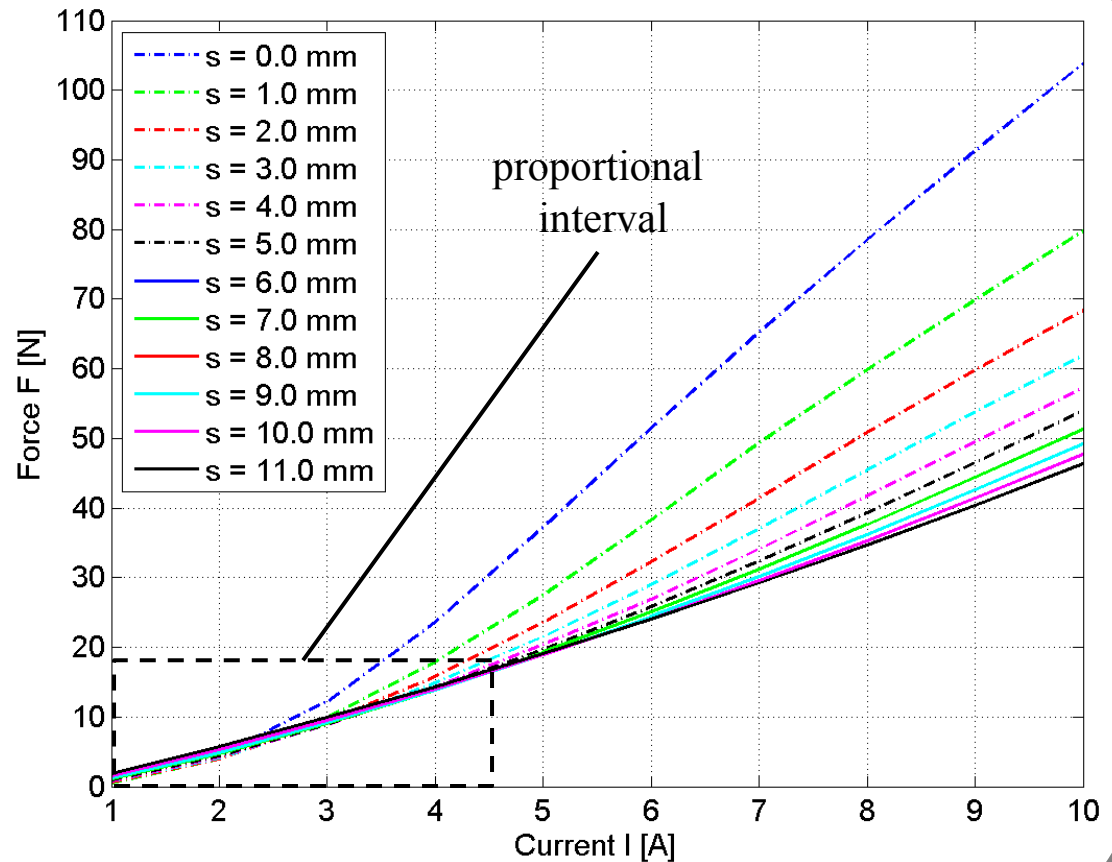
- ▶ Proper bypass dimensions provided force-stroke-curves show interval of approximately proportional interrelationship.
- ▶ Typical nonlinear ascent of force near minimum and maximum air gap.



## 4. Finite Element Approach

### Force-Current-Curves:

- ▶ Very good proportional interrelationship between force  $F$  and coil current  $I$  for  $2 \text{ mm} < s < 11 \text{ mm}$ .
- ▶ Nonlinear ascent of force near minimum main air gap.
- ▶ Nonlinear ascent of force for high currents.



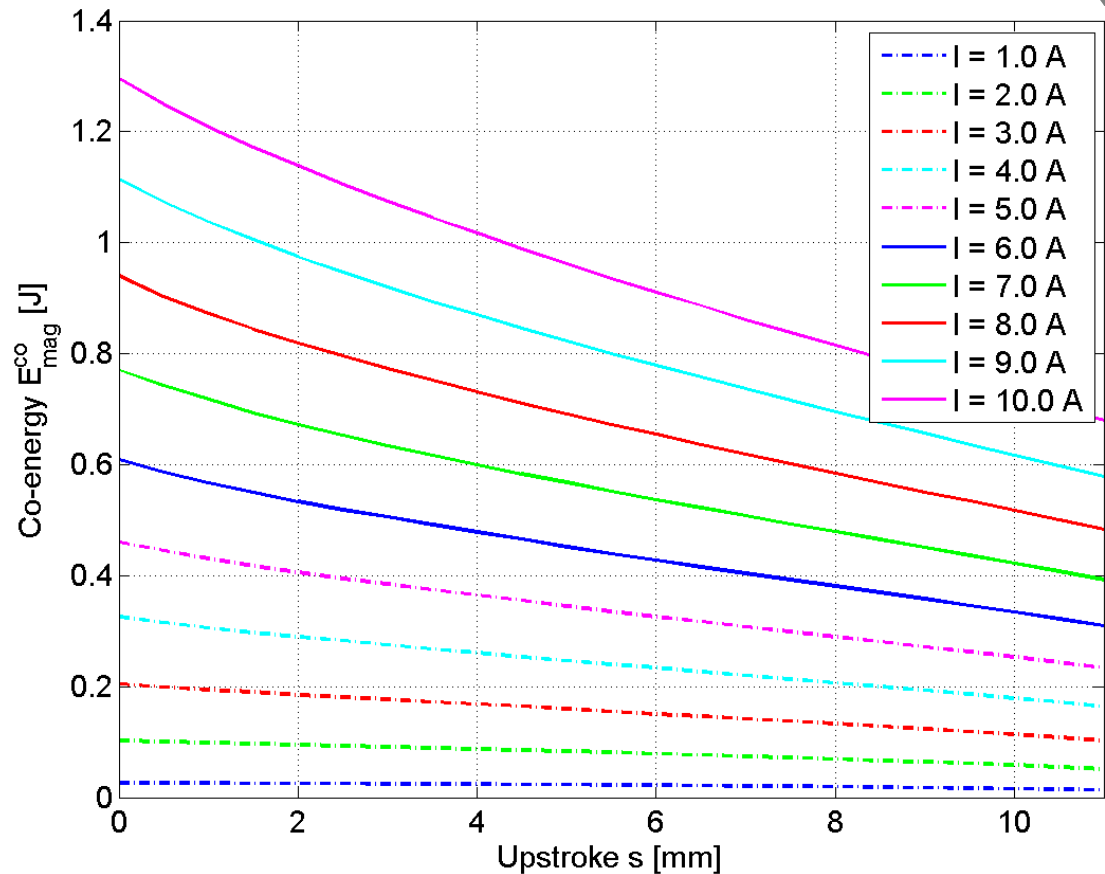
## 4. Finite Element Approach

### Force-Co-Energy-Curves:

- ▶ Magnetic energy/co-energy can be used for force calculation via method of virtual displacement (see paper for more details).

$$F = - \frac{dE_{\text{mag\_co}}}{ds}$$

- ▶ Constant growth of energy/co-energy  $\rightarrow$  constant force.



## 5. Summary

### Analytic model

- ▶ Analytic approach works fine for quite simple geometries. Influence of geometric parameters can easily be identified.
- ▶ Geometries becoming more complex and materials having nonlinear characteristics analytic approach rapidly gets complicated. Simplifications help to reduce efforts but also reduce accuracy of the model.

### FEM model

- ▶ Almost no limits to complexity of geometry and material properties exist. Accuracy of results in regions of special interest can be increased on demand by refining and improving mesh quality.
- ▶ Immense postprocessing capabilities.
- ▶ Influence of geometric parameters can not be seen directly. Therefore systematic Parameter studies have to be performed.

Simultaneous application of both approaches lead to a better and deeper understanding!



**Thank you very much  
for your kind attention!**